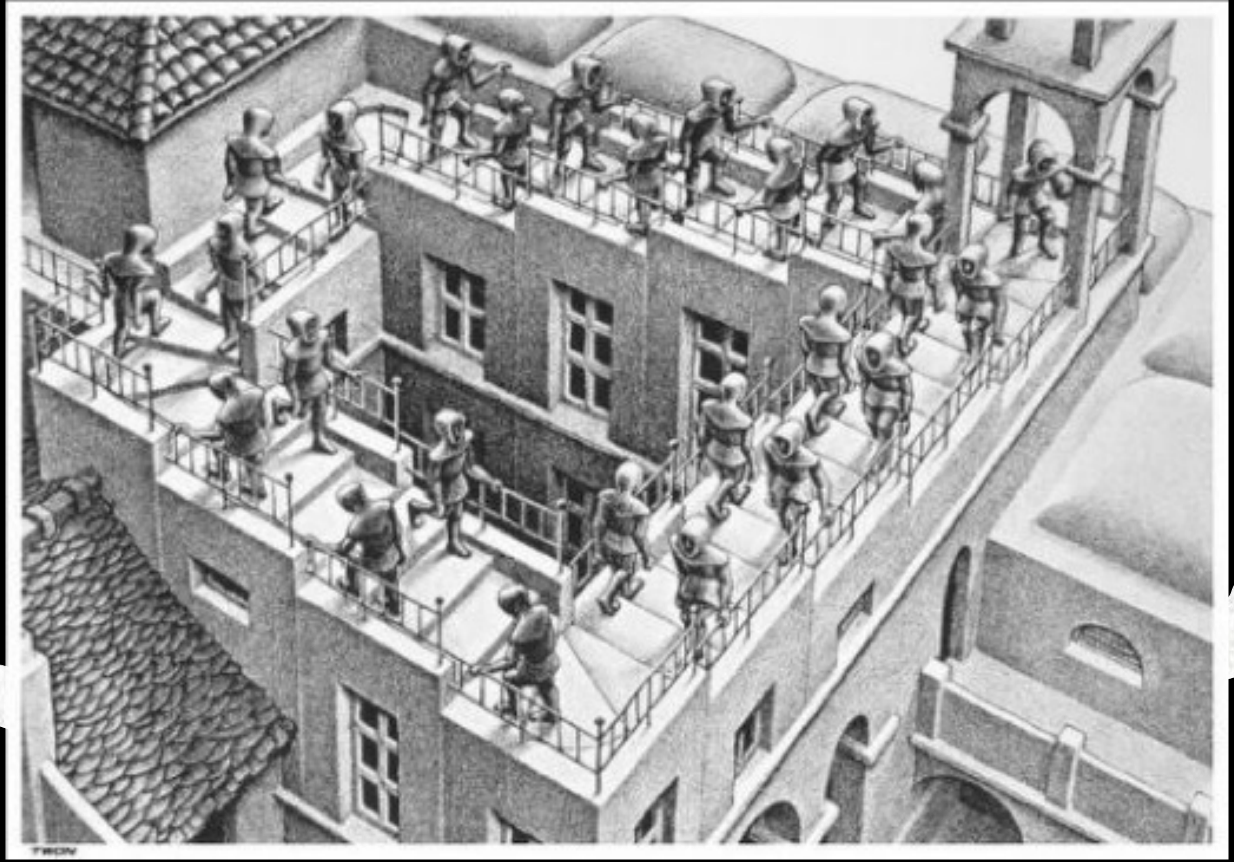


The Complexity of the Cosmos: A possible path to make the Universe Simple

Raul Jimenez

In this talk we propose two possible quantum signatures of the gravitons at early Universe that could lead to observational imprints of the quantum nature of the inflationary period:

1). In "An Entangled Universe," Tejerina-Pérez, Daniele Bertacca, Raul Jimenez, 2403.15742,

In this paper we consider

- a graviton production from the presence of a classical, coherent state of the inflaton scalar field results in entangled states in the gravitons' polarizations.
- At horizon crossing, interactions between the gravitons and (lower scale) inflatons, together with the gathering of "which-path information" from the cosmological horizon, perform the required Bell experiments leading to a definitive measure.
- This measure can be imprinted in the scalar correlation four-point function. This is because of a non-trivial effect due to the derivatives on two scalar fluctuations.
- This scalar fluctuations provides a fingerprint that depends on the polarization of the graviton that Alice and/or Bob measured in their patch.

In this talk we propose two possible quantum signatures of the gravitons at early Universe that could lead to observational imprints of the quantum nature of the inflationary period:

1) In “Inflation without an Inflaton,” Daniele Bertacca, Raul Jimenez, Matarrese & Ricciardone, 2412.14265:

- We propose a novel scenario in which scalar perturbations, that seed the large-scale structure of the Universe, are generated without relying on a scalar field (the inflaton).
- In this framework, inflation is driven by a de Sitter space-time (dS), where tensor metric fluctuations (i.e., gravitational waves) naturally arise from quantum vacuum oscillations, and scalar fluctuations are generated via second-order tensor effects.

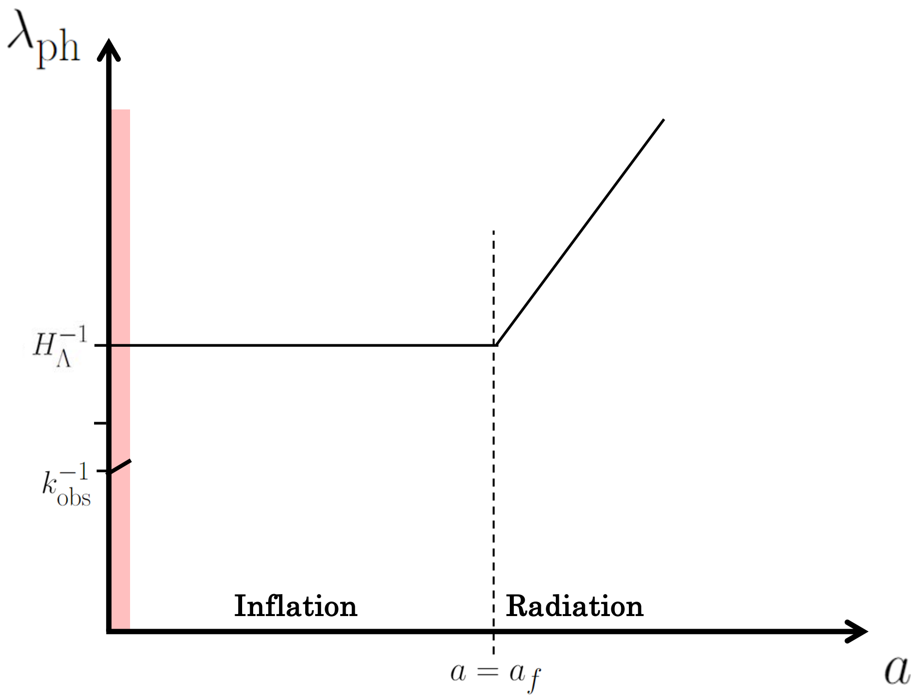
An Entangled Universe

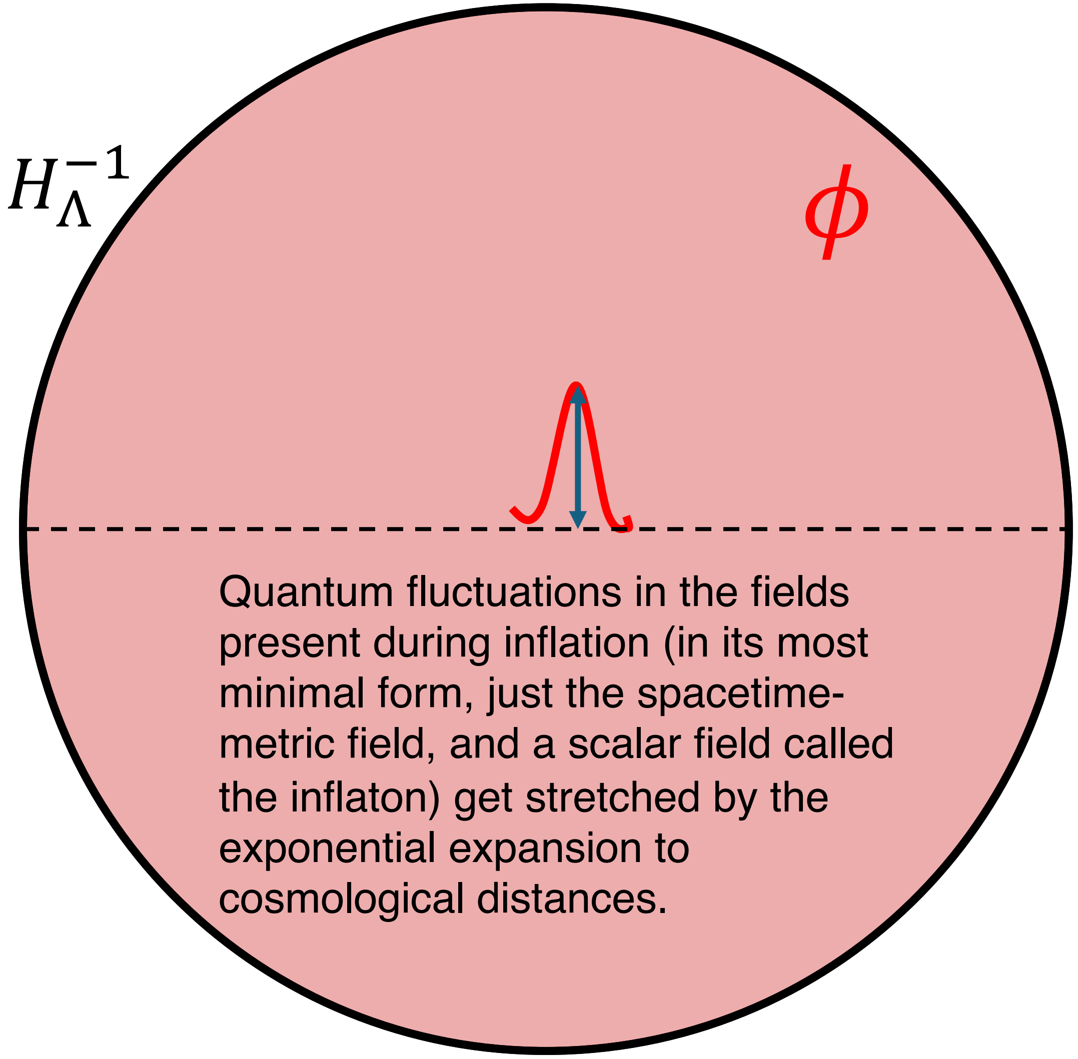
Project by:

Pablo Tejerina-Pérez, Daniele Bertacca, Raul Jiménez

Inflation (general)

- Scalar field – Scalar perturbations
- Quasi - de Sitter metric – Tensor perturbations





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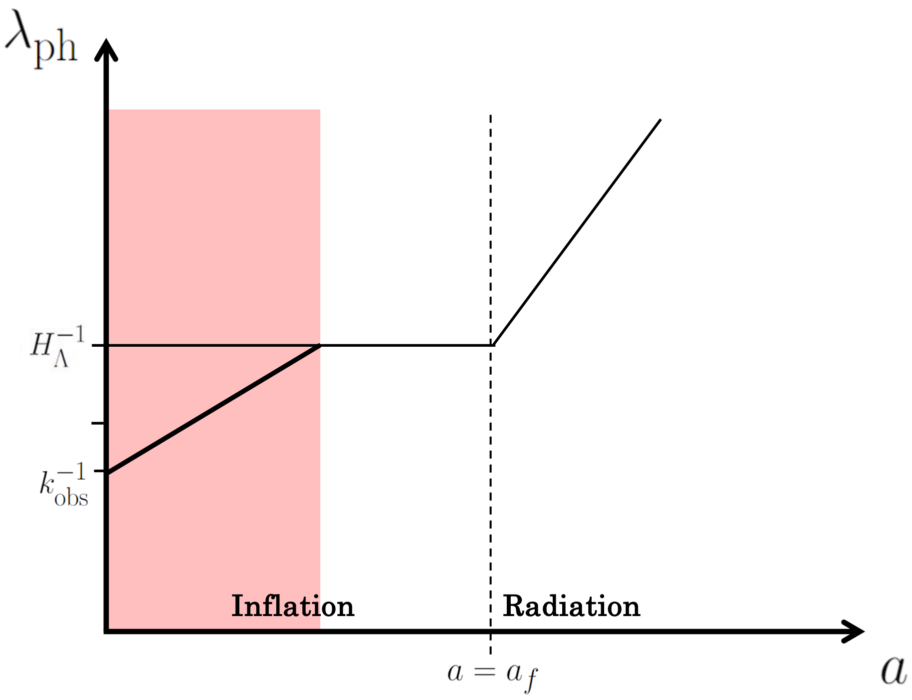

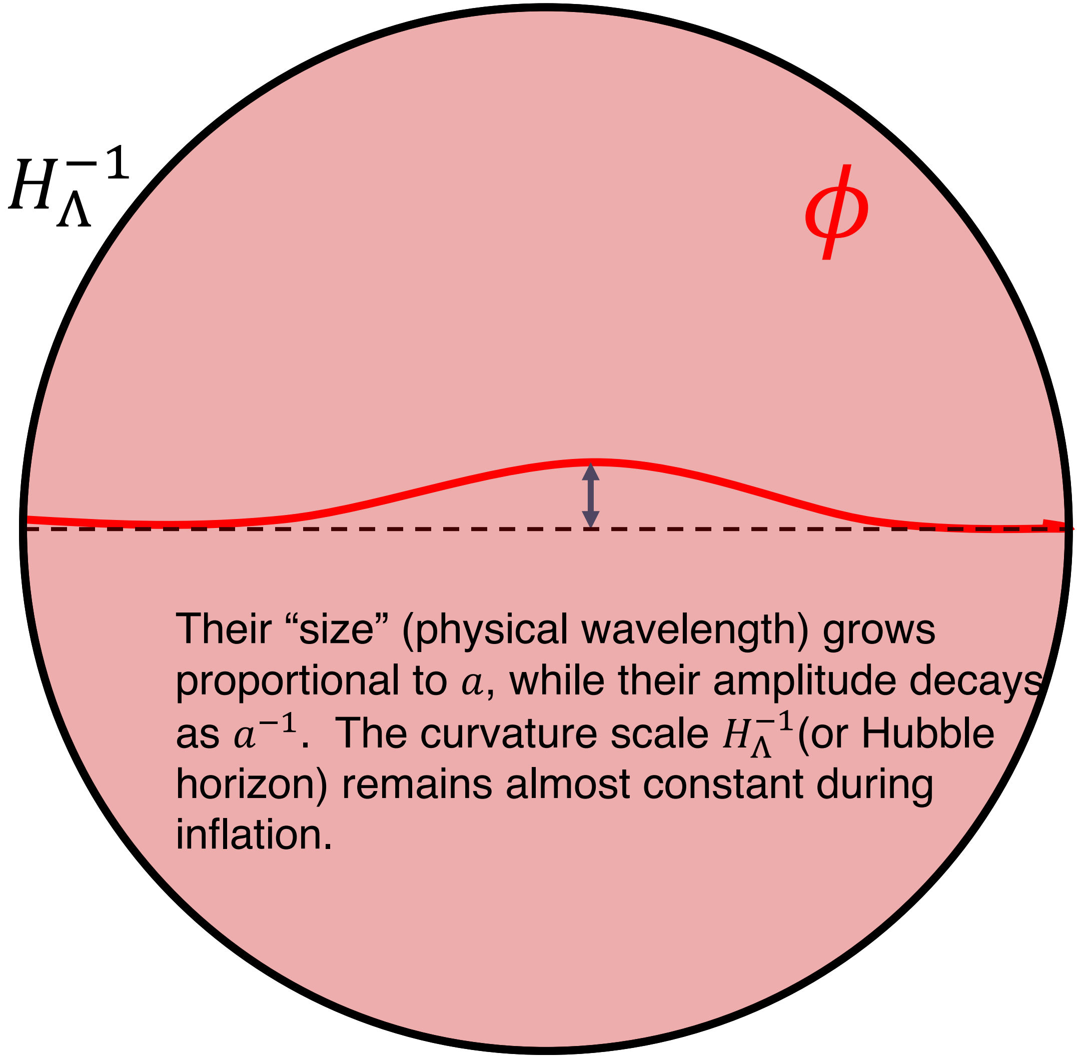

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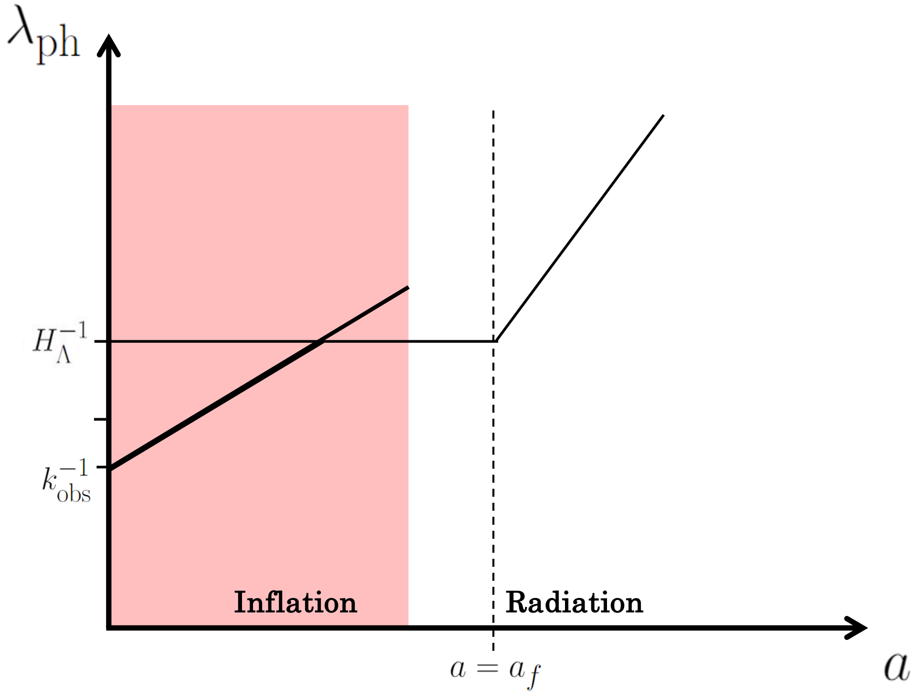
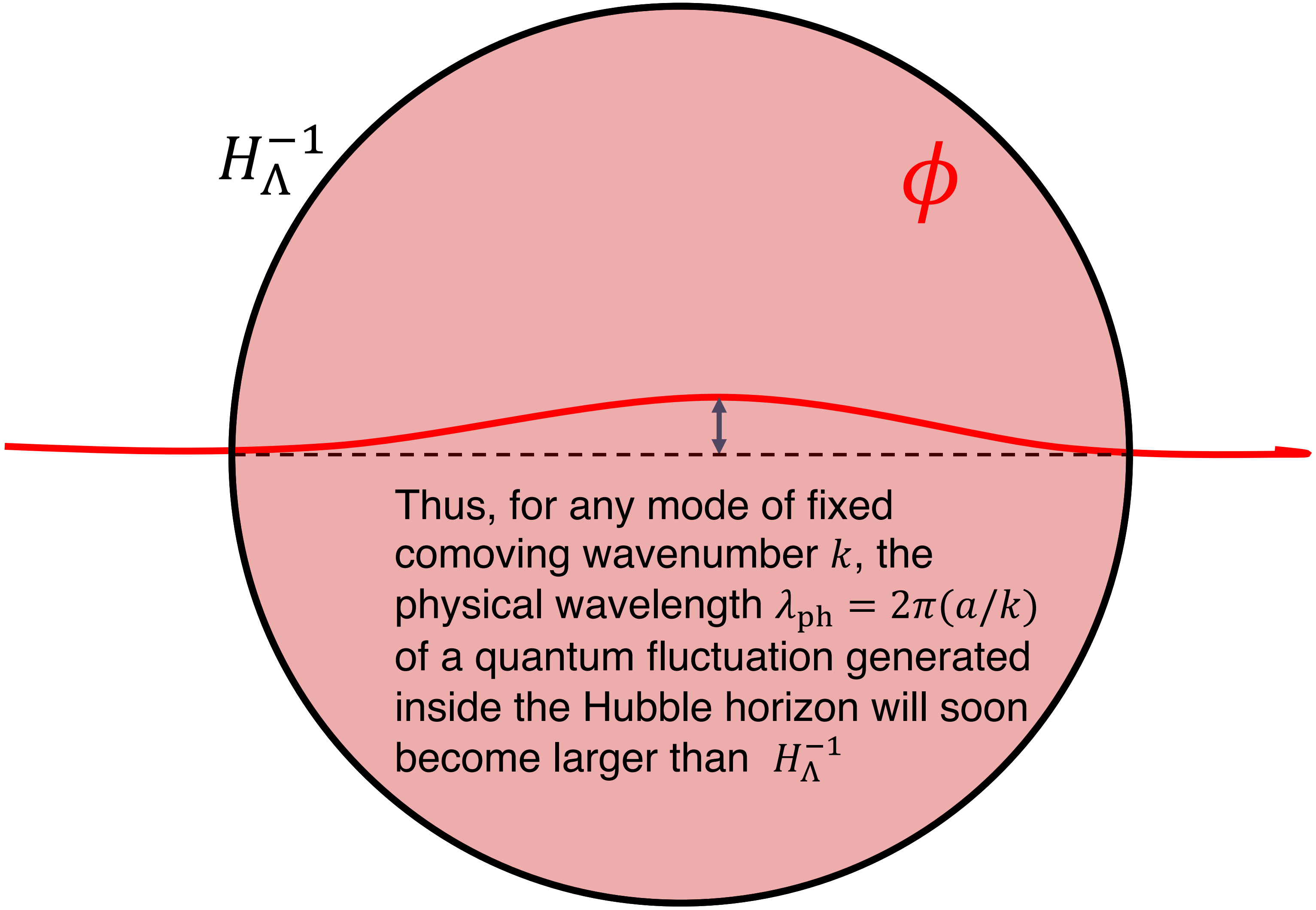


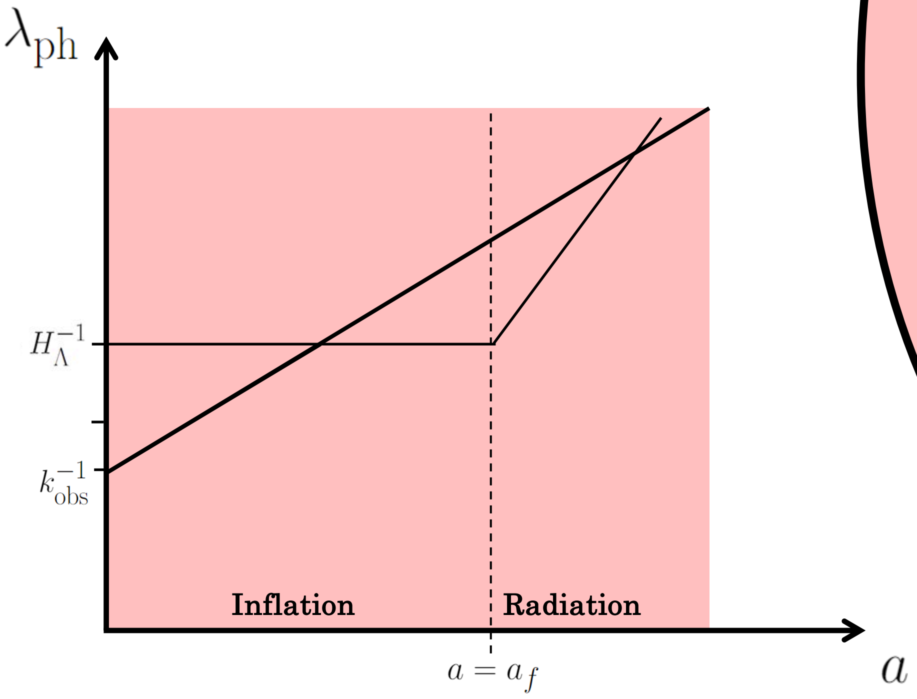


Thus, for any mode of fixed comoving wavenumber k, the physical wavelength $\lambda_{\rm ph} = 2\pi(a/k)$ of a quantum fluctuation generated inside the Hubble horizon will soon become larger than H_Λ^{-1}

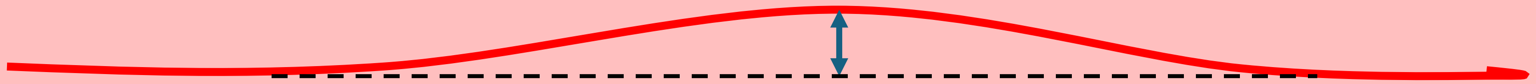
Therefore, the corresponding mode leaves the horizon, and starts “feeling” the curvature of spacetime.
At horizon crossing, its amplitude freezes and remains almost constant until the end of the inflationary stage.

Inflation (general)

- Radiation epoch



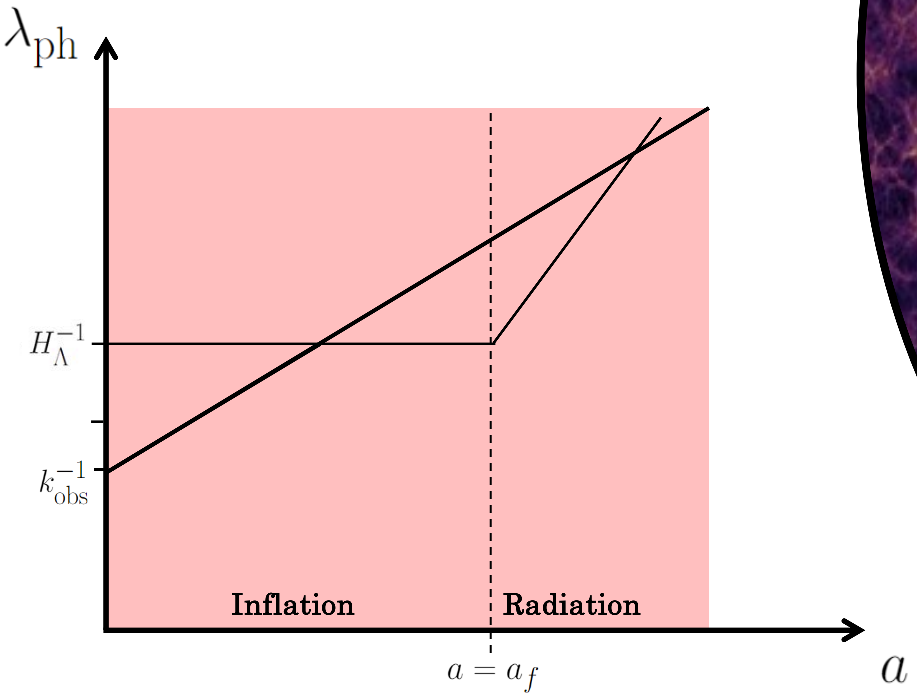
H_Λ^{-1}



- Once inflation is over, the radiation dominated era
- starts (decelerating Hubble expansion), during which the curvature scale $H_{\rm rad}^{-1}$ starts growing at a rate $a/\dot{a}$.
- At this point, the now large fluctuations re-enter the Hubble horizon as density perturbations of cosmological scale, and as gravitational waves.

Inflation (general)

- Gravitational seeds for LSS



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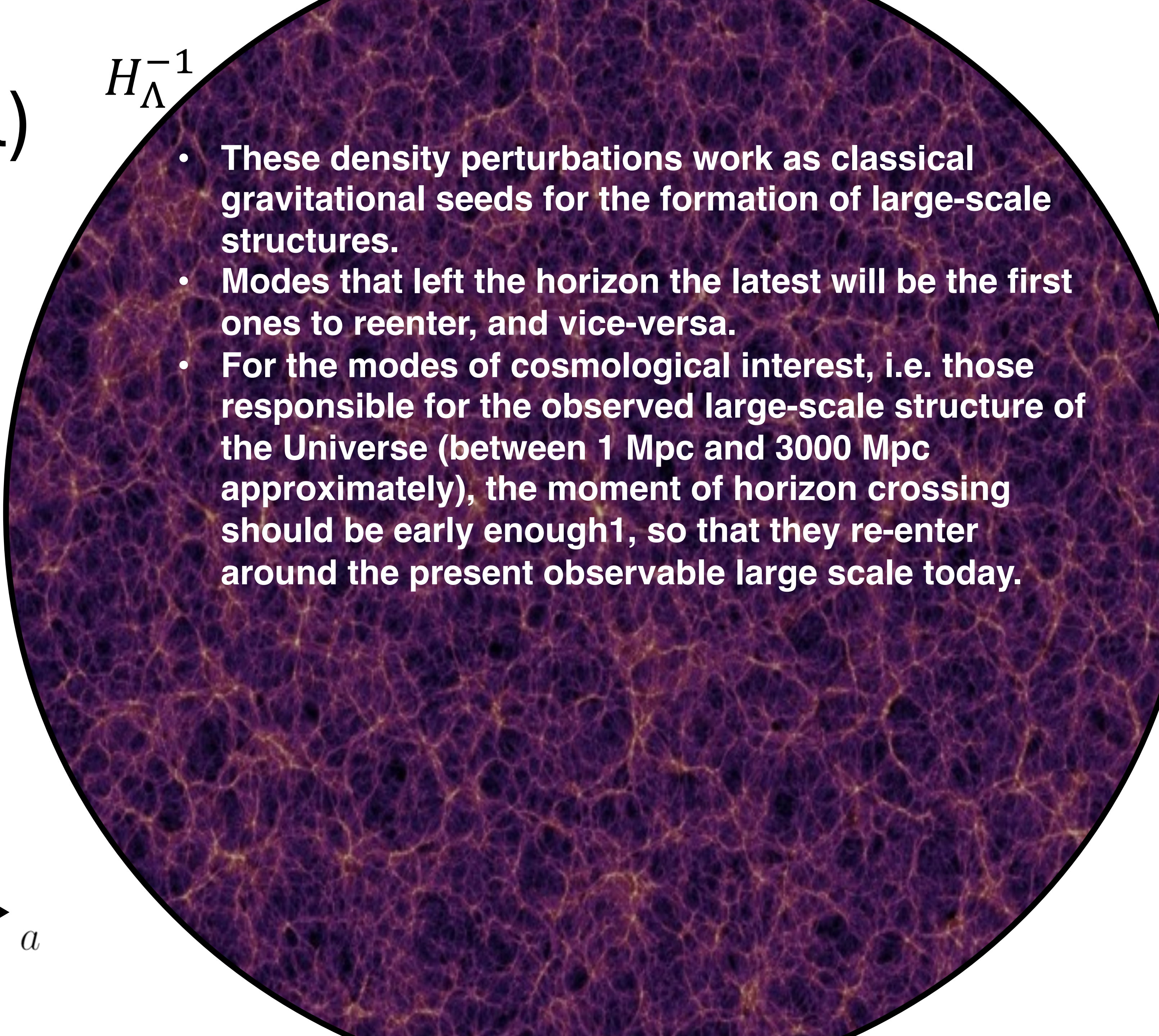

- **These density perturbations work as classical gravitational seeds for the formation of large-scale structures.**
- **Modes that left the horizon the latest will be the first ones to reenter, and vice-versa.**
- **For the modes of cosmological interest, i.e. those responsible for the observed large-scale structure of the Universe (between 1 Mpc and 3000 Mpc approximately), the moment of horizon crossing should be early enough1, so that they re-enter around the present observable large scale today.**

Inflation (general)

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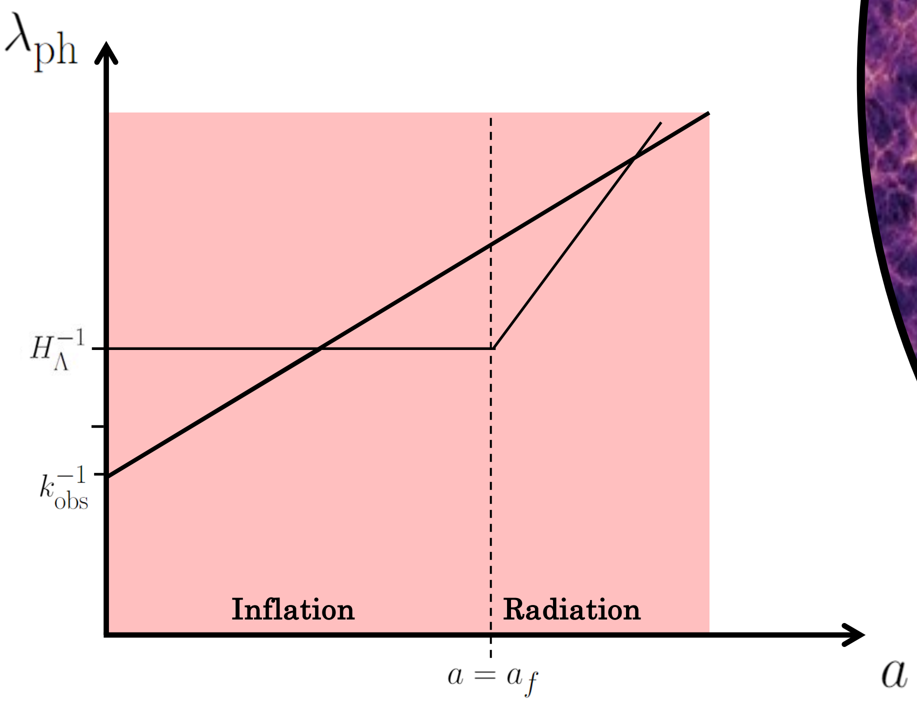

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Therefore, Inflation introduces a <u>natural way</u> to explain the inhomogeneities and anisotropies in the Universe that gave rise to the formation of the cosmological structures we observe today.

Motivations

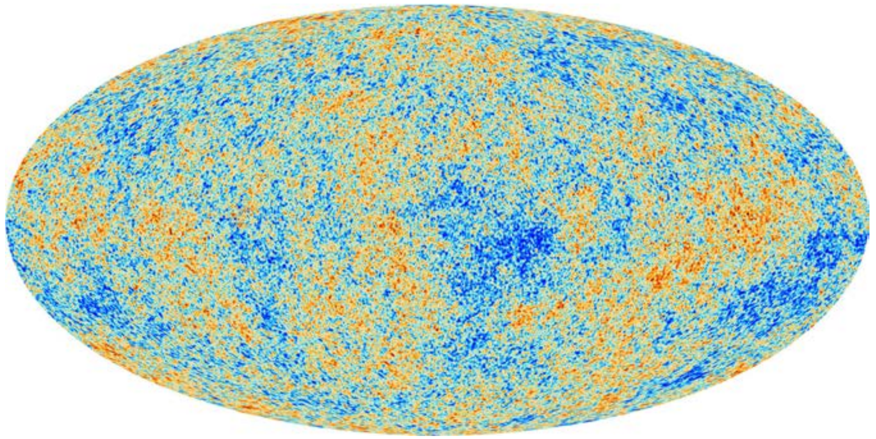
We treat this perturbations as classical:

When did the quantum-to-classical transition occur?

Can we find some trace of the initial quantumness?

In this context, there is a somewhat natural question that emerges: at what point does a quantum fluctuation generated during inflation stop being of quantum nature?

Motivations



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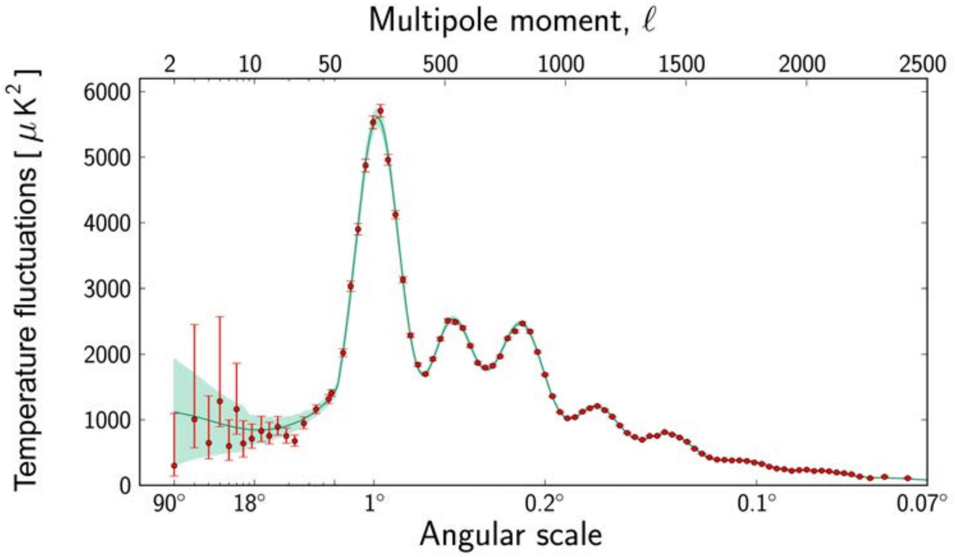

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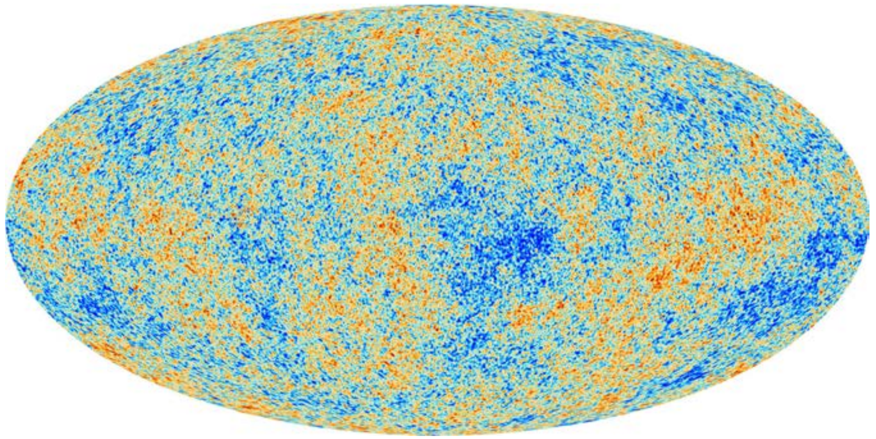

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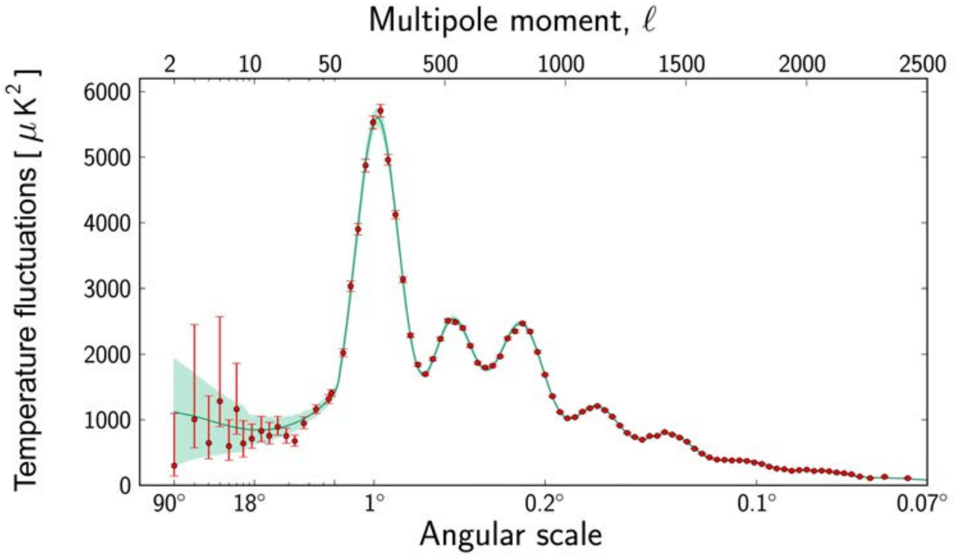

- **It is still an open question how this classicalization of quantum fluctuations occurs.**
- **If any signal of the early nature of the quantum fluctuation remains, this should be imprinted in some current observable, maybe in the form of non-gaussianities in the CMB anisotropies, or in higher-order correlation functions of galaxy distributions, or some other observable.**

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- Then, we describe plausible processes by which the quantum nature of the tensor fluctuations of the metric field (i.e. gravitons) during inflation is made explicit.
- We study how, through interaction with their environment, gravitons may imprint this quantumness into some observable quantity.
- We then propose what this observable quantity might be.

Entanglement as a quantum property

Let me briefly review what is known to be the “most quantum aspect” in quantum mechanics and quantum optics: **entanglement.**

This effect makes the quantum nature of a system appear in a very explicit manner; if we are hoping to find traces of quantumness in some observable, **entanglement** is what we should aim for.

Entanglement as a quantum property

- **Entanglement** between two quantum states is present when the quantum state describing the whole system cannot be written as a product of the states describing each subsystem separately, i.e. it is non-separable.

- **Entanglement** is exclusively a consequence of the principle of superposition present in quantum mechanics and makes the non-locality of the theory explicit.

Entanglement as a quantum property

$$|\Psi\rangle_i = \alpha\,|\psi_1\rangle_A \otimes |\psi_1\rangle_B + \beta\,|\psi_2\rangle_A \otimes |\psi_2\rangle_B$$

$$\alpha,\ \beta \in \mathbb{C},$$
$$|\alpha|^2 + |\beta|^2 = 1$$

- *Entangled* states are **pure** (quantum) **states** (of a bipartite system), since they can be described by a vector state $|\Psi\rangle$ in a Hilbert space and, more importantly, they cannot be described by a product of separable states.

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- If we choose to describe each subsystem individually, due to the nature of the entangled state, we cannot write its state as a vector state, and we have what we call a **mixed state** (when looking at each subsystem with disregard of the state of the full system).

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- **Mixed states** express only *partial knowledge of the full quantum state of a system*.

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Here $|\psi_i\rangle_j$ represents particle j (the index j could be a particle, momenta, directions, etc...) in state $|\psi_i\rangle$.

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A

B

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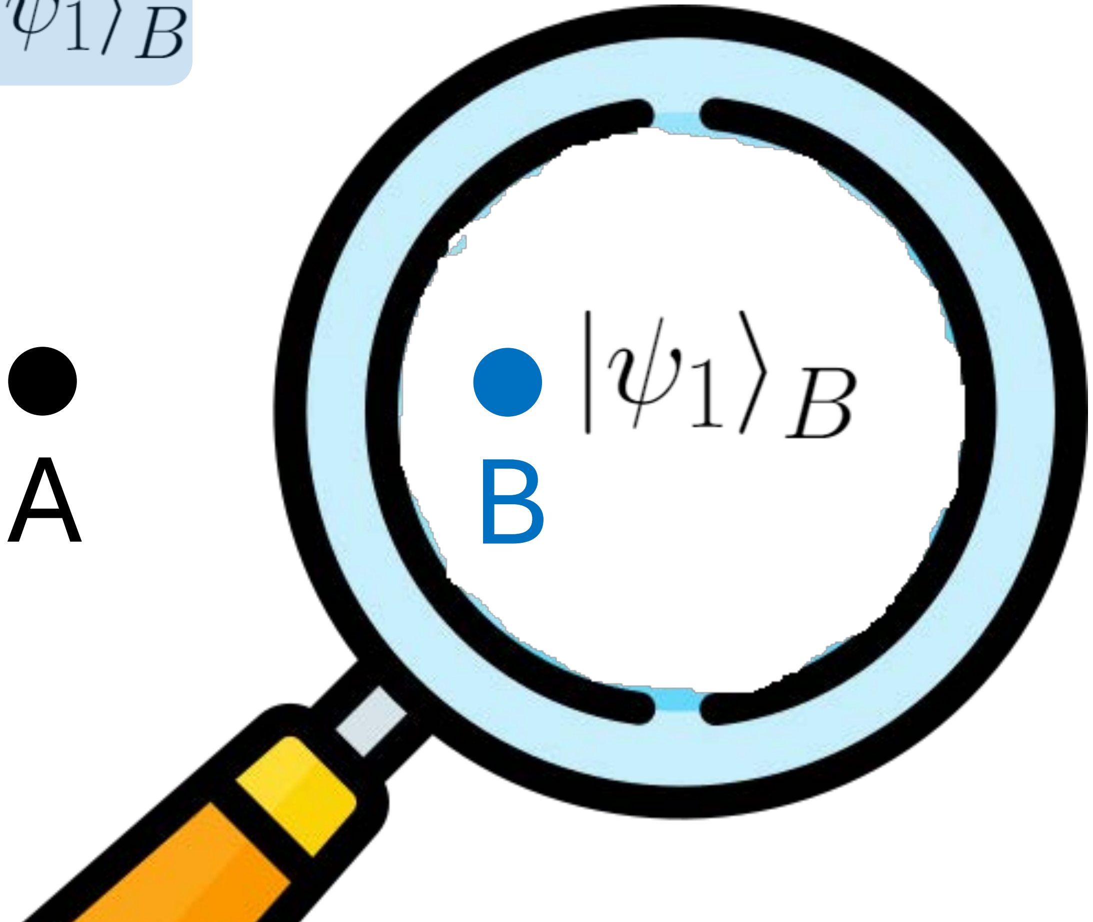

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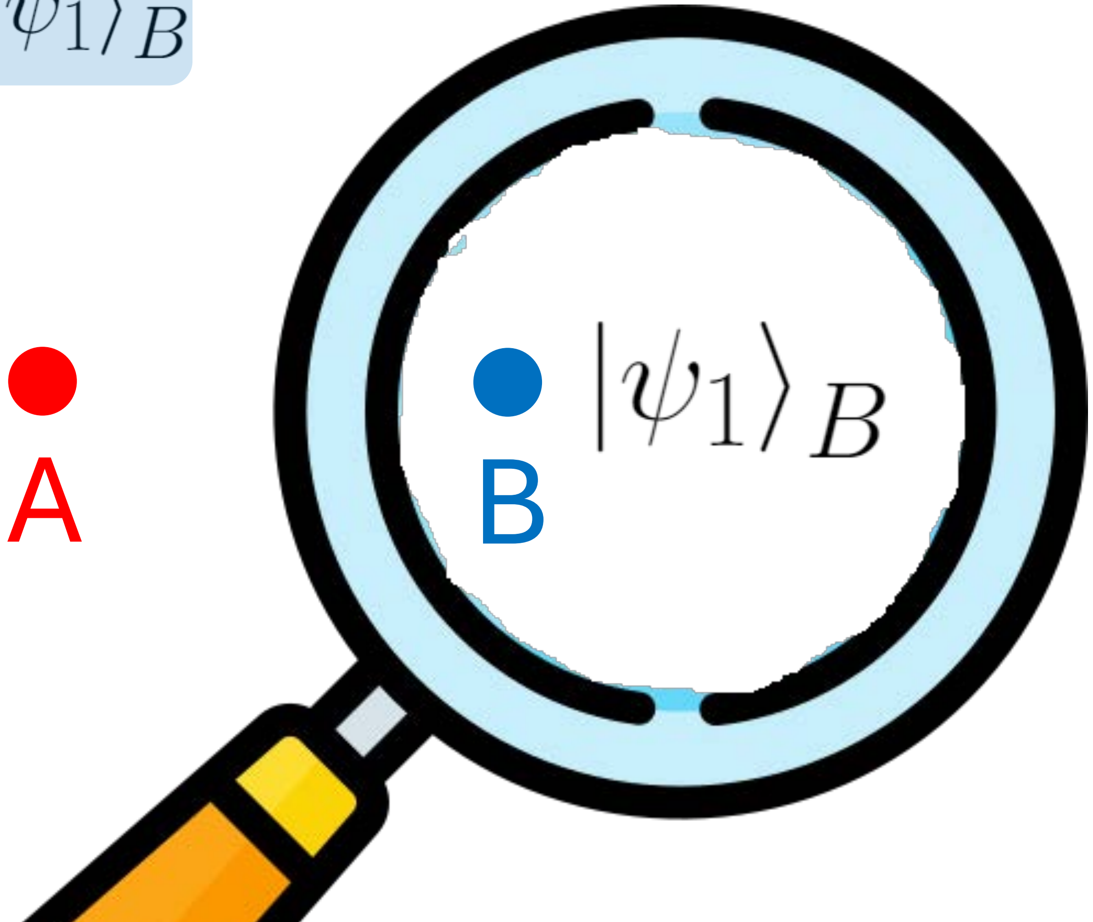

$|\psi_1\rangle_B$ B

By performing a measure on one particle, we instantaneously change the state of the other!

Bell Experiment and Inequalities

- In 1964, the physicist John Stewart Bell proposed his famous theorem, which showed a clear **mathematical difference between** any description by a **classical**, *local hidden variables theory*, or by **the quantum-mechanical theory** that gave rise to *the non-local effect of the entanglement.*
- Extensive work has been done over the years concerning all theoretical and experimental aspects of Bell's theorem and experiment [e.g. see Clauser + (1969), Shimony & Clauser (1978)].

Bell Experiment and Inequalities

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- Two possible measurements of some physical observable, whose result is dependent on some local variable θ_i(or on some random choice between θ_i and θ'_i), at each of the spatial locations (denoted by $i = 1, 2$).
- Each observation (/measurement) is represented by non-commuting operators, call them $A(\theta_1)$ and $A(\theta'_1)$, and $B(\theta_2)$ and $B(\theta'_2)$, respectively.

Bell Experiment and Inequalities

Then we can define the observable:

$$S = C(\theta_1, \theta_2) + C(\theta_1', \theta_2) + C(\theta_1, \theta_2') - C(\theta_1', \theta_2')$$

where

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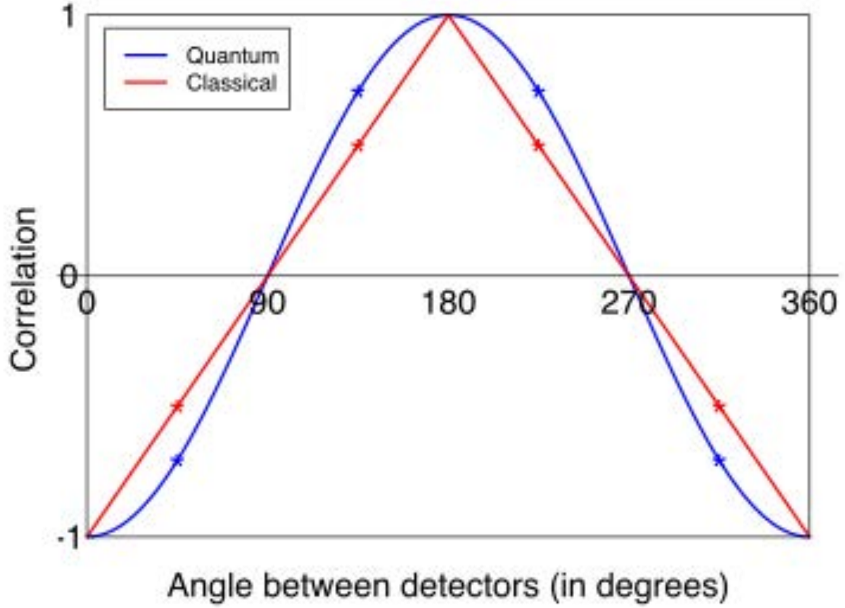

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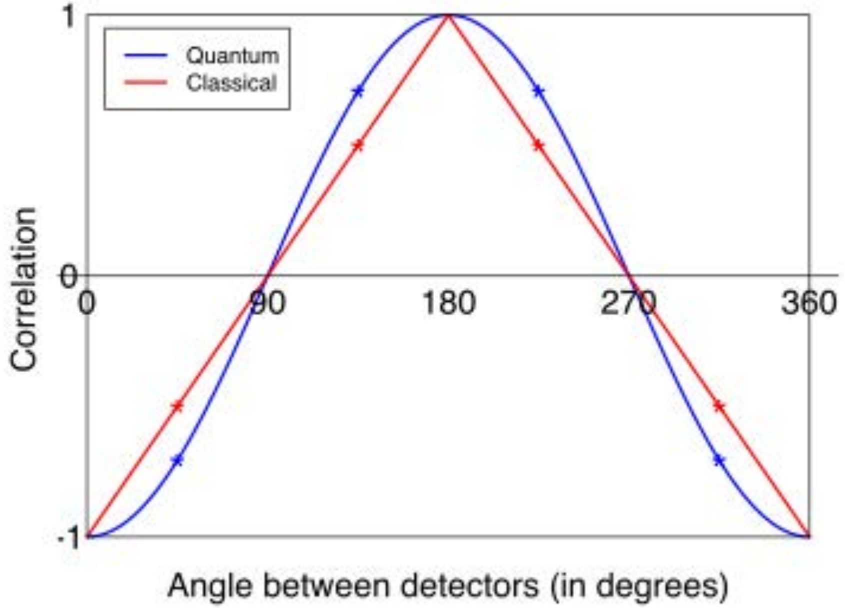

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By exceeding the former inequality in experimental measures, one proves the non-locality as an intrinsic property of entanglement and quantum mechanics.
This particular Bell inequality is known as the Clauser-Horne-Shimony-Holt inequality, see Clauser + (1969)

Bell Experiment and Inequalities

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- Definite results for the quantum measurement of the operators. For the inequality in S, both $A(\theta_1)$ and $B(\theta_2)$ measured on the state $(*)$ can yield ± 1.
- A classical channel to transmit the results of the measurements to a common location where they can be correlated.

Bell Experiment and Inequalities

$$S = C(\theta_1, \theta_2) + C(\theta_1', \theta_2) + C(\theta_1, \theta_2') - C(\theta_1', \theta_2')$$

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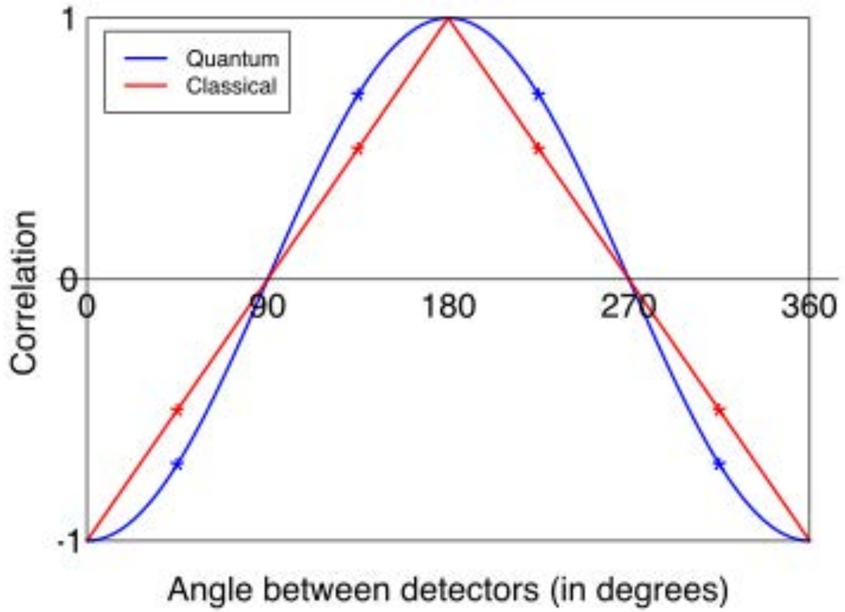


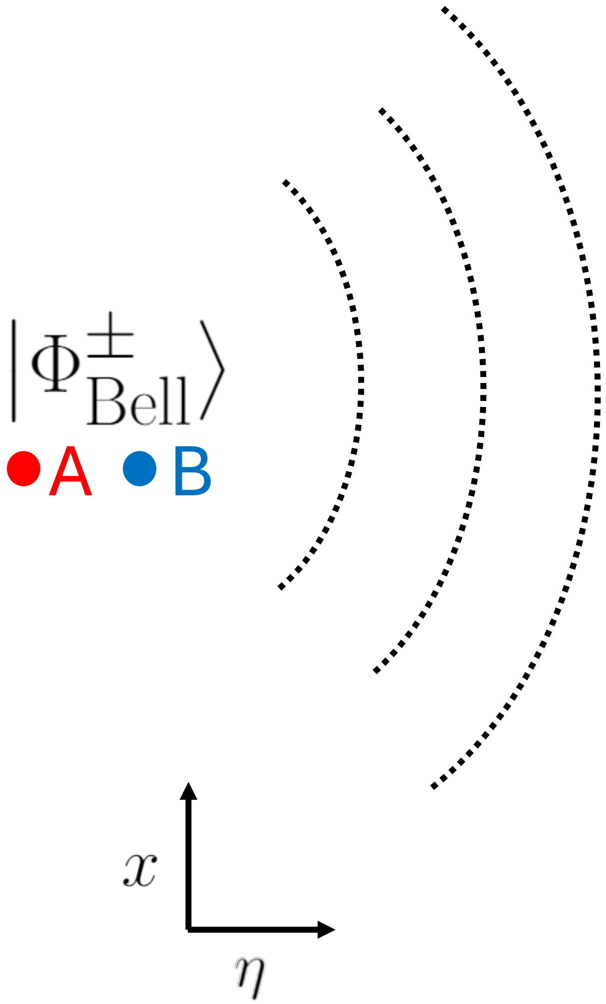
It is useful to work with 2-states known as Bell states, which are defined as

$$|\Psi^{\pm}_{\mathrm{Bell}}\rangle = \frac{1}{\sqrt{2}}(|0\rangle_A |1\rangle_B \pm |1\rangle_A |0\rangle_B)$$

$$|\Phi^{\pm}_{\mathrm{Bell}}\rangle = \frac{1}{\sqrt{2}}(|0\rangle_A |0\rangle_B \pm |1\rangle_A |1\rangle_B)$$

As we will see later, let us cosider these Bell states

Elements of a Bell experiment



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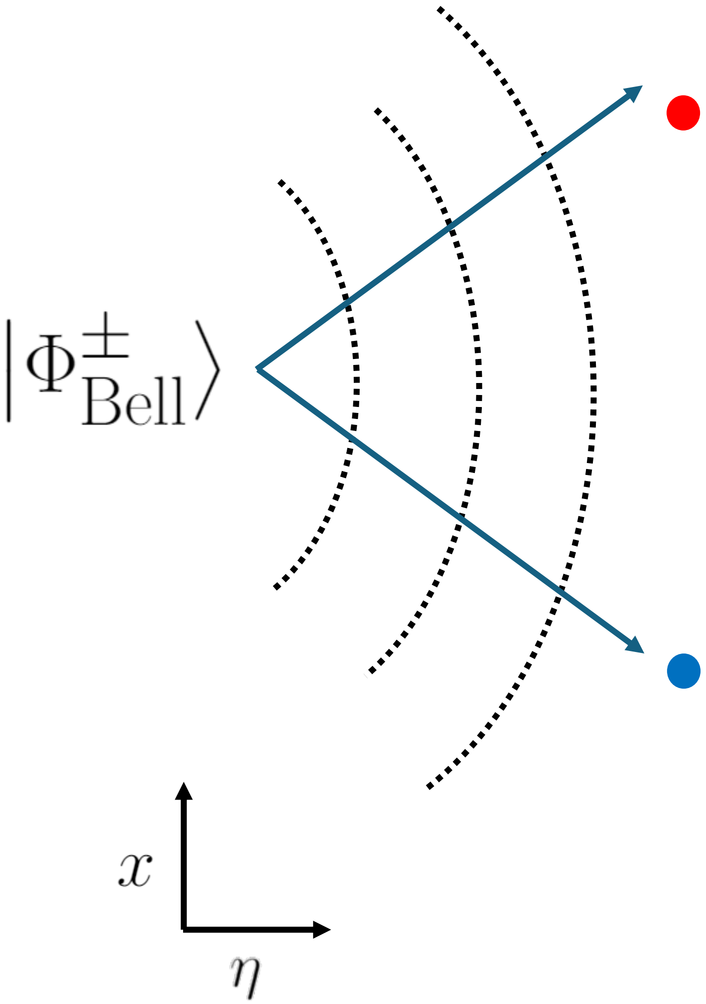

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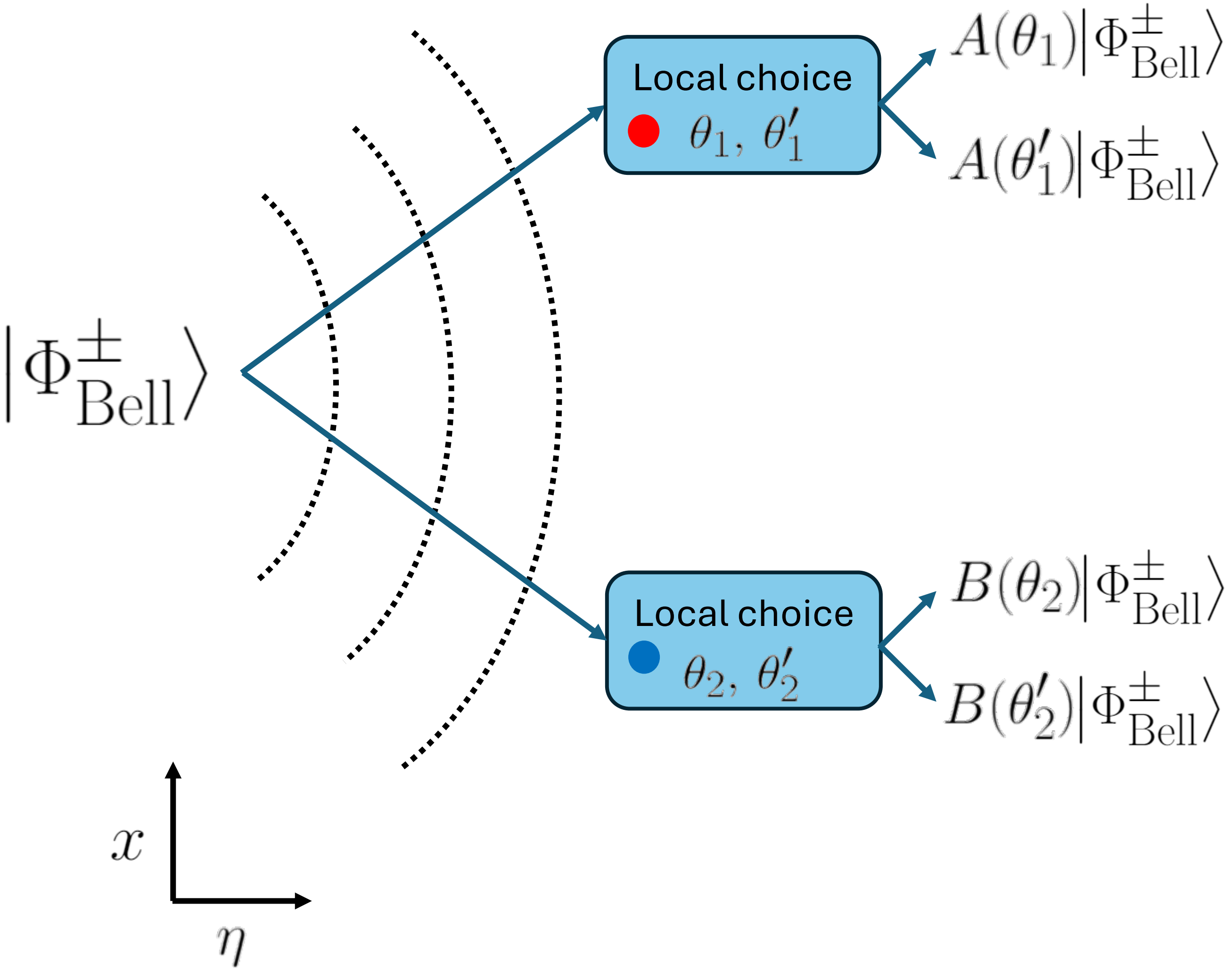

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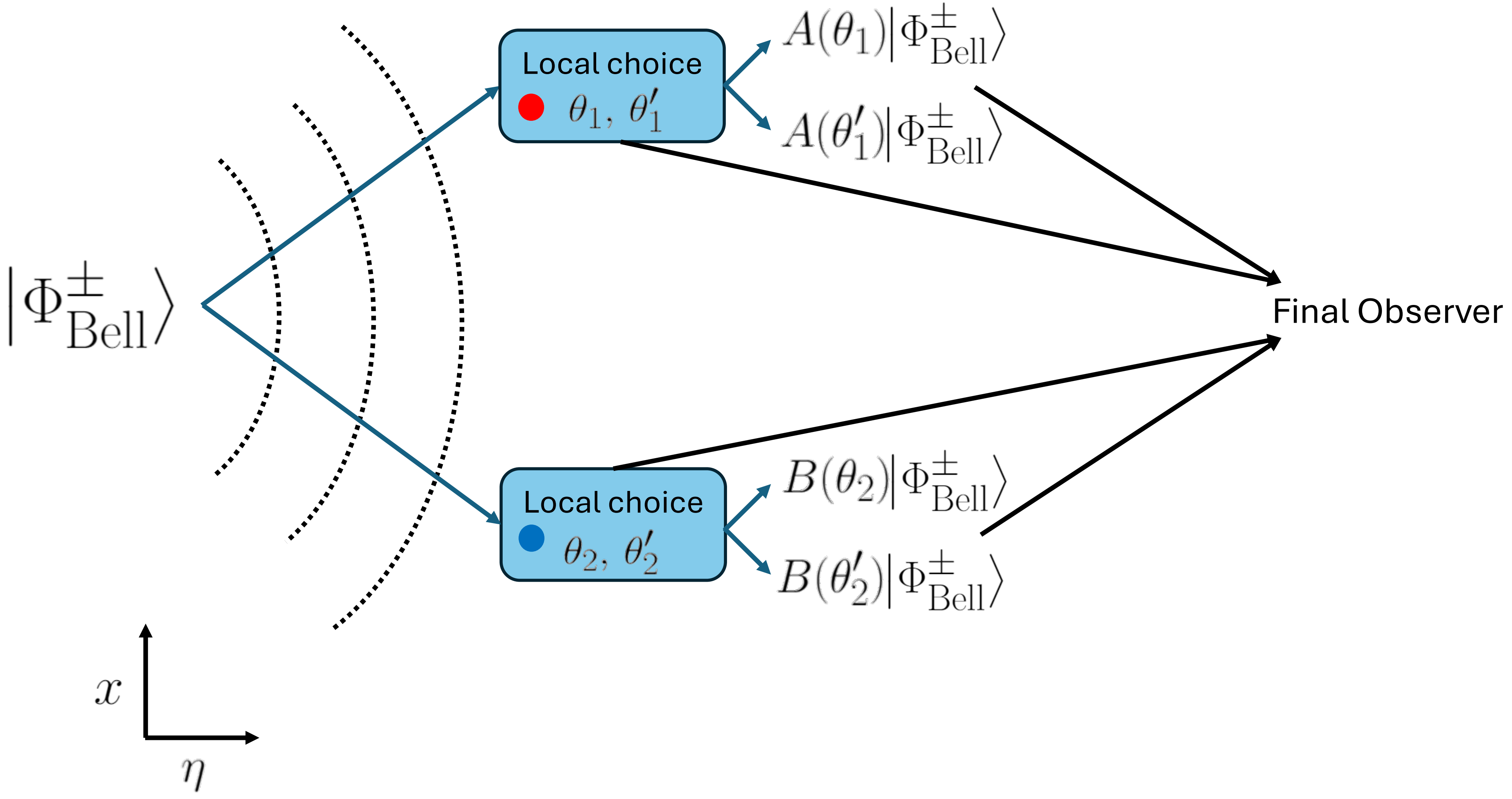

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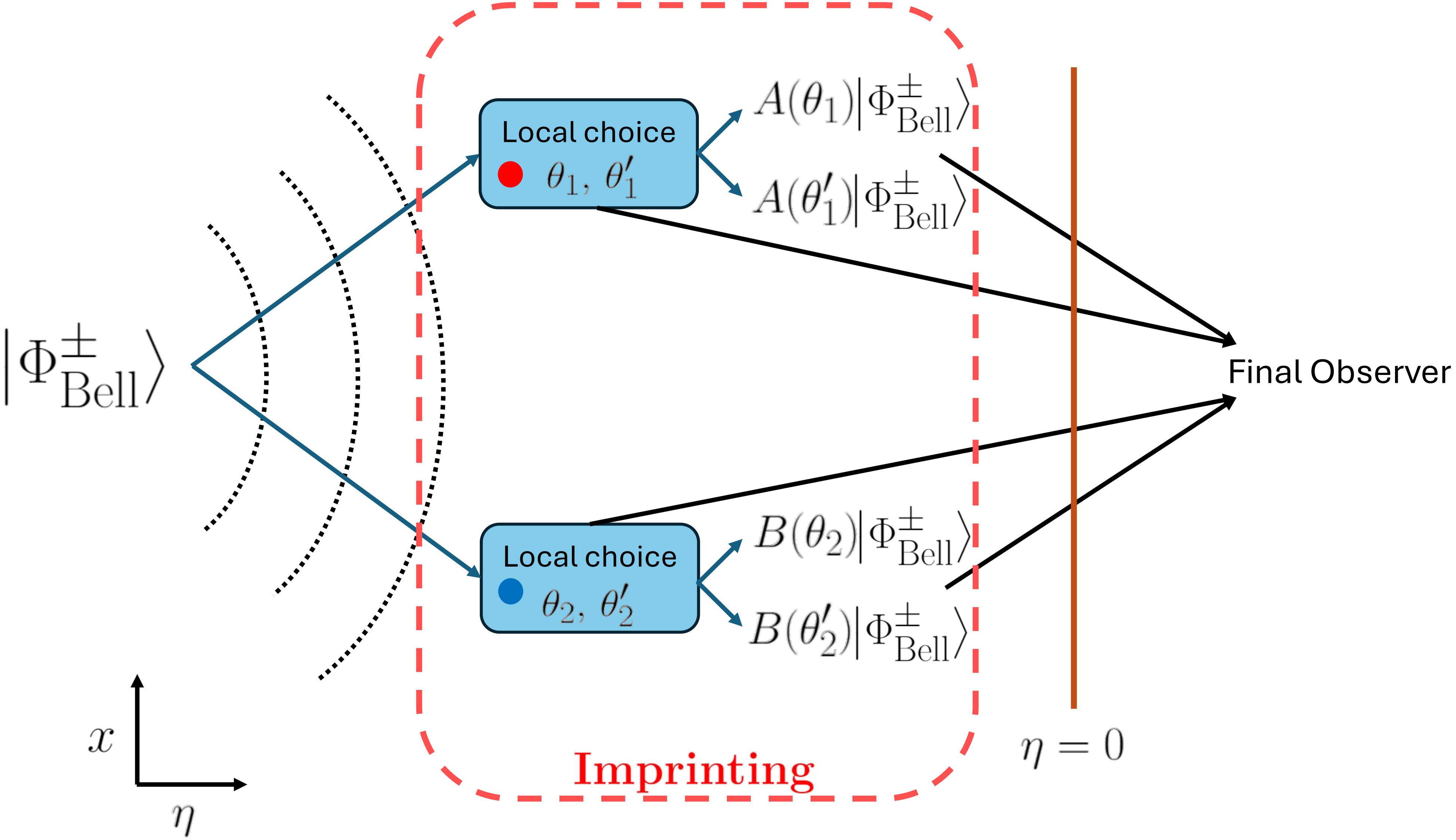
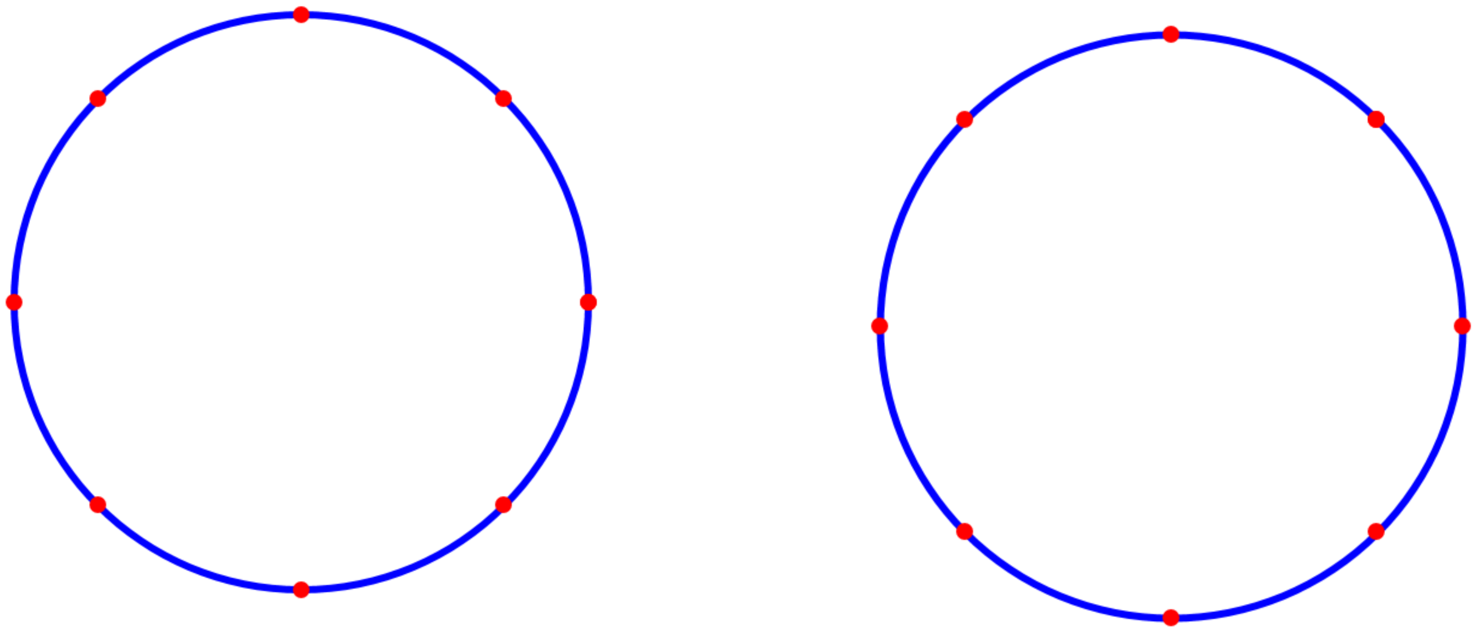


Entangled state of gravitons

- Observable with discrete spectrum – tensor polarizations: +,×

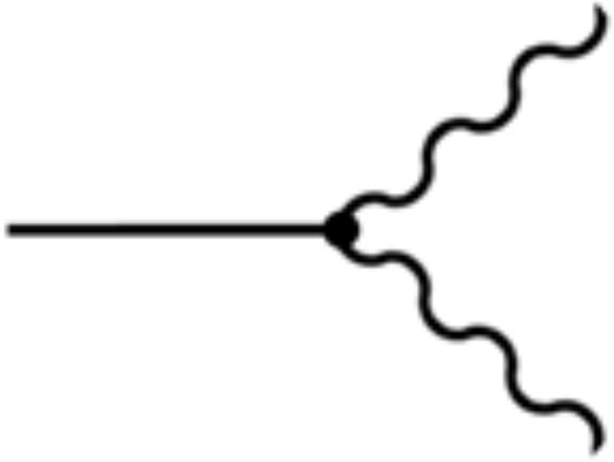


$$|\Phi^-_{\mathrm{Bell}}\rangle \propto \left(|+\rangle_{\mathbf{A}}\,|+\rangle_{\mathbf{B}} - |\times\rangle_{\mathbf{A}}\,|\times\rangle_{\mathbf{B}}\right)$$

Entangled state of gravitons

- We assume a **coherent, classical state for the inflaton field** at an early stage of inflation – “pump” field:

$$|\Phi_0\rangle = |\alpha\rangle^{\zeta} \otimes |0\rangle^{\gamma}$$



- Action to 3rd order in perturbations:

$$I^{(3)}_{\rm int} = \frac{M_p^3}{8} \int dt d^3\mathbf{x}\, a\epsilon_1 \zeta \partial_l \gamma_{ij} \partial_l \gamma_{ij}$$

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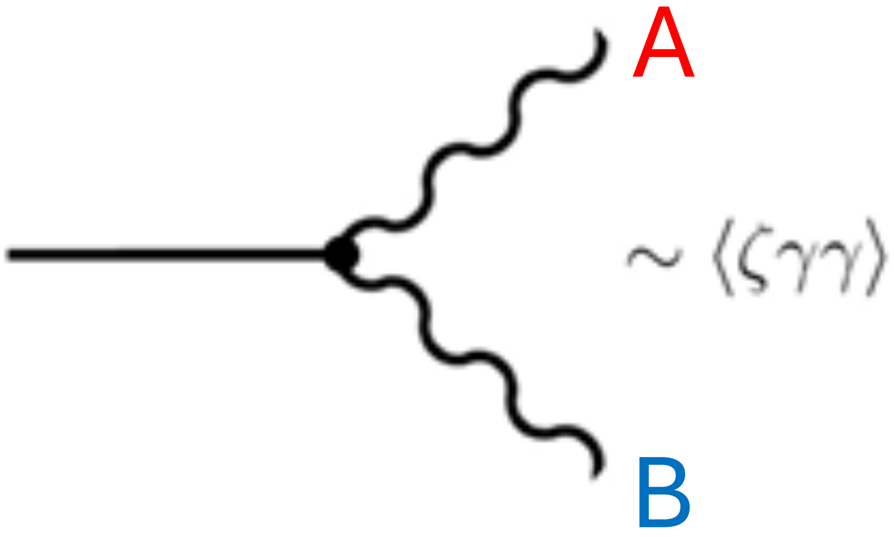
$$|\Phi_0\rangle = |\alpha\rangle^{\zeta} \otimes |0\rangle^{\gamma}$$



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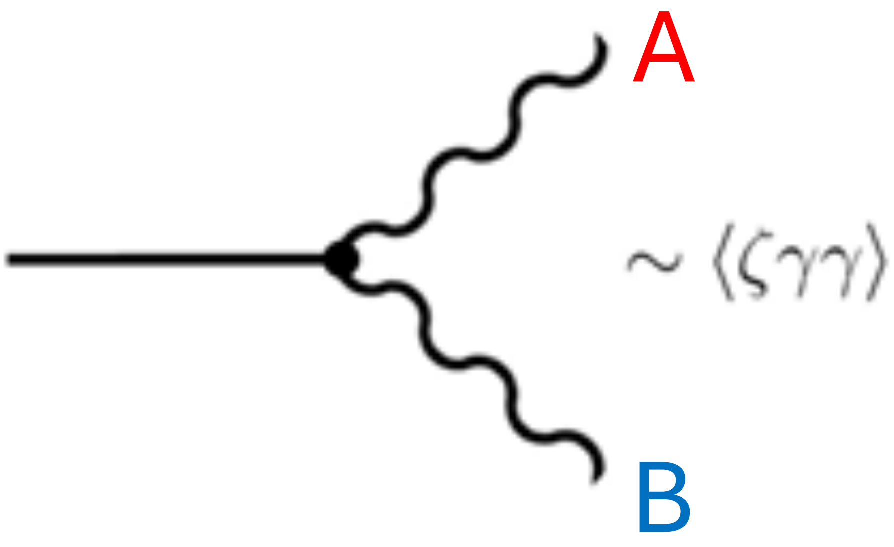

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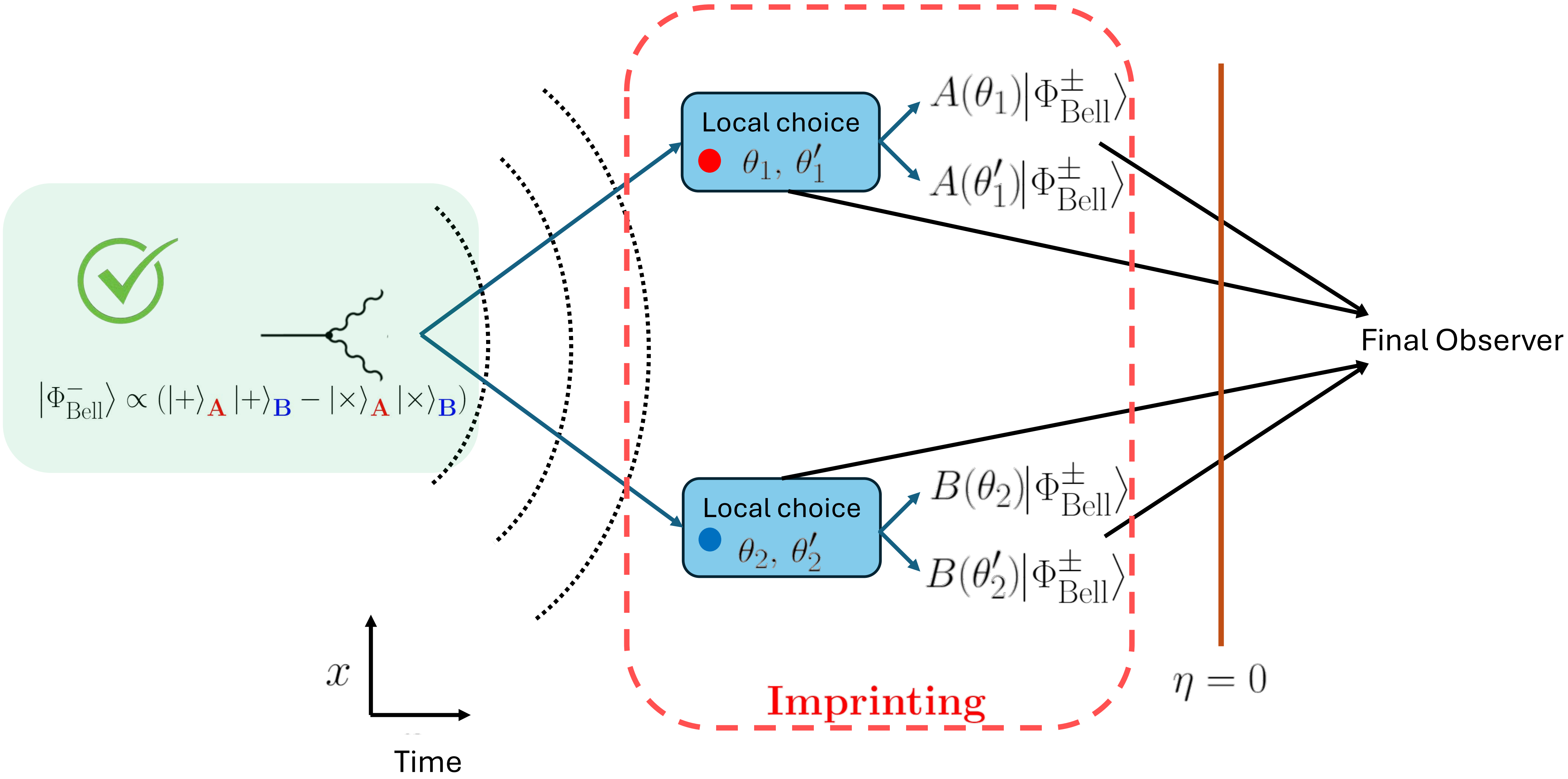
Elements of a Bell experiment



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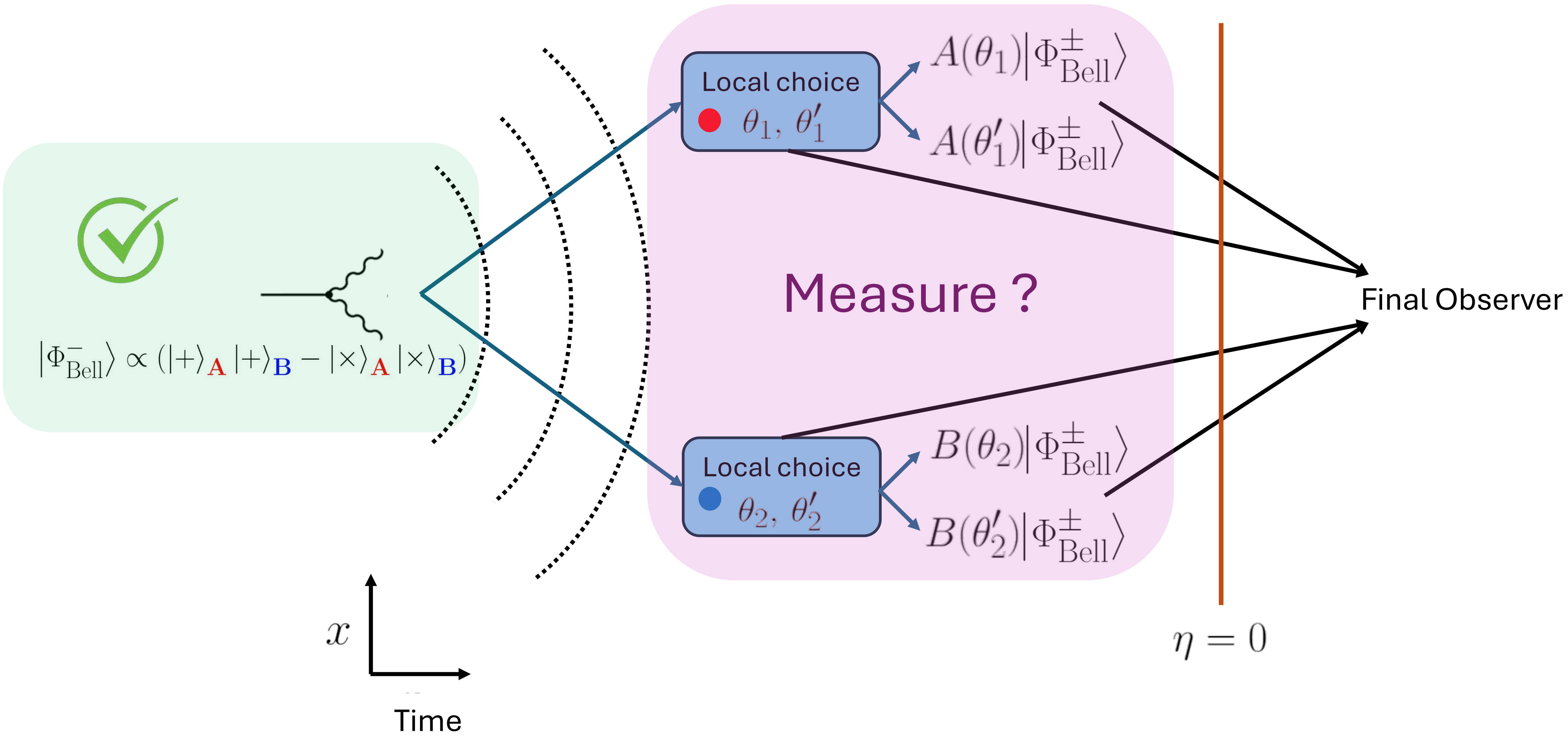

Decohere Quantum Superpositions: Wald approach

$$\mathcal{D} = 1 - e^{-\frac{1}{2}\langle N \rangle}$$

$\langle N \rangle \equiv$ Average # of soft gravitons (gravitational field) radiated through H^+

- It is shown that this same result applies to **Killing horizons**
  [D. Danielson, G. Satishchandran, R. Wald, arXiv:2301.00026]

$$\langle N \rangle \sim \frac{m^2 d^4}{R_H^5} T$$

- The cosmological horizon of de Sitter is a Killing horizon – we take the **cosmological horizon** to **perform the measure** and **induce a collapse**

$$\left|\Phi^-_{\mathrm{Bell}}\right\rangle \propto \left(|+\rangle_{\mathbf{A}} |+\rangle_{\mathbf{B}} - |\times\rangle_{\mathbf{A}} |\times\rangle_{\mathbf{B}}\right) \xrightarrow[\mathcal{D}]{H_\Lambda} \left|\Phi^-_{\mathrm{Bell}}\right\rangle_f \propto |\times\rangle_{\mathbf{A}} |\times\rangle_{\mathbf{B}}$$

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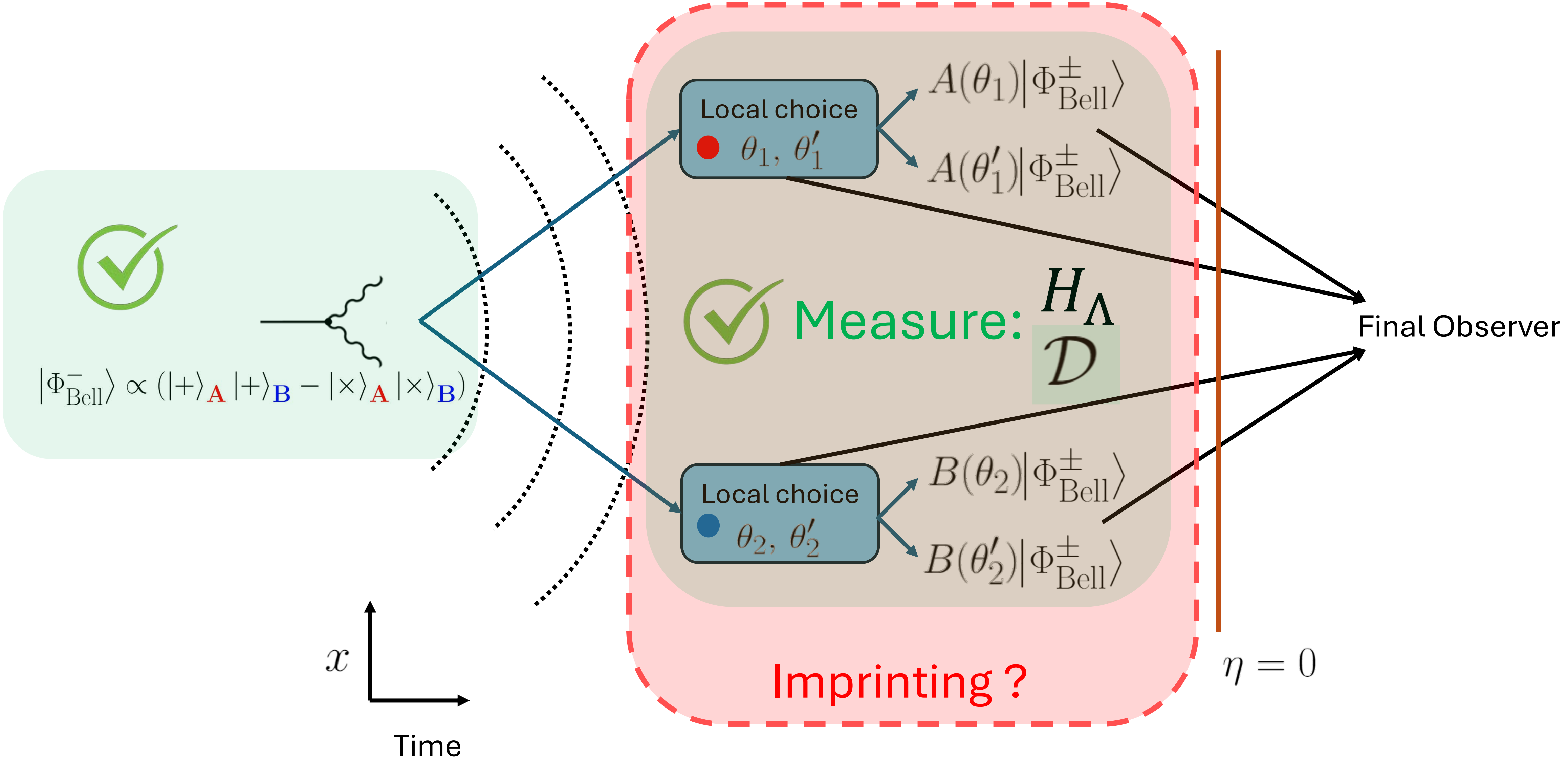

late time observables

Results get classically transmitted to after inflation through horizon-exit and later deceleration of spacetime, which makes fluctuations re-enter.

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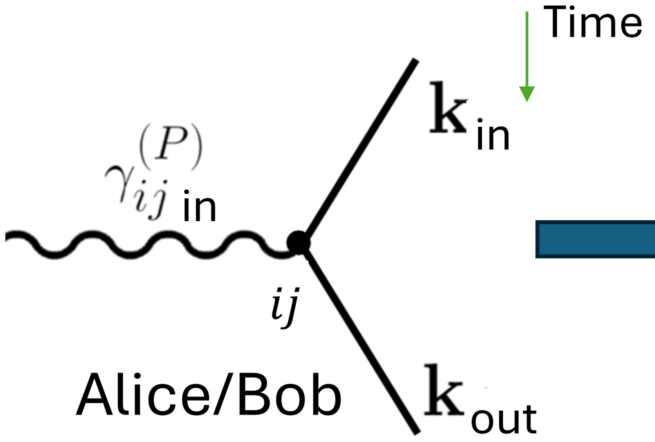

An alternative way for the measure

$$I^{(3)} \supset \int \mathrm{d}t\mathrm{d}^3\mathbf{x}\, M_p^2 \epsilon_1 a \gamma^{ij} \partial_i \zeta \partial_j \zeta$$

It can give a measurable signal in the **halo bias** (in the non-Gaussian part of the 2- and 3-point functions of dark matter halos).

Additional derivatives may imprint on **intrinsic alignment** between subhalos in the same patch (i.e. a tidal effect between these two subhalos linked to the polarization of the graviton)

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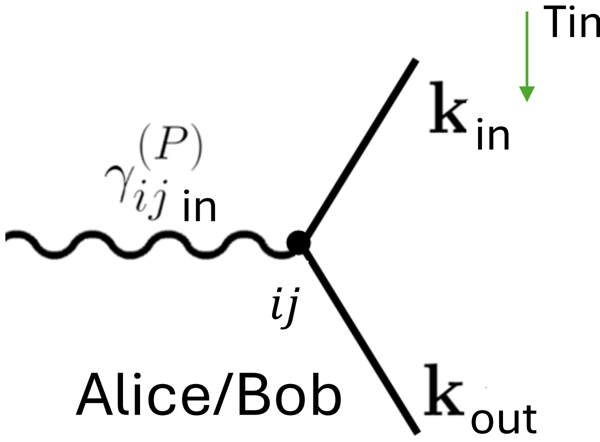

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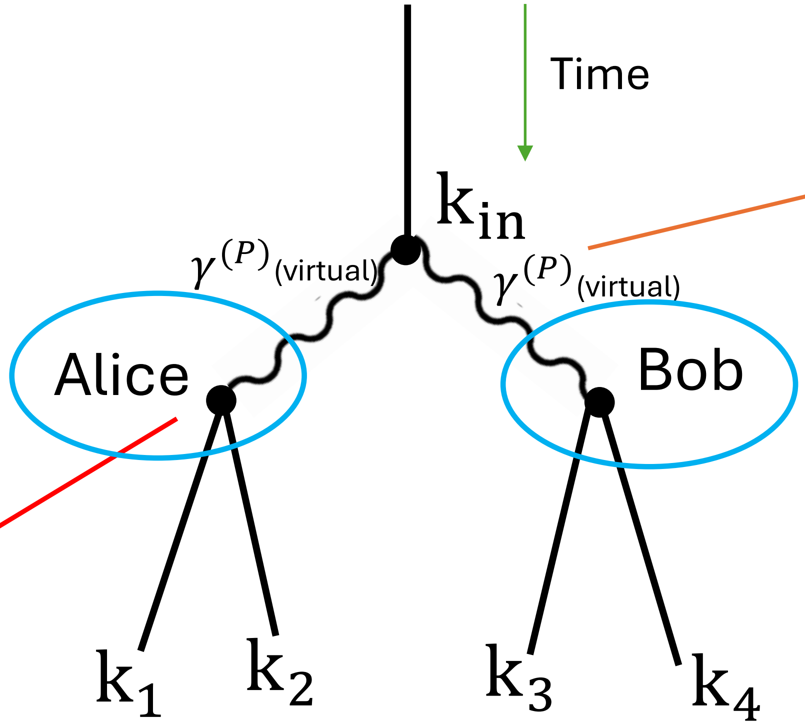

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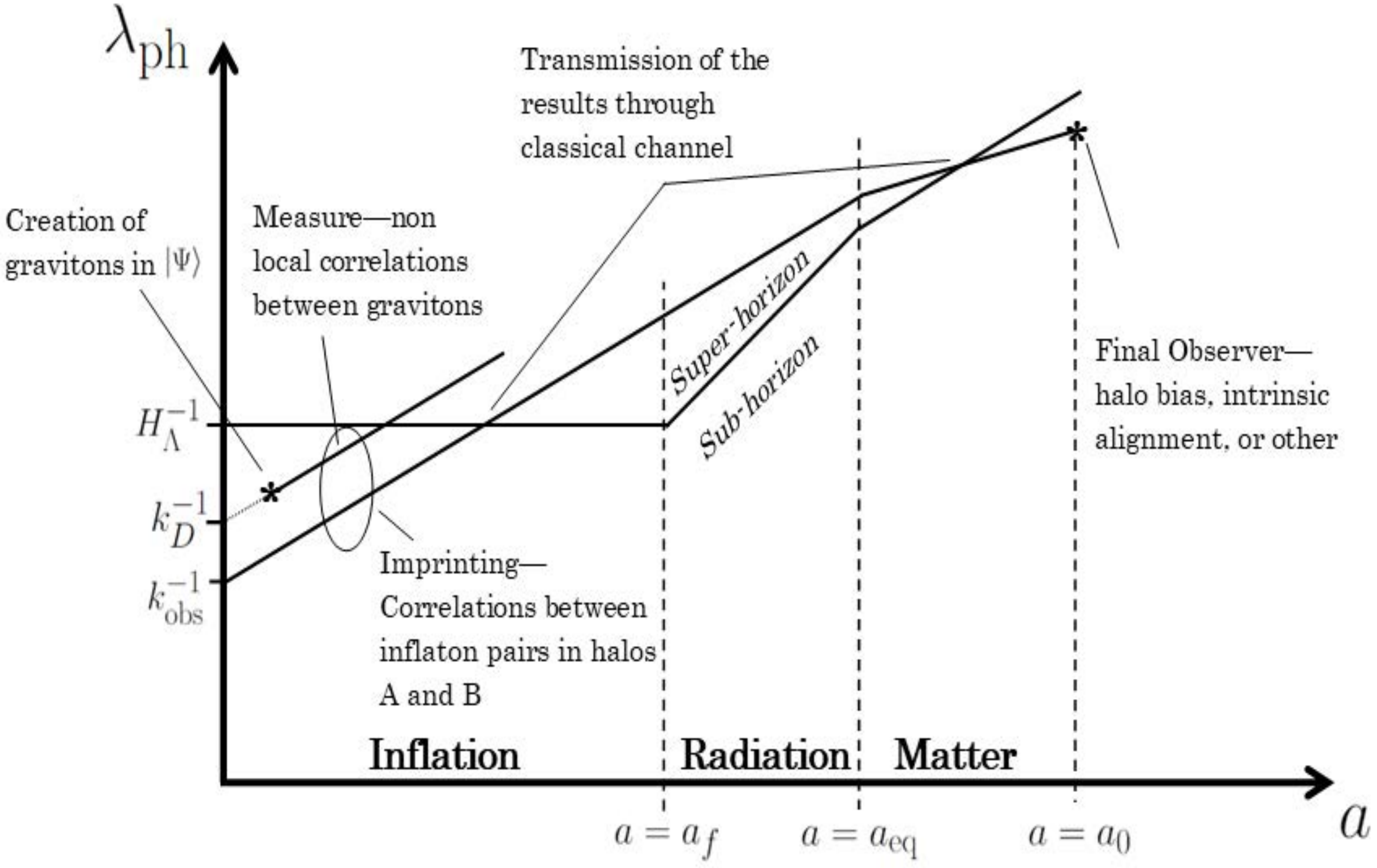

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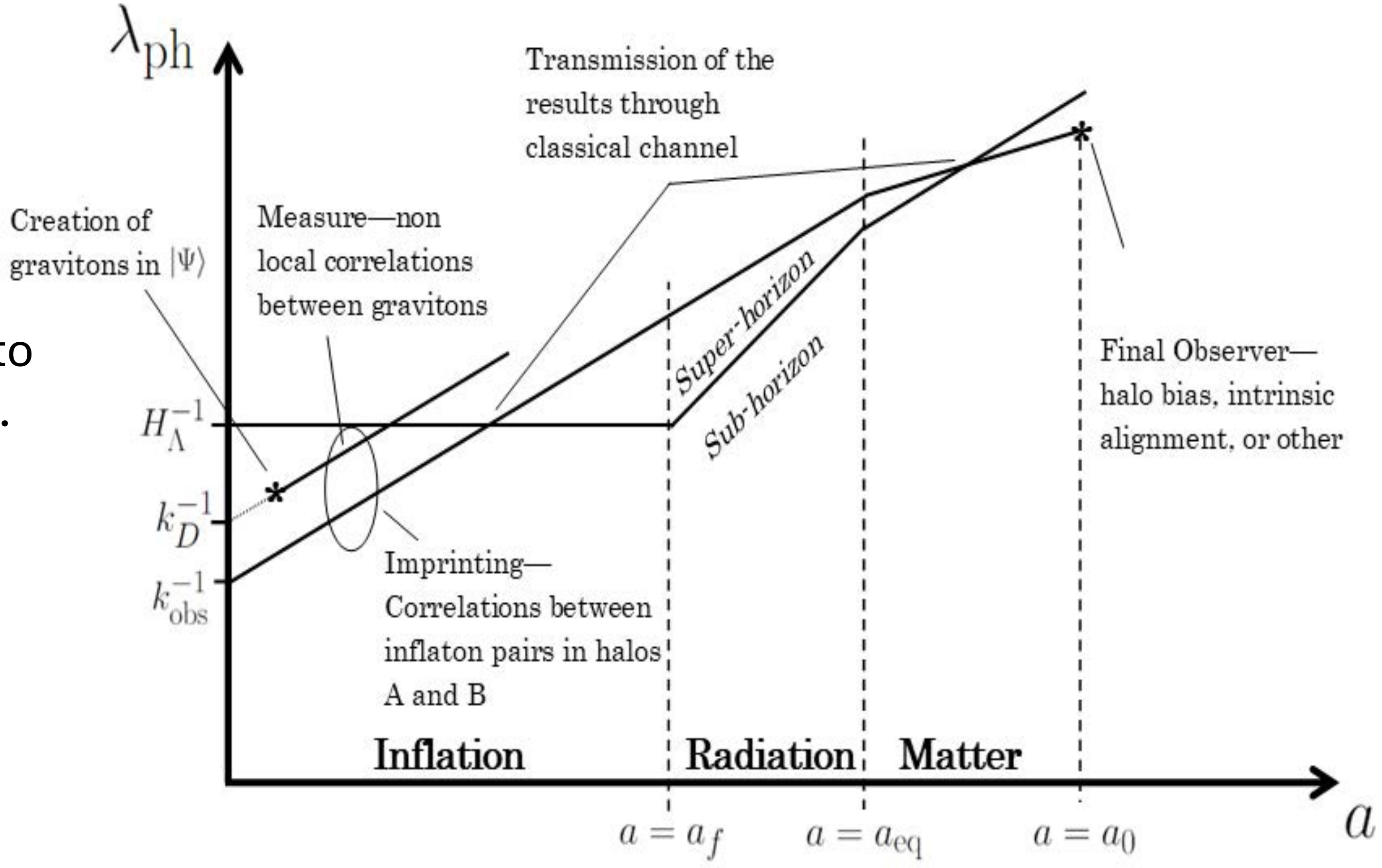

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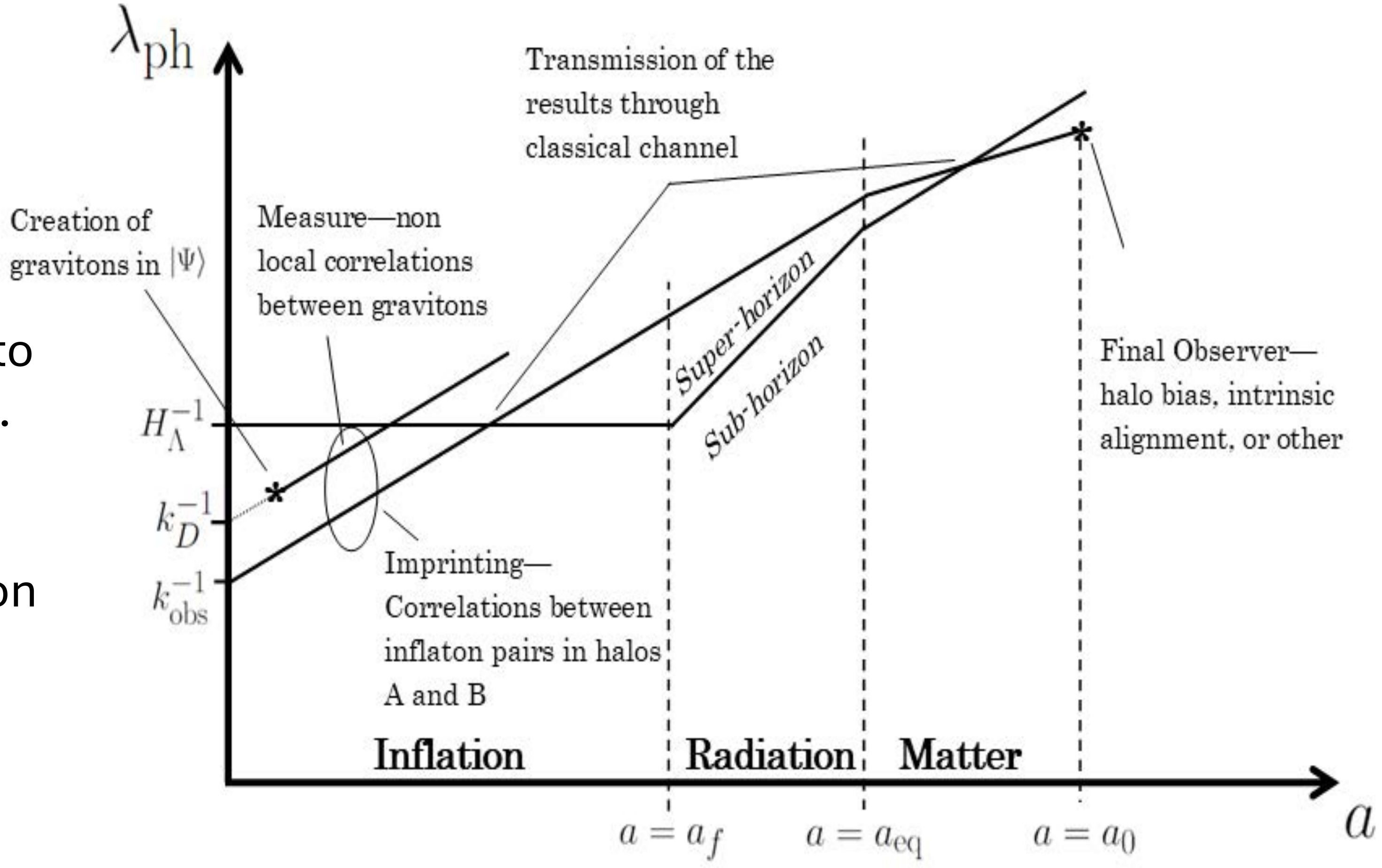

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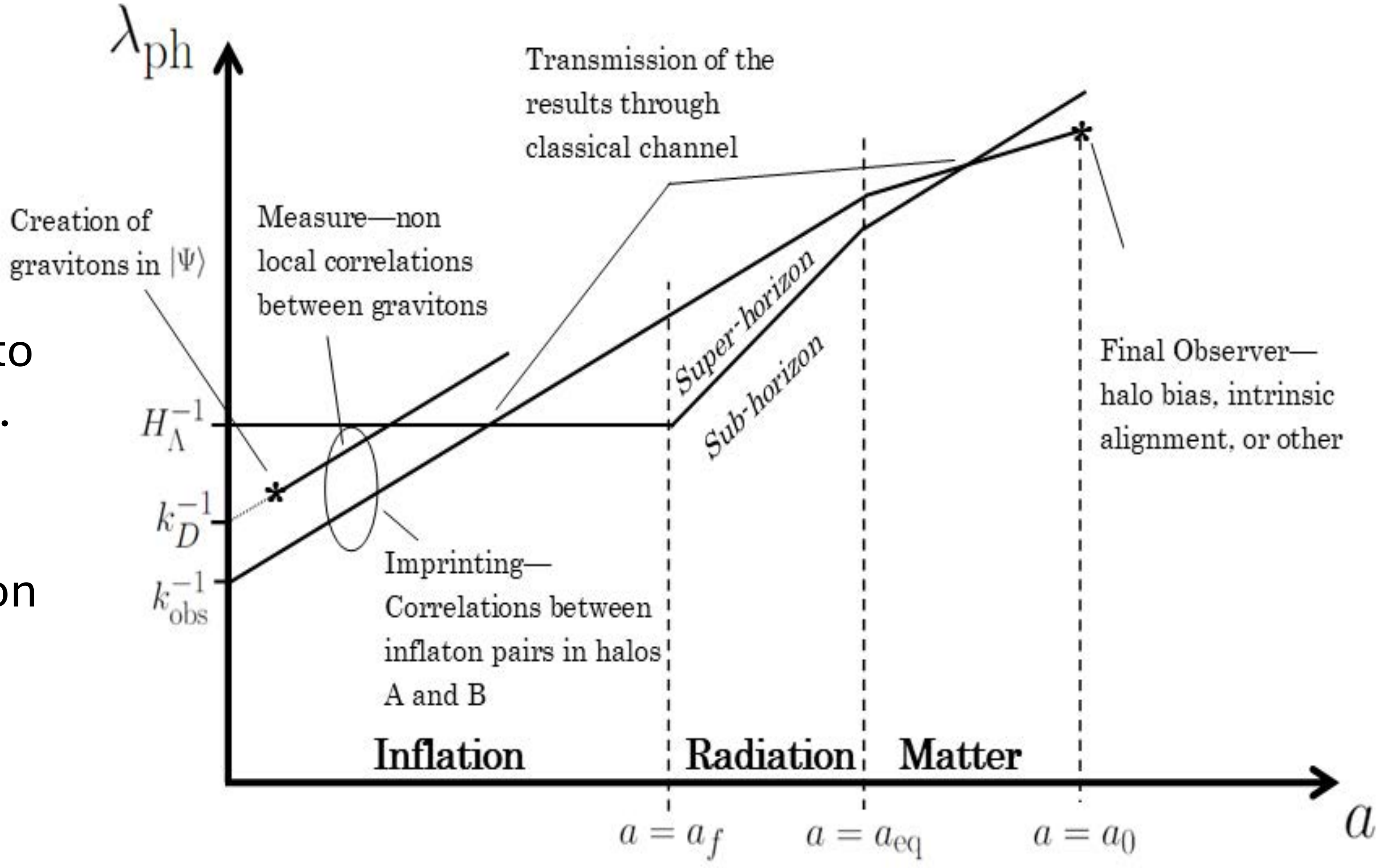
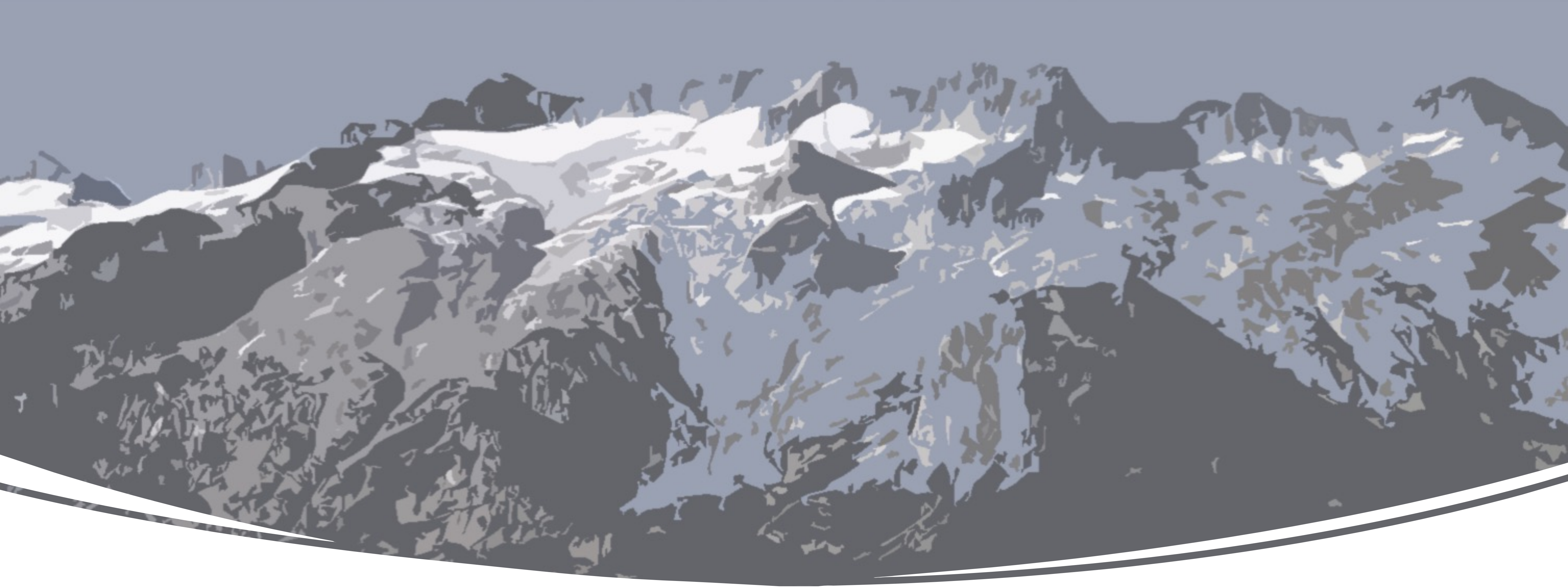
- Suggestion of late-time observables that may carry the imprinting from the 4-point scalar correlation function.





Inflation without an Inflaton

ArXiv: 2412.14265 in Physical Review

Project by:

Daniele Bertacca, Raul Jiménez, Sabino Matarrese and Angelo Ricciardone

The inflationary paradigm

- The inflationary paradigm [see, e.g., Starobinsky (1979, 1980, 1982), Mukhanov & Chibisov (1981), Guth (1981), Linde (1983, 1987), Albrecht and P. J. Steinhardt (1982), Kofman, A. D. Linde and A. A. Starobinsky (1985)] remains the most successful candidate to explain the origin and evolution of the Universe and its large-scale structure [Mukhanov and G. V. Chibisov 1981].

- This has led to the search among fundamental particle physics for an inflaton scalar field.

Has there been any definitive theoretical argument leading to a single inflaton candidate? So far, no...

- In fact, it is possible to build any inflationary potential, even in fundamental theories, like string theory, that fits the observational data: namely the observed tilt of the spectrum of scalar fluctuations, the upper limit on the energy scale of inflation (the yet to be discovered tensor modes) [see, e.g., Aghanim+ [Planck] (2020) and Galloni+ (2023)] and the non-Gaussian $1 - n_s$ signature.

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- **This is perhaps the main weakness of the inflation paradigm, as it depends on a model-dependent construction.**
- **It is therefore interesting to search for scenarios that are fully model-independent, so that inflation becomes a theory rather than a model.**

and… an inflation without the inflaton?

- In DB, Jimenez, Matarrese & Ricciardone, 2412.14265, we focus on how scalar perturbations are generated in a model-independent fashion, within a purely quantum physics framework.
- We propose a novel scenario in which scalar perturbations, that seed the large-scale structure of the Universe, are generated without relying on a scalar field (the inflaton).

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- In this framework, inflation is driven by a de Sitter space-time (dS), where tensor metric fluctuations (i.e., gravitational waves) naturally arise from quantum vacuum oscillations, and scalar fluctuations are generated via second-order tensor effects

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- In this framework, inflation is driven by a de Sitter space-time (dS), where tensor metric fluctuations (i.e., gravitational waves) naturally arise from quantum vacuum oscillations, and scalar fluctuations are generated via second-order tensor effects
- We show that scalar perturbations arise as a second-order effect from tensor perturbations and can become significantly enhanced, allowing them to dominate over the linear tensor modes, which are inherently present in dS.

Generation of second-order scalar modes from tensor perturbations

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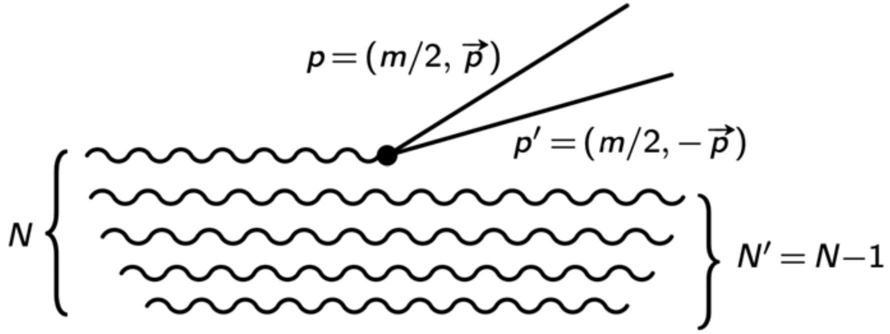
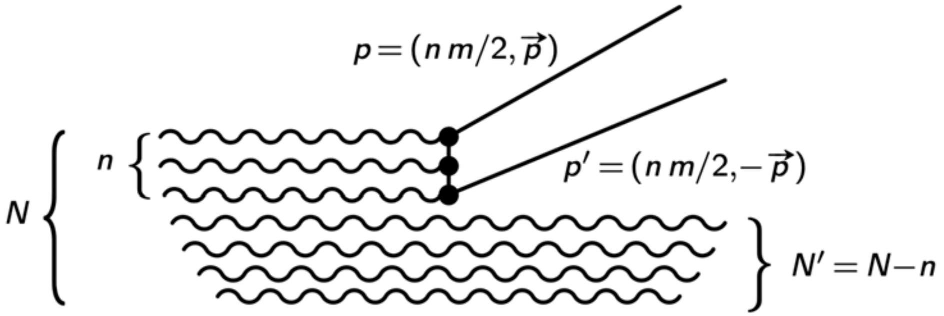
- Recently, a quantitative analysis of such tensor-induced scalar perturbations was done in Bari+(2022, 2023) for post-inflationary epochs. **Our scenario relies on a similar mechanism to generate the scalar perturbations.**
- In addition, the instability of dS space [see, e.g., Mottola (1985), Antoniadis+ (2007), Polyakov (1982, 2007, 2012), Dvali+ (2007, 2014, 2017), Alicki+ (2023a, b)] provides both a natural way for a graceful exit from inflation and also the means to end into a radiation dominated epoch.

If dS metric as process of scattering and decay of the gravitons: Dvali et al. point of view

- As it has been pointed out in Dvali+ (2007, 2014, 2017), if the quantum picture of the dS metric can be described as a Hamiltonian process of scattering and decay of the gravitons composing the coherent state of gravitons, then the self-coupling of gravitons and their coupling with other relativistic particle species leads to metric processes quantum scattering and decay of the constituent gravitons of dS.



Particle production as graviton decay: One of the N initial gravitons decays and produces 2 external particles of 4-momentum p and p'.



Leading order process to produce particles with high total energy $E \gg m$. At least $n > E/m$ of the initial N gravitons have to decay to produce the two external particles of 4-momentum p and p'.

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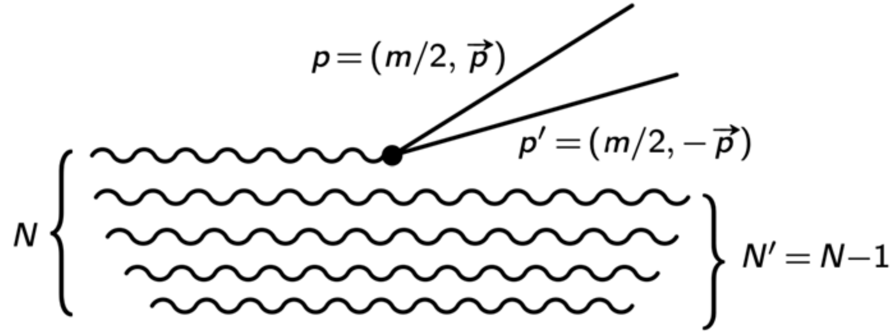

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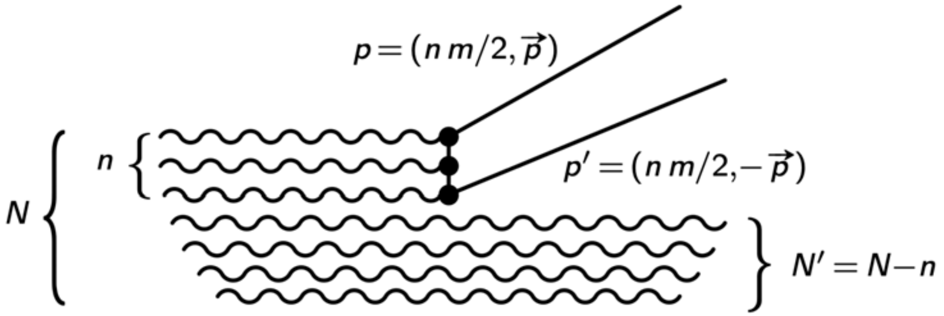

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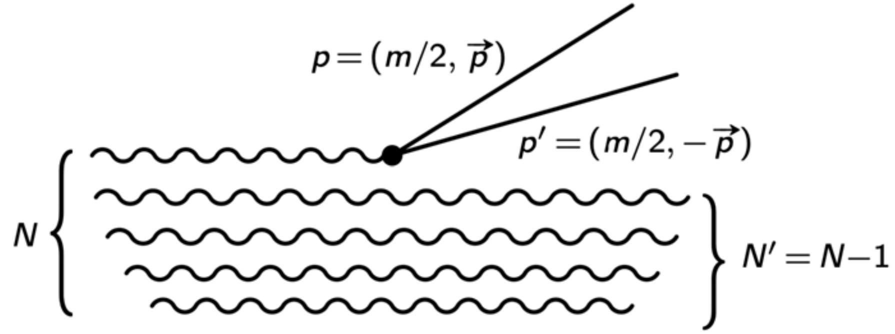
- In Dvali+ 2017 they called it as a "Quantum Break-Time of de Sitter of a system " and it is the time-scale after which its true quantum evolution departs from the classical mean field evolution.



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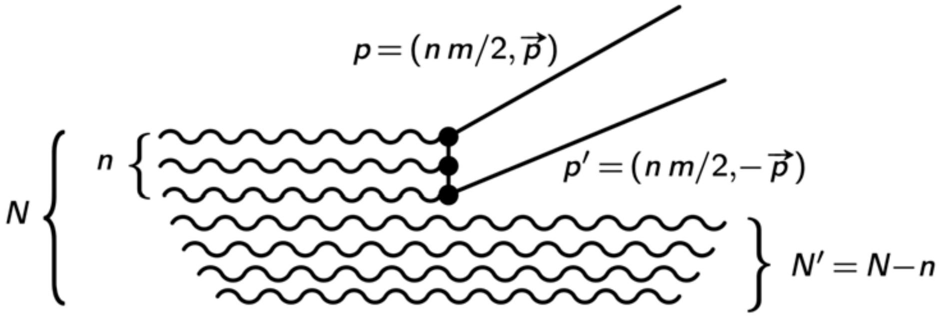

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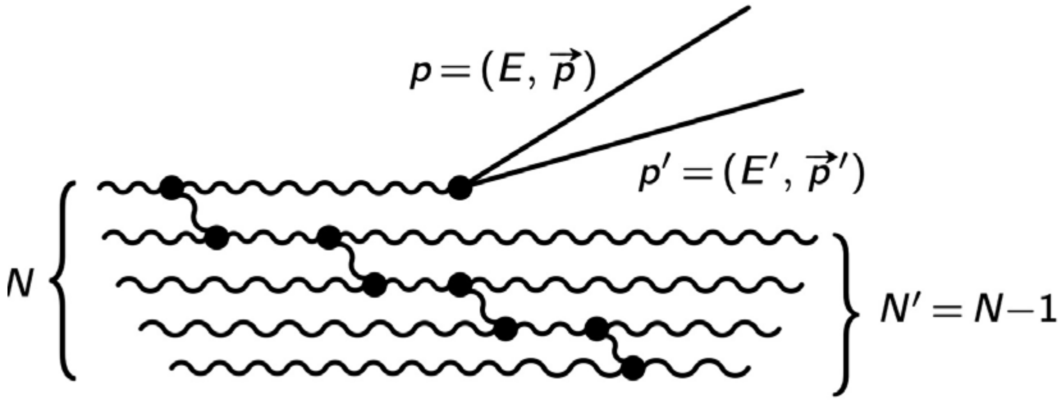
Let me precise the last point:

- If one describes dS as a quantum coherent state composite of gravitons, then the self-coupling of gravitons— as well as their coupling to other relativistic particle species, such as those in the Standard Model (SM), which must always be present—leads to quantum scattering and decay of the constituent gravitons of dS.



Higher order process of particle production, in which the produced particles recoil against all remaining gravitons. In particular, this allows for produced particles of low energies E, E′ ≪ m/2.

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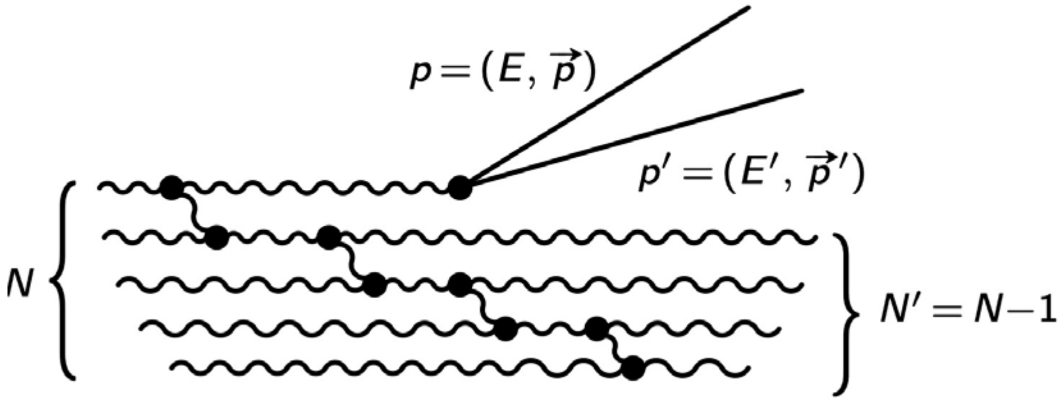

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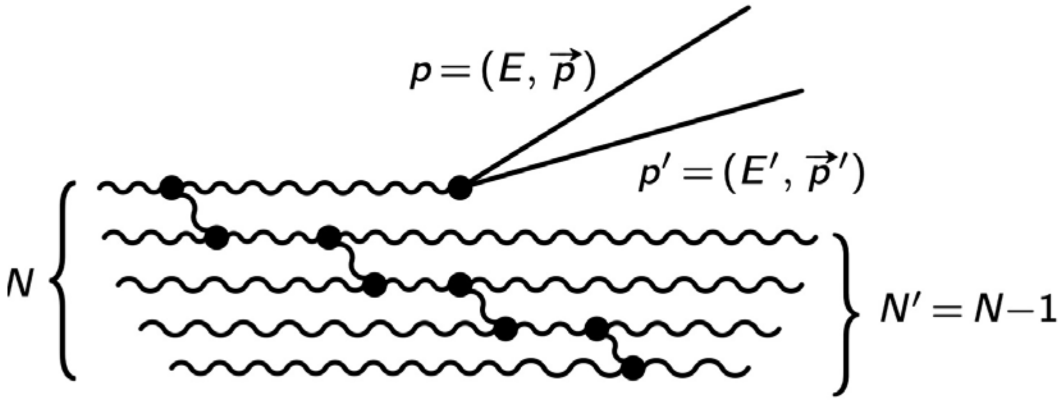
- In this case, the final quantum state cannot be described as a coherent state, and there will no longer be the dispersion relations of the free quanta propagating on a classical dS background.



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Instability of dS? [Mottola point of view]

- As Antoniadis+ (2007) points out, based on particle creation Mottola (1985) and the fluctuation-dissipation theorem Mottola (1986a) or, equivalently, on thermodynamic considerations Mottola (1986b), dS spacetime is unstable and the time scale of this instability can be exponentially large given any initial perturbation.

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This occurs because the geometry of the gravitational field is coupled to the energy-momentum stress tensor and this quantity, in turn, governs the dynamics of the gravitational field.
Consequently, the quantum fluctuations of the vacuum or, equivalently, the effects of particle creation are linked to a background gravitational field.

Instability of dS: singular behavior for low frequencies [Mottola point of view]

- By analyzing the polarization function corresponding to the perturbations in scalar/metric traces of the de Sitter background, the fluctuations of quantum matter possess a spectrum with a singular behavior for low frequencies Mottola (1985).

[Note that this describes a system response to perturbations on length scales of the order of the horizon size or larger.]

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- Then, low frequency divergence determines the particle creation rate in the adiabatic limit of slowly varying backgrounds, and, at the same time, this singular behavior means that the background is unstable to small perturbations.

This, indeed, is the **typical signal due to the spontaneous breaking of the time reversal symmetry** [see Antoniadis+ (2007)].
If we are initially in a de Sitter spacetime, the coherent vacuum energy converts into matter/radiation mode on the time scales relevant to cosmology.

Instability of dS: thermodynamic considerations [Mottola point of view]

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- Indeed, considering a small fluctuation in the Hawking temperature of the horizon which (as in the case of the black hole) **this causes a small net heat exchange between the inner region and the horizon and its surroundings**.
- This leads to having a system with negative heat capacity always between the internal region and the horizon.

Instability of dS: thermodynamic considerations [Mottola point of view]

- This negative heat capacity highlights the impossibility of a stable system in thermodynamic equilibrium and corresponds to an out-of-control process in which any infinitesimal heat exchange between the regions inside and outside the horizon further distances the system from its equilibrium configuration.

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- The entire space is unstable to quantum/thermal fluctuations in its Hawking temperature, nucleating a sort of vacuum bubble at an arbitrary point, breaking global dS invariance, see Mottola (1986b).

E.g., Quantum Fisher Cosmology (QFC)

- Along this line of reasoning, an attempt is done in Gomez and R. Jimenez (2020, 2021a, 2021b), in a framework termed QFC. It has been shown that the tilt of the primordial scalar power spectrum can be predicted to be $n_s = 0.9672$ just by considering the Heisenberg uncertainty principle in measuring time in dS

Instability of dS - thermodynamic considerations: Alicki point of view

In Alicki+ (2023a, b)] using an approach related to quantum thermodynamics:

- they provide a natural mechanism for the irreversible relaxation of the cosmological constant,
- and dS decay, in order to escape the inflationary epoch, without the need for subsequent reheating, and
- suggest an alternative way to generate the primordial perturbations which could arise from the thermal fluctuations described by the power spectrum.

About perturbations?

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- In DB, Jimenez, Matarrese & Ricciardone, 2412.14265 we introduce a novel mechanism where we derive the exact expressions for the second-order scalar potentials and the scalar power spectrum resulting from second-order tensor perturbations.
- We demonstrate that the latter agrees with the expected nearly scale-invariance from observations, opening the way for numerous potential follow-up studies and extensions.

Note that here the considered fluid unavoidably arises from the vacuum expectation value of the second-order contribution to the Einstein's tensor from gravitational waves (GW), which on sub-horizon scales leads to non-vanishing energy, pressure and anisotropic stress (**this point is raised also in Dvali + 2013!!**).

SCALAR PERTURBATIONS FROM TENSOR MODES

- We consider pure dS metric, which is in Cartesian coordinates, $ds^2 = -dt^2 + e^{2t/\alpha}(dx^2 + dy^2 + dz^2)$,

where $\alpha \equiv (3/\Lambda)^{1/2}$ and $\Lambda/8\pi G$ is the vacuum energy.

- We assume Einstein gravity and the following perturbed second order metric

$$g_{00} = -a^2(1+\psi_2),$$
$$g_{0i} = \frac{a^2}{2}\omega_{2i},$$
$$g_{ij} = a^2\left[(1-\phi_2)\delta_{ij} + \chi_{1ij} + \frac{1}{2}\chi_{2ij}\right],$$

where $a(\eta) = -1/H_\Lambda \eta$.

Now a first question that we can make…

- Unless the "background" vacuum energy (by Mottola et al.) or coherent states (by Dvali et al.) $\Lambda/8\pi G$ and if we exclude any (perturbative) contributions on the right-hand side of Einstein's equations, is it possible to obtain solutions for, e.g., ψ_2 and ϕ_2, which depend only on χ_{1ij}?

The answer is…

Now a first question that we can make...

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No, we can't...

Indeed, if we try to solve the Einstein Equations for ψ_2 and ϕ_2 we get as inconsistence on the solution for ϕ_2 (and, at the same time ψ_2=0).

SCALAR PERTURBATIONS FROM TENSOR MODES

For generality, on the RHS of Einstein's equations, we allow for the presence of a stress-energy tensor which accounts for the sum of the cosmological constant driving our dS expansion plus a generic fluid, with energy density ρ, isotropic pressure p, four-velocity u^μ and anisotropic stress tensor π^μ_ν, namely

$$T^\mu_\nu = -\frac{\Lambda}{8\pi G}\delta^\mu_\nu + (\rho + p)u^\mu u_\nu + p\delta^\mu_\nu + \pi^\mu_\nu \; .$$

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NOTE: by using the idea by Dvali et al 2013 and 2017 this seemingly-classical system quantum mechanically as the mixture of the two "Bose-gases":

1) the quantum coherent state that describes Λ;
2) a cosmological fluid that plays the role of a cosmic clock! Precisely, this compositeness acts as a quantum clock (that becomes classical at scales larger than horizon) that imprints measurable effects into cosmological observables.

These effects are cumulative and gather the information throughout the entire duration of inflation.

SCALAR PERTURBATIONS FROM TENSOR MODES

Considering only the tensor contribution we have, in the comoving frame of such a GW fluid,

$$u^\mu = \frac{1}{a}\left(\delta^\mu_0 + \frac{1}{2}v^\mu_2\right) \ ,$$

from which we can compute u_μ as:

$$u_0 = -a\left(1 + \frac{1}{2}\psi_2\right) ; \quad u_i = \frac{a}{2}\left(\omega_{2i} + v_{2i}\right)$$

SCALAR PERTURBATIONS FROM TENSOR MODES

For the 00 component at second order, we find

$$\frac{1}{2}\delta^2 T^0_0 = -\frac{1}{2}\delta^2\rho \ .$$

The other components of T^{μ}_{ν} are given by

$$\frac{1}{2}\delta^2 T^i_0 = -(1+w)\bar{\rho}\left[\frac{1}{2}v^i_2\right] \ ,$$
$$\frac{1}{2}\delta^2 T^i_j = +\frac{1}{2}\delta^2 p\delta^i_j + \frac{1}{a^2}\frac{1}{2}\delta^2\pi^i_j$$

where $\bar{\rho}$ and w are, respectively, the energy density and the equation of state at the background level.

We split the peculiar velocity $v_{2i}= v_{2,i} + v_{\perp 2i}$as, where $v_{\perp 2i}$ is a solenoidal (divergence-free or transverse) vector.

SCALAR PERTURBATIONS FROM TENSOR MODES

- The anisotropic stress tensor can be decomposed into a trace-free scalar part, Π , a vector part, Π_i, and a tensor part, Π_{ij}, at each order according to

$$\frac{1}{a^2}\delta^2\pi_{ij} = \left[\Pi_{2,ij} - \frac{1}{3}\nabla^2\Pi_2\delta_{ij} + \frac{1}{3}\left(\Pi_{2i,j} + \Pi_{2j,i}\right) + \Pi_{2ij}\right] .$$

Equation for ϕ_2

Considering a graviton perturbation, with generic w and $c_s$cs, where $\delta^2 p = c_s^2 \delta^2 \rho$ and manipulating the Einstein equations, we find the Master Equation for ϕ_2

$$
\begin{aligned}
&\phi_2'' + 3\left(1+c_s^2\right)\mathcal{H}\phi_2' + \left[2\mathcal{H}' + \left(1+3c_s^2\right)\mathcal{H}^2\right]\phi_2 - c_s^2\nabla^2\phi_2 = 8\pi G a^2\left[2\mathcal{H}' + 3\left(1+c_s^2\right)\mathcal{H}^2\right]\Pi_2 \\
&\quad + \frac{8\pi G}{3}a^2\nabla^2\Pi_2 + 8\pi G a^2\mathcal{H}\Pi_2' - \frac{3}{2}\left[2\mathcal{H}' + \left(1+3c_s^2\right)\mathcal{H}^2\right]\nabla^{-4}\partial_i\partial^j\mathcal{A}^i_j - \frac{3}{2}\mathcal{H}\nabla^{-4}\partial_i\partial^j{\mathcal{A}^i_j}' - \frac{1}{2}\nabla^{-2}\partial_i\partial^j\mathcal{A}^i_j \\
&\quad - \left[2\mathcal{H}' + \left(1+3c_s^2\right)\mathcal{H}^2\right]\nabla^{-2}\Bigg[-\frac{3}{4}\chi_1^{lk,m}\chi_{1kl,m} - \frac{1}{2}\chi_1^{kl}\nabla^2\chi_{1lk} + \frac{1}{2}\chi^{km}_{1,l}\chi^l_{1m,k} + \frac{1}{2}\chi'_{1km}\chi'_{1km} - \frac{3}{2}\nabla^{-2}\partial_i\partial_j\left({\chi^i_{1k}}'\chi'_{1kj}\right)\Bigg] \\
&\quad + \frac{3}{8}\chi^{ml}_{1,k}\chi^{,k}_{1lm} - \frac{3}{8}{\chi_1^{kl}}'\chi'_{1kl} - \frac{1}{4}\chi^m_{1k,l}\chi^{lk}_{1,m} - \mathcal{H}\nabla^{-2}\Bigg[-\frac{3}{2}{\chi_1^{lk,m}}'\chi_{1kl,m} + \frac{1}{2}{\chi_1^{kl}}'\nabla^2\chi_{1lk} - \frac{1}{2}\chi_1^{kl}\nabla^2{\chi_{1lk}}' + \frac{1}{2}{\chi^{km}_{1,l}}'\chi^l_{1m,k} \\
&\quad + \frac{1}{2}\chi^{km}_{1,l}{\chi^l_{1m,k}}' - 2\mathcal{H}{\chi_{1km}}'\chi'_{1km}\Bigg] + 3\mathcal{H}\nabla^{-4}\left(-2\mathcal{H}{\chi^i_{1k,j}}'{\chi^j_{1k,i}}' + {\chi^j_{1k,i}}'\nabla^2\chi^i_{1k,j}\right) + \frac{1}{2}\nabla^{-2}\partial_i\partial_j\left({\chi^i_{1k}}'\chi'_{1kj}\right) \\
&\quad + c_s^2\Bigg[-\frac{1}{8}{\chi_1^{ik}}'\chi'_{1ki} - \mathcal{H}\chi_1^{ik}\chi'_{1ki} + \frac{1}{2}\chi_1^{mk}\nabla^2\chi_{1mk} + \frac{3}{8}\chi^{ml}_{1,k}\chi^{,k}_{1lm} - \frac{1}{4}\chi^m_{1k,l}\chi^{lk}_{1,m}\Bigg],
\end{aligned}
$$

where

$$
\mathcal{A}^i_j = \frac{1}{2}\chi_1^{lk,i}\chi_{1kl,j} + \chi_1^{kl}\chi^{,i}_{1lk,j} - \chi_1^{kl}\chi^i_{1l,jk} - \chi_1^{kl}\chi^{,i}_{1lj,k} + \chi_1^{kl}\chi^i_{1j,kl} + \chi^{ik}_{1,l}\chi^{,l}_{1jk} - \chi^{ki}_{1,l}\chi^l_{1j,k}\,.
$$

Equation for ψ_2

Using the traceless part of the ij components we find

$$\psi_2 = \phi_2 - 8\pi G a^2 \Pi_2 - \frac{\mathcal{F}_\chi}{4}\,,$$

where

$$\mathcal{F}_\chi = 4\nabla^{-2}\left(\frac{3}{4}\chi_1^{lk,m}\chi_{1kl,m} + \frac{1}{2}\chi_1^{kl}\nabla^2\chi_{1lk} - \frac{1}{2}\chi_{1,l}^{km}\chi_{1m,k}^{l}\right) - 6\nabla^{-4}\partial_i\partial^j\mathcal{A}_j^i\,,$$

and, considering only the tensor contribution, the gauge-invariant curvature perturbation on uniform density hypersurfaces is

$$\zeta_2 = -\phi_2 - \frac{\mathcal{H}}{\bar{\rho}'}\delta^2\rho\,.$$

Then using the energy constraint equation we get

$$\zeta_2 = -\phi_2 + \frac{\mathcal{H}}{4\pi G a^2\rho'}\Bigg[3\mathcal{H}\phi_2' + 3\mathcal{H}^2\psi_2 - \nabla^2\phi_2 + \frac{1}{8}\chi_1^{ik'}\chi_{1ki}' + \mathcal{H}\chi_1^{ik}\chi_{1ki}' - \frac{1}{2}\chi_1^{mk}\nabla^2\chi_{1mk} - \frac{3}{8}\chi_{1,k}^{ml}\chi_{ml}^{1,k} + \frac{1}{4}\chi_{1k,l}^{m}\chi_{1,m}^{lk}\Bigg].$$

Focusing only modes larger than the horizon…

- Focusing only modes larger than the horizon, neglecting terms that contain ${\chi_{1kj}}'$, and considering only terms with total number of spatial derivatives equal (or less) than zero, we find

$$\phi_2'' - \frac{3\left(1+c_s^2\right)}{\eta}\phi_2' + \frac{3(1+c_s^2)}{\eta^2}\phi_2 = +8\pi G\frac{\left(5+3c_s^2\right)}{H_\Lambda^2\eta^4}\Pi_2 - \frac{8\pi G}{H_\Lambda^2\eta^3}\Pi_2'$$
$$-\frac{3\left(1+c_s^2\right)}{\eta^2}\left[\frac{3}{2}\nabla^{-4}\partial_i\partial^j\mathcal{A}^i_j + \nabla^{-2}\left(-\frac{3}{4}\chi_1^{lk,m}\chi_{1kl,m} - \frac{1}{2}\chi_1^{kl}\nabla^2\chi_{1lk} + \frac{1}{2}\chi_{1,l}^{km}\chi_{1m,k}^{l}\right)\right]$$

where we set, in de Sitter Universe, $H = H_\Lambda$ is constant and $\mathcal{H}' = \mathcal{H}^2$, $\eta = -\exp[-H_\Lambda t]\,/H_\Lambda$, i.e. $a(\eta) = -1/H_\Lambda\eta$ and $\mathcal{H} = -1/\eta$.

Focusing modes larger than the horizon for ϕ_2 is const.

In this case we need an extra relation to close the equations and find a general solution.
Un possible way is to look at the particular solution in which ϕ_2 is constant on very large scales. In this case we get

$$\phi_2 = +8\pi G\frac{\left(5+3c_s^2\right)}{3\left(1+c_s^2\right)H_\Lambda^2\eta^2}\Pi_2 - \frac{8\pi G}{3\left(1+c_s^2\right)H_\Lambda^2\eta}\Pi_2' + \frac{1}{4}\mathcal{F}_\chi = \text{const.}$$

and let us write the traceless part as

$$\psi_2 = \phi_2 - 8\pi G a^2\Pi_2 + \frac{3}{2}\nabla^{-4}\partial_i\partial^j\mathcal{A}_j^i + \nabla^{-2}\left[-\frac{3}{4}\chi_1^{lk,m}\chi_{1kl,m} - \frac{1}{2}\chi_1^{kl}\nabla^2\chi_{1lk} + \frac{1}{2}\chi_{1,l}^{km}\chi_{1m,k}^{l}\right]$$
$$= \phi_2 - \frac{8\pi G}{H_\Lambda^2\eta^2}\Pi_2 - \frac{\mathcal{F}_\chi}{4}\ ,$$

and combine them we find

$$\psi_2 = \frac{8\pi G}{H_\Lambda^2\eta^2}\left[\frac{\left(5+3c_s^2\right)}{3\left(1+c_s^2\right)} - 1\right]\Pi_2 - \frac{8\pi G}{3\left(1+c_s^2\right)H_\Lambda^2\eta}\Pi_2'\ .$$

Focusing modes larger than the horizon for ϕ_2 is const.

- Then using the energy-momentum constraint equation we get

$$\frac{8\pi G}{H_\Lambda^2}\Pi_2 = \eta_{\rm in}^2\left[\frac{8\pi G}{\eta_{\rm in}^2 H_\Lambda^2}\Pi_{2\rm in} - \left(\phi_2 - \frac{1}{4}\mathcal{F}_\chi\right)\right]\left(\frac{\eta}{\eta_{\rm in}}\right)^{\left(5+3c_s^2\right)} + \left(\phi_2 - \frac{1}{4}\mathcal{F}_\chi\right)\eta^2$$

$$\psi_2 = \left[-\frac{8\pi G}{\eta_{\rm in}^2 H_\Lambda^2}\Pi_{2\rm in} + \left(\phi_2 - \frac{1}{4}\mathcal{F}_\chi\right)\right]\left(\frac{\eta}{\eta_{\rm in}}\right)^{3\left(1+c_s^2\right)} .$$

We can quickly verify that the previous two equations can be combined again to recover the usual relation

$$\frac{8\pi G}{\eta_{\rm in}^2 H_\Lambda^2}\Pi_2 + \psi_2\left(\frac{\eta}{\eta_{\rm in}}\right)^2 = \left(\phi_2 - \frac{1}{4}\mathcal{F}_\chi\right)\left(\frac{\eta}{\eta_{\rm in}}\right)^2 .$$

For ϕ_2 is const., is it possible to have $\psi_2 \neq 0$?

These relations

$$\psi_2 = \left[-\frac{8\pi G}{\eta_{\rm in}^2 H_\Lambda^2}\Pi_{2\rm in} + \left(\phi_2 - \frac{1}{4}\mathcal{F}_\chi\right)\right]\left(\frac{\eta}{\eta_{\rm in}}\right)^{3\left(1+c_s^2\right)}$$

and

$$\frac{8\pi G}{\eta_{\rm in}^2 H_\Lambda^2}\Pi_2 + \psi_2\left(\frac{\eta}{\eta_{\rm in}}\right)^2 = \left(\phi_2 - \frac{1}{4}\mathcal{F}_\chi\right)\left(\frac{\eta}{\eta_{\rm in}}\right)^2 .$$

suggest that if

$$\left(\phi_2 - \frac{1}{4}\mathcal{F}_\chi\right) = \frac{8\pi G}{\eta_{\rm in}^2 H_\Lambda^2}\Pi_{2\rm in} ,$$

we have $\psi_2 = 0$ when $\phi_2 = \mathcal{F}_\chi/4$.

- Let us point out that $\psi_2 = 0$ on very large scales is a **possible solution** but it is not sufficient if we want to promote these solutions to those describing the scalar primordial power spectrum.

A comment:

As can be seen from the metrics we have considered, with our approach we can analytically study vectors and tensors at the second order.

These contributions will also be analysed in detail in a subsequent paper.

When $\psi_2 = 0$ and $\phi_2 = \mathcal{F}_\chi/4$

Let me focus only on the first terms in

$$\mathcal{F}_\chi = 4\nabla^{-2}\left(\frac{3}{4}\chi_1^{lk,m}\chi_{1kl,m} + \frac{1}{2}\chi_1^{kl}\nabla^2\chi_{1lk} - \frac{1}{2}\chi_{1,l}^{km}\chi_{1m,k}^{l}\right) - 6\nabla^{-4}\partial_i\partial^j\mathcal{A}_j^i\,,$$

then we have

$$\Delta_\phi(k) = \frac{k^3}{2\pi^2}\frac{1}{64(2\pi)^3}\frac{1}{k^4}\int \mathrm{d}^3k_1\mathrm{d}^3k_2\,\delta^{(3)}[\mathbf{k}-(\mathbf{k}_1+\mathbf{k}_2)]\,\mathcal{K}_h(\mathbf{k}_1,\mathbf{k}_2)\left[\frac{2\pi^2}{k_1^3}\frac{16}{\pi}\left(\frac{H_{\rm inf}}{m_{\rm pl}}\right)^2\right]\left[\frac{2\pi^2}{k_2^3}\frac{16}{\pi}\left(\frac{H_{\rm inf}}{m_{\rm pl}}\right)^2\right]$$

where

$$\mathcal{K}_h(\mathbf{k}_1,\mathbf{k}_2,k^2) = \left\{\left(k_1^2+k_2^2+3\mathbf{k}_1\cdot\mathbf{k}_2\right)^2\left[\left(1-\hat{\mathbf{k}}_1\cdot\hat{\mathbf{k}}_2\right)^4+\left(1+\hat{\mathbf{k}}_1\cdot\hat{\mathbf{k}}_2\right)^4\right]\right.$$
$$\left.+8\left(\mathbf{k}_1\cdot\mathbf{k}_2\right)\left(k_1^2+k_2^2+3\mathbf{k}_1\cdot\mathbf{k}_2\right)\left(\hat{\mathbf{k}}_1\times\hat{\mathbf{k}}_2\right)^2\left[3+\left(\hat{\mathbf{k}}_1\cdot\hat{\mathbf{k}}_2\right)^2\right]+8k_1^2k_2^2\left(\hat{\mathbf{k}}_1\times\hat{\mathbf{k}}_2\right)^4\left[1+\left(\hat{\mathbf{k}}_1\cdot\hat{\mathbf{k}}_2\right)^2\right]\right\}$$

For ϕ_2 is const., is it possible to have $\psi_2 \neq 0$?

For $\psi_2 \neq 0$ we can say more…

1) when $\psi_2 \neq 0$, ψ_2 is not constant!

2) … and what about ζ ? What can we say?

… or let me rephrase with other questions…

- This results show how the second-order scalar fluctuation grows as $k\eta \rightarrow 0$ and thus, dominates over the tensor modes.
- In this set up the curvature perturbation does not stay constant, which is a crucial property of this variable ζ?
- What is the justification for not having constant ζ ?
- Why does the elimination of this characteristic not matter in this case?

For ϕ_2 is const., is it possible to have $\psi_2 \neq 0$?

For $\psi_2 \neq 0$ we can say more…

1) when $\psi_2 \neq 0$, ψ_2 is not constant!

2) … and what about ζ ? What can we say?

Possible answer(s):

- This effect is very important (for the framework we are considering in this paper) to obtain at the end of inflation an amplitude for the power spectrum of P_ζ that is larger than the tensor one.
- This is a specific feature of our scenario, which allows us to reach constraints imposed, for instance, at CMB scales. Otherwise, we are not able to reach the constraint imposed, for example, by CMB measurements. As for the smallest (comoving) scales, they should be very small compared to the (comoving) cosmological horizon.
- Of course, there are several non-trivial effects that can be directly observed via Master equation that we have obtained.

For ϕ_2 is const., is it possible to have $\psi_2 \neq 0$?

For $\psi_2 \neq 0$ we can say more…

1) when $\psi_2 \neq 0$, ψ_2 is not constant!

2) … and what about ζ ? What can we say?

Note that the non-constant part of ζ is CRUCIAL if we want to measure **quantum mechanics effects**!

Indeed, in this case, during the inflation epoch

$$\left[\zeta_{\vec{k}}, \dot{\zeta}_{-\vec{k}}\right] \neq 0!!!$$

See also the discussion made in Madalcena (2015)

E.g. via a similar prescription made in the first part of the talk!

For ϕ_2 is const., is it possible to have $\psi_2 \neq 0$?

Note that here, $\eta_{\rm in}$ is a suitable initial time in which the hypothesis that ϕ_2 is const., i.e on very large scale (where our hypothesis is correct).

Then, considering just a mode with a given comoving k, we know that this assumption is reasonably correct just after horizon crossing.

Then $\eta_{\rm in}$ must depend on comoving k (i.e., for different k we should have a suitable $\eta_{\rm in}$). In this case, as
a first approximation, we can set $|\eta_{\rm in}(k)| \approx 1/c_s k$.

Therefore, if we Fourier transform all these variables we obtain

$$\tilde{\psi}_2 \simeq \left[-\frac{8\pi G}{H_\Lambda^2} c_s^2 k^2 \tilde{\Pi}_{2\rm in} + \left(\tilde{\phi}_2 - \frac{1}{4}\tilde{\mathcal{F}}_\chi\right)\right] (c_s k|\eta|)^{3\left(1+c_s^2\right)}$$

For ϕ_2 is const., is it possible to have $\psi_2 \neq 0$?

At super-horizon scales, still assuming ϕ_2 is const. , where we left w and c_s generic

$$\bar{\rho}(\eta) = \bar{\rho}_{\text{in}} \left(\frac{\eta}{\eta_{\text{in}}}\right)^{3(1+w)} = \bar{\rho}_{\text{in}} \left(c_s k|\eta|\right)^{3(1+w)}$$

for ζ_2 we find

$$\tilde{\zeta}_2 - \tilde{\phi}_2 = -\frac{H_\Lambda^2}{4\pi G(1+w)\bar{\rho}}\tilde{\psi}_2 = -\frac{H_\Lambda^2}{4\pi G(1+w)\bar{\rho}_{\text{in}}}\left[-\frac{8\pi G}{H_\Lambda^2}c_s^2 k^2 \tilde{\Pi}_{2\text{in}} + \left(\tilde{\phi}_2 - \frac{1}{4}\tilde{\mathcal{F}}_\chi\right)\right]\left(c_s k|\eta|\right)^{3\left(c_s^2 - w\right)}$$

Now we need to know the value of c_s and $\bar{\rho}_{\text{in}}$ at a given mode k.
It is transparent that

$$w - c_s^2 > 0$$

for the scalar fluctuations to be larger than the tensor ones. This can be achieved by the same gravitons produced during de Sitter phase. For this particle (fluid) radiation, the scalar perturbations will no longer grow [Dvali + 2013, 2017].

This provides a natural route to end inflation!

About Rehating

- At a usual end of inflation, however it came to be, there must be some reheating process which repopulates the observable patch with the standard model matter. In this scenario, the reheating is gravity mediated (since there is no scalar field to couple to the SM).
- Instead, in our scenario, particle production is produced by de Sitter decaying into radiation as postulated in the Mottola, Polyakov, Dvali works...
  This out-of-equilbrium transition to a radiation phase is what provides a replenishment of the patch with SM particles. We can use Kolb & Turner book to be more quantitative. For out of equilibrium decay TRH = $10^{-10}(E/GeV)^{3/2}$ GeV, where E is the scale of inflation. Using the value derived from the level of CMB fluctuations (10^{14-15} GeV) leads to $10^{11} < T_{RH}/GeV < 10^{12}$, which allows for baryogenesis, and it is below the current CMB limits on the energy scale of inflation.
- This will result in a particle density of $10^{90} < n_\chi < 10^{100}$, perfectly consistent with the current number of particles in the horizon of 1090 and entropy generated out of equilibrium between 1−3 ($S_f / S_i = 10^7(E/GeV)^{-1/2}$), which guarantees no extra injection of entropy.

Some final comments (1):

- The power spectrum is nearly scaling invariant;
- We have shown that it is possible to generate nearly scale-invariant scalar adiabatic perturbations in pure de Sitter.
- This is a scenario where the inflaton does not exist, and thus opens up the possibility to provide a picture of inflation that is model independent.

Some final comments (2):

There are some interesting features in our calculation. First, note that the

- scalar fluctuations are generated outside the horizon from the tensor perturbations. Assuming these fluctuations are adiabatic, as in the standard picture of inflation, the perturbations induced in the potential ϕ persist after the transition from the dS phase to a radiation dominated epoch through the decay of dS.

- Similarly to scalar modes, vector perturbations are also generated from second order tensor fluctuations.

- As expected, the fluctuations deviate from Gaussianity and exhibit a certain intrinsic level of non-Gaussian features, the quantification of which will be addressed in a future publication.

Some comments (3):

- However, we note the following: for any non-conformally invariant field (with mass $m \ll H$) the quantum generation of particles from the vacuum state (or quantum depletion of the Bose-Einstein condensate, see Dvali et al.) will add extra contributions to the second-order scalar perturbation mode, hence contributing to a gaussianization by the central limit theorem (see also the point of Dvali et al.).
- In our scenario, the magnitude of fluctuations which rely exclusively on dS features and potential physical mechanisms that enhance scalar perturbations.
- These factors suggest a possible explanation for why the fluctuations are at the 10^{-5} level.

Working progress...

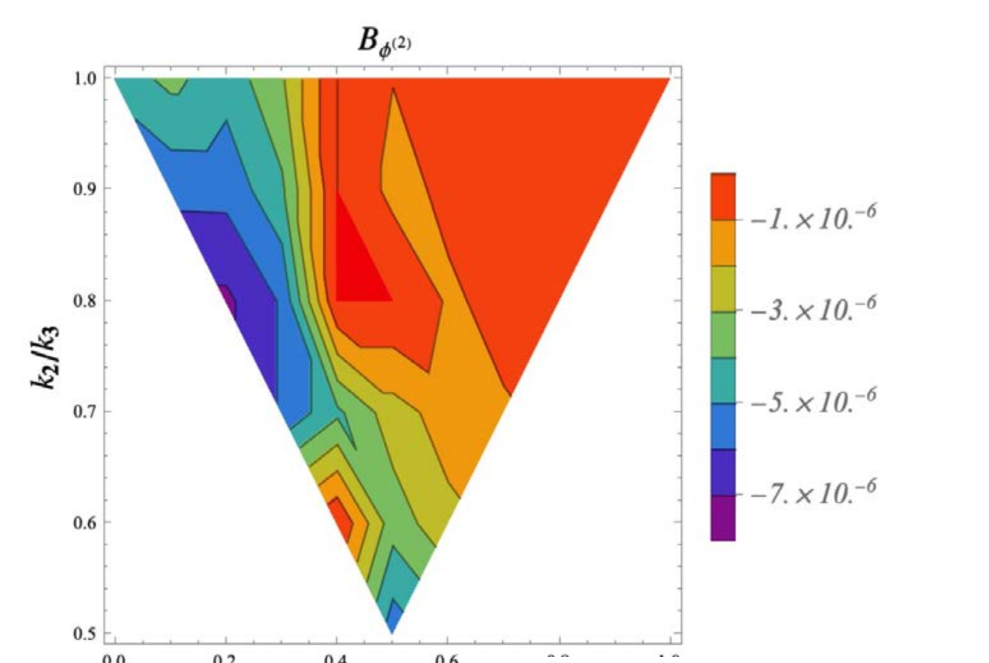
Marisol Traforetti
PhD student at ICC Barcelona

She is completing the account with the full kernel and is calculating the power spectrum on larger scales equal to the horizon until the end of inflation.

$$
\begin{aligned}
\mathcal{P}_\phi(k) = &\frac{1}{(2\pi)^3}\frac{9}{64}\frac{1}{k^8}\int d^3k_1 d^3k_2\, \delta^{(3)}(\mathbf{k}-(\mathbf{k}_1+\mathbf{k}_2))\mathcal{P}_h(k_1)\mathcal{P}_h(k_2) \\
&\left\{\left[3(k_1^2+k_2^2)\mathbf{k}_1\cdot\mathbf{k}_2 + k_1^2k_2^2\left(1+3(\hat{\mathbf{k}}_1\cdot\hat{\mathbf{k}}_2)^2\right)+k_1^4+k_2^4\right]^2\left[\left(1-\hat{\mathbf{k}}_1\cdot\hat{\mathbf{k}}_2\right)^4+\left(1+\hat{\mathbf{k}}_1\cdot\hat{\mathbf{k}}_2\right)^4\right]-\right. \\
&-8\left[3(k_1^2+k_2^2)\mathbf{k}_1\cdot\mathbf{k}_2 + k_1^2k_2^2\left(1+3(\hat{\mathbf{k}}_1\cdot\hat{\mathbf{k}}_2)^2\right)+k_1^4+k_2^4\right](k_1^2+k_2^2+\mathbf{k}_1\cdot\mathbf{k}_2) \\
&\qquad(\mathbf{k}_1\cdot\mathbf{k}_2)(\hat{\mathbf{k}}_1\times\hat{\mathbf{k}}_2)^2\left[1+3(\hat{\mathbf{k}}_1\cdot\hat{\mathbf{k}}_2)^2\right]+ \\
&\left.+8(k_1^2+k_2^2+\mathbf{k}_1\cdot\mathbf{k}_2)^2k_1^2k_2^2(\hat{\mathbf{k}}_1\times\hat{\mathbf{k}}_2)^4\left[1+(\hat{\mathbf{k}}_1\cdot\hat{\mathbf{k}}_2)^2\right]\right\}
\end{aligned}
$$

Mariam Abdelaziz
PhD student in SSM Naples

She is also calculating bispectrum with a view to a more in-depth study of Non-Gaussianity for this type of model.



$$B_{\phi_2} = -\frac{8^3}{k_1^2k_2^2k_3^2}\left(\frac{H_{\rm inf}}{m_{\rm pl}}\right)^6\int d^3p_1\mathcal{K}\frac{1}{p_1^3p_2^3p_3^3}$$

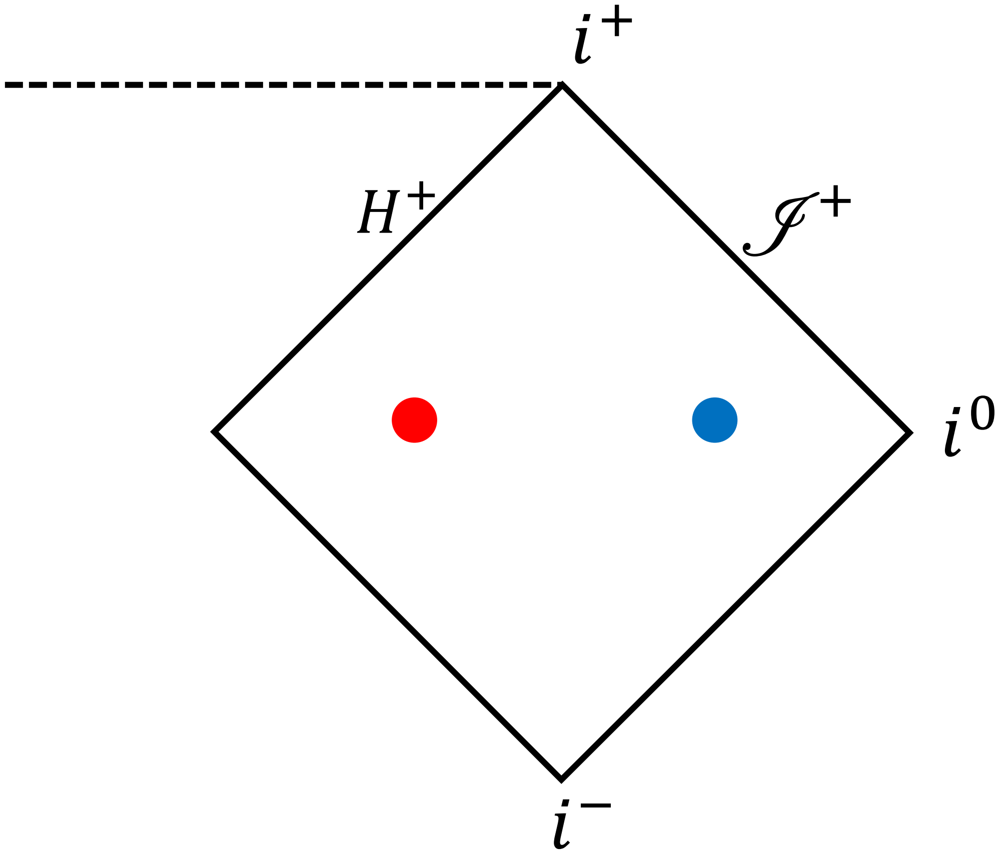
BACKUPS

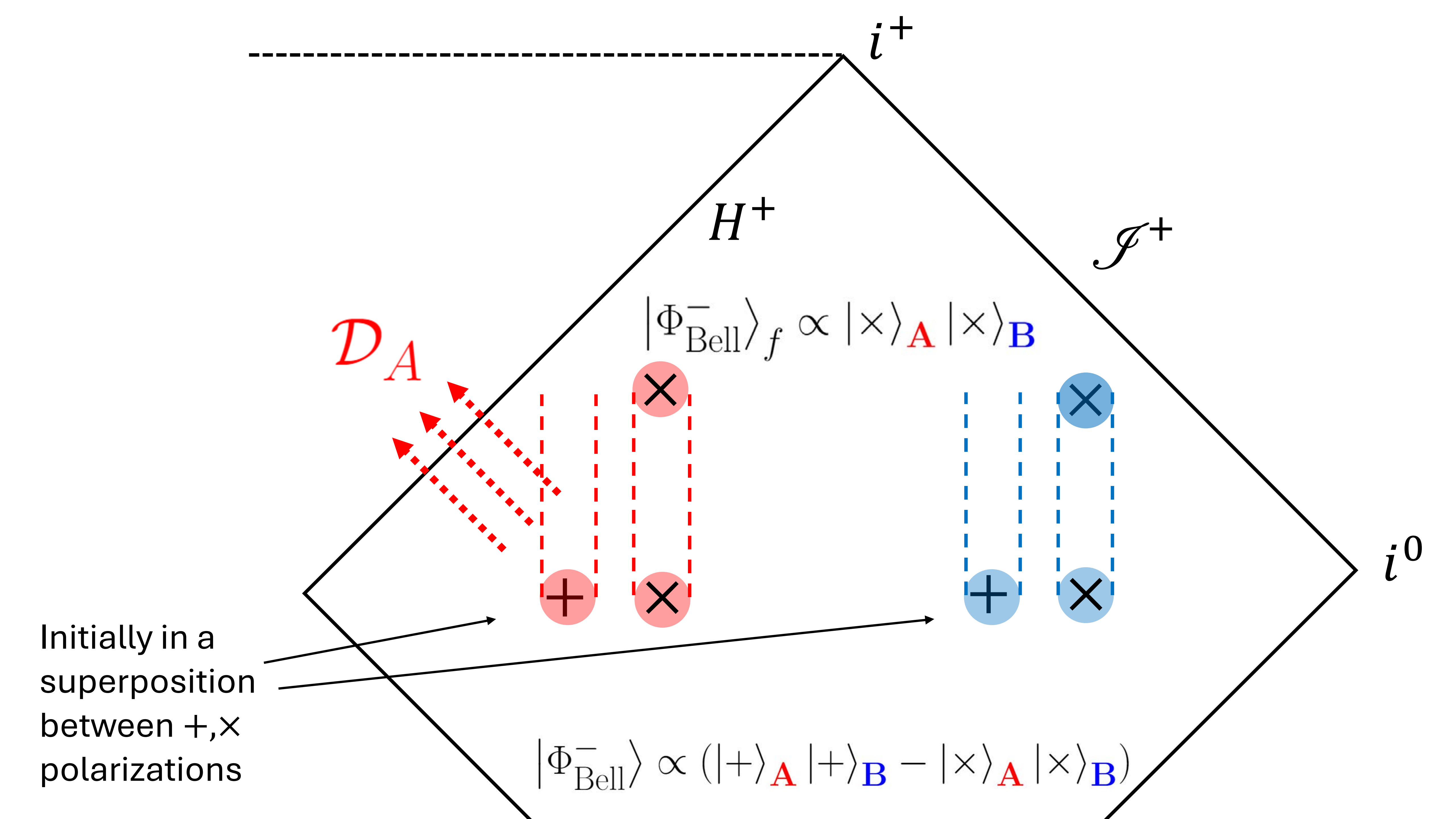
BACKUPS

First part

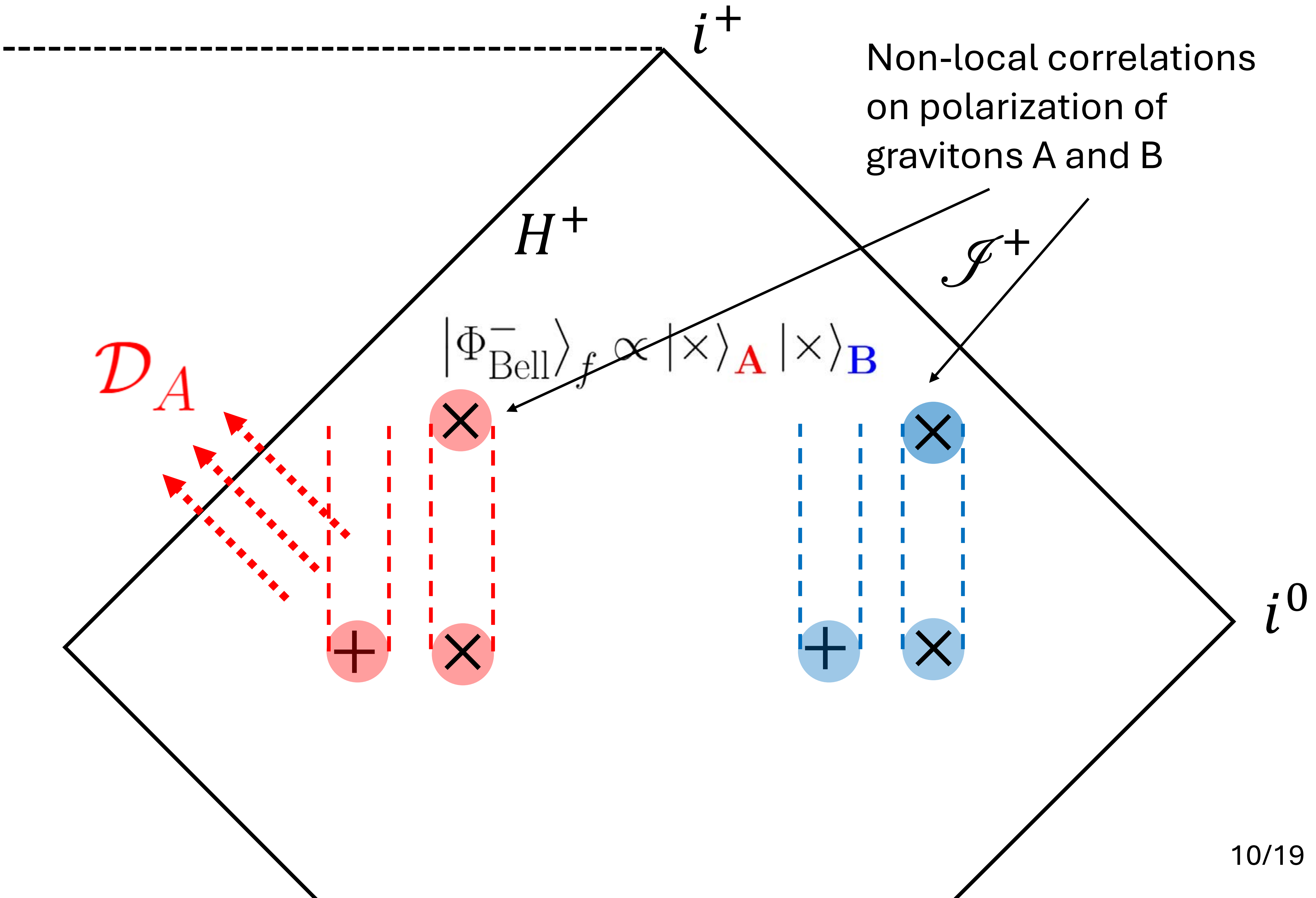
Measure/Decoherence by the Cosmological Horizon

- [R. Wald, *Killing Horizons decohere quantum superpositions*]





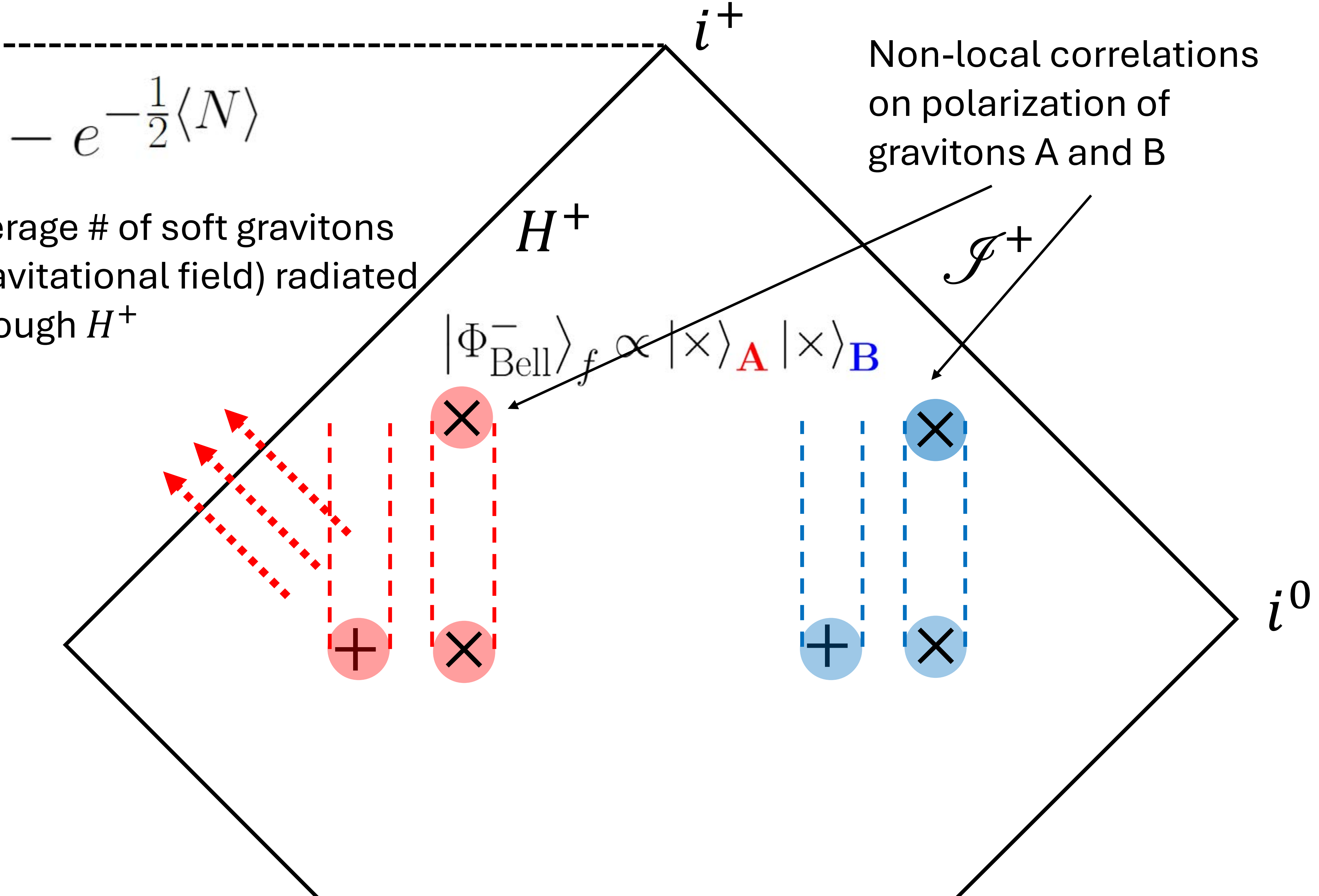
i^+
H^+
$\mathscr{I}^+$
i^0
$\mathcal{D}_A$
$|\Phi^-_{\rm Bell}\rangle_f \propto |\times\rangle_{\mathbf{A}} |\times\rangle_{\mathbf{B}}$
Initially in a superposition between +,× polarizations
$|\Phi^-_{\rm Bell}\rangle \propto (|+\rangle_{\mathbf{A}} |+\rangle_{\mathbf{B}} - |\times\rangle_{\mathbf{A}} |\times\rangle_{\mathbf{B}})$



i^+
Non-local correlations on polarization of gravitons A and B
H^+
$\mathscr{I}^+$
$|\Phi^-_{\rm Bell}\rangle_f \propto |\times\rangle_{\mathbf{A}} |\times\rangle_{\mathbf{B}}$
$\mathcal{D}_A$
i^0

$$\mathcal{D} = 1 - e^{-\frac{1}{2}\langle N \rangle}$$

$\langle N \rangle \equiv$ Average # of soft gravitons (gravitational field) radiated through H^+



$$\mathcal{D} = 1 - e^{-\frac{1}{2}\langle N \rangle}$$

$\langle N \rangle \equiv$ Average # of soft gravitons (gravitational field) radiated through H^+

- It is shown that this same result applies to **Killing horizons**

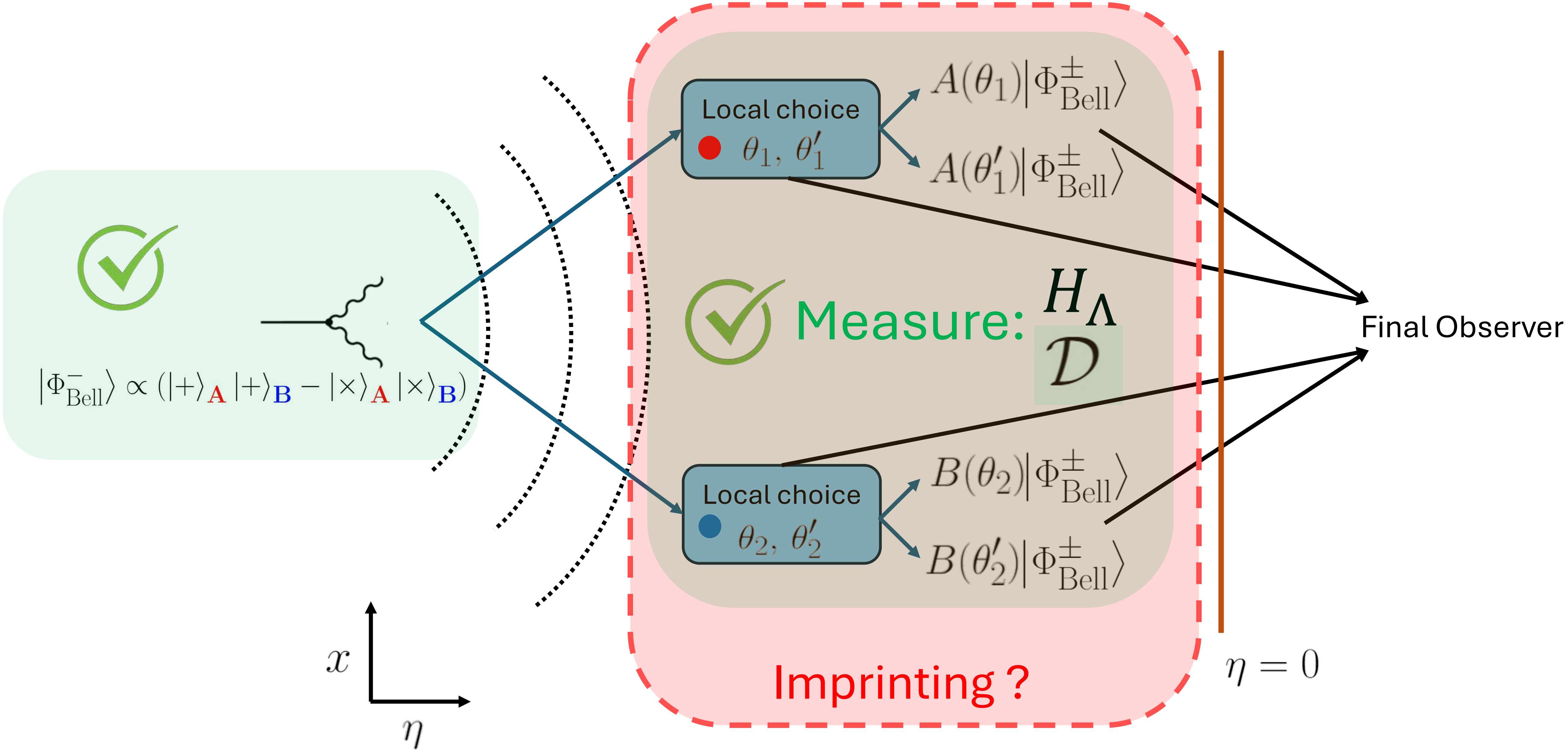
  [D. Danielson, G. Satishchandran, R. Wald, arXiv:2301.00026]

$$\langle N \rangle \sim \frac{m^2 d^4}{R_H^5} T$$

- The cosmological horizon of de Sitter is a Killing horizon – we take the **cosmological horizon** to **perform the measure** and **induce a collapse**

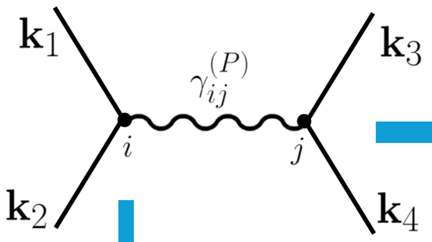
$$\left|\Phi^-_{\rm Bell}\right\rangle \propto \left(|+\rangle_{\mathbf{A}} |+\rangle_{\mathbf{B}} - |\times\rangle_{\mathbf{A}} |\times\rangle_{\mathbf{B}}\right) \xrightarrow[\mathcal{D}]{H_\Lambda} \left|\Phi^-_{\rm Bell}\right\rangle_f \propto |\times\rangle_{\mathbf{A}} |\times\rangle_{\mathbf{B}}$$

Elements of a Bell experiment



Late time observables

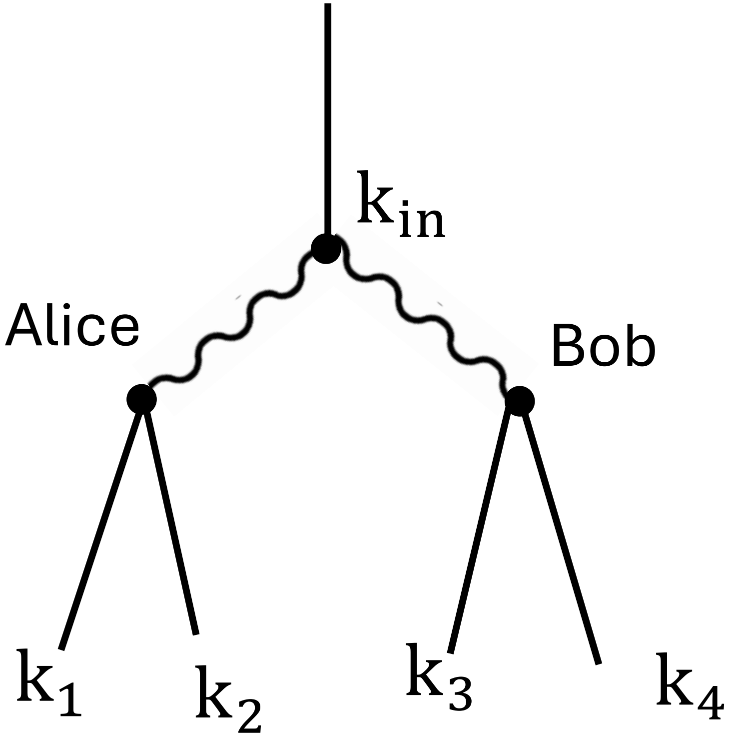
Results get classically transmitted to after inflation through horizon-exit and later deceleration of spacetime, which makes fluctuations re-enter. Then:



$\langle \zeta_{\mathbf{k}_1}\zeta_{\mathbf{k}_2}\zeta_{\mathbf{k}_3}\zeta_{\mathbf{k}_4}\rangle^{\mathrm{GE}}$ can give a measurable signal in the **halo bias** (in the non-Gaussian part of the 2- and 3-point functions of dark matter halos).

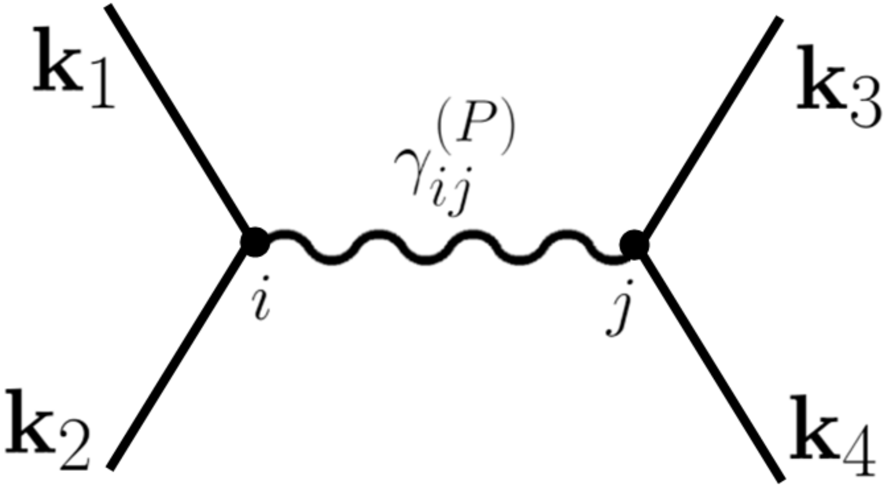
$I^{(3)} \supset \int \mathrm{d}t\mathrm{d}^3\mathbf{x}\, M_p^2 \epsilon_1 a \gamma^{ij}\partial_i\zeta\partial_j\zeta$: Additional derivatives may imprint on **intrinsic alignment** between subhalos in the same patch (i.e. a tidal effect between these two subhalos linked to the polarization of the graviton)

Back del Bakup della prima parte



Imprinting by Graviton Exchange (GE)

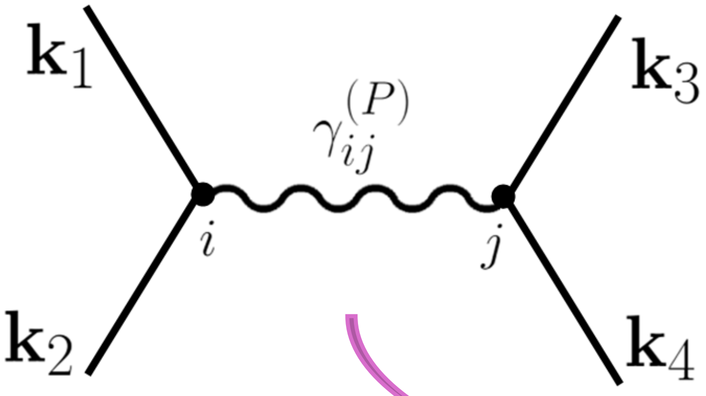
$$I^{(3)} \supset \int \mathrm{d}t\mathrm{d}^3\mathbf{x}\, M_p^2 \epsilon_1 a \gamma^{ij} \partial_i \zeta \partial_j \zeta$$



$$\sim \langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \zeta_{\mathbf{k}_4} \rangle^{\mathrm{GE}} (P)$$

Imprinting by Graviton Exchange (GE)

$$I^{(3)} \supset \int \mathrm{d}t\mathrm{d}^3\mathbf{x}\, M_p^2 \epsilon_1 a \gamma^{ij} \partial_i \zeta \partial_j \zeta$$



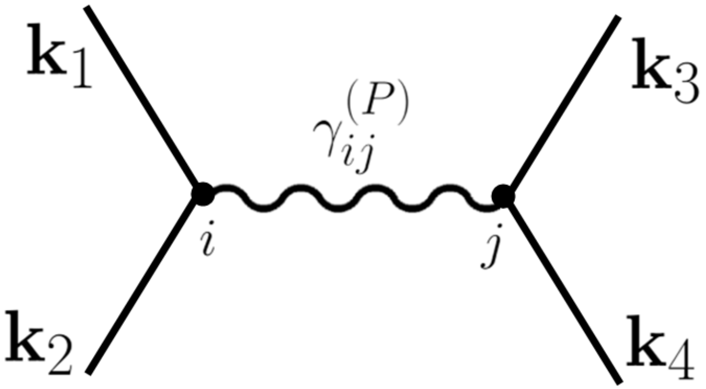
$$\sim \langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \zeta_{\mathbf{k}_4} \rangle^{\mathrm{GE}}(P) \;\propto\; \sum_{P=+,\times} \epsilon_{ij}^{P}(\mathbf{k}_{12}) k_1^i k_2^j \, \epsilon_{lm}^{P}(\mathbf{k}_{34}) k_3^l k_4^m$$

$$\mathbf{k}_{12} \equiv \mathbf{k}_1 + \mathbf{k}_2$$
$$\mathbf{k}_{34} \equiv \mathbf{k}_3 + \mathbf{k}_4$$

This interaction is **sensitive** to the **polarization** of the **graviton**(s).
The **collapse** reduces the **sum** to **1 term** only .

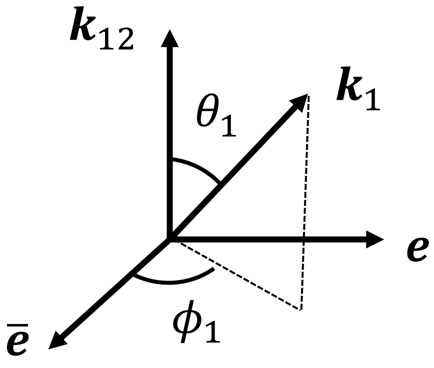
Imprinting by Graviton Exchange (GE)

$$I^{(3)} \supset \int \mathrm{d}t\mathrm{d}^3\mathbf{x}\, M_p^2 \epsilon_1 a \gamma^{ij} \partial_i \zeta \partial_j \zeta$$

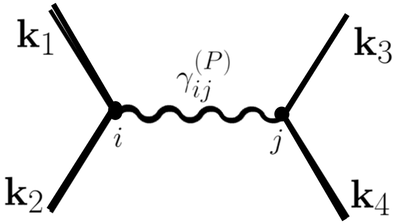


$$\sim \langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \zeta_{\mathbf{k}_4} \rangle^{\mathrm{GE}}(P) \propto \sum_{P=+,\times} \underbrace{\epsilon_{ij}^P(\phi_1) k_1^i k_2^j}_{A(\phi_1)} \underbrace{\epsilon_{lm}^P(\phi_3) k_3^l k_4^m}_{B(\phi_3)}$$

$$\mathbf{k}_{12} \equiv \mathbf{k}_1 + \mathbf{k}_2$$
$$\mathbf{k}_{34} \equiv \mathbf{k}_3 + \mathbf{k}_4$$



Bell observable S^{QM}



$$S^{QM} = \langle A(\phi_1)B(\phi_3)\rangle + \langle A(\phi_1{}')B(\phi_3)\rangle + \langle A(\phi_1)B(\phi_3{}')\rangle - \langle A(\phi_1{}')B(\phi_3{}')\rangle$$

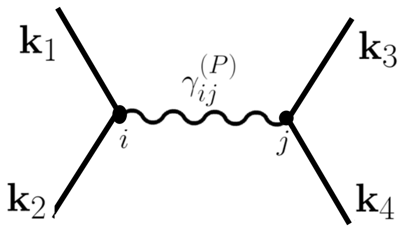
$$= \left\langle \epsilon_{ij}^{P}(\phi_1^{A})k_1^{i}k_2^{j}\,\epsilon_{lm}^{P}(\phi_3^{B})k_3^{l}k_4^{m}\right\rangle + \left\langle \epsilon_{ij}^{P}(\phi_1^{A'})k_1^{i}{}'k_2^{j}{}'\,\epsilon_{lm}^{P}(\phi_3^{B})k_3^{l}k_4^{m}\right\rangle +$$

$$+ \left\langle \epsilon_{ij}^{P}(\phi_1^{A})k_1^{i}k_2^{j}\,\epsilon_{lm}^{P}(\phi_3^{B'})k_3^{l}{}'k_4^{m}{}'\right\rangle - \left\langle \epsilon_{ij}^{P}(\phi_1^{A'})k_1^{i}{}'k_2^{j}{}'\,\epsilon_{lm}^{P}(\phi_3^{B'})k_3^{l}{}'k_4^{m}{}'\right\rangle$$

since:

$A(\theta) = \epsilon_{ij}^{P_A}(\phi_1^{A})k_1^{i}k_2^{j}$; $B(\phi) = \epsilon_{lm}^{P_B}(\phi_3^{B})k_3^{l}k_4^{m}$, and $P_A = P_B = P_{A'} = P_{B'} \equiv P$

Bell observable S^{QM}



$$S^{QM} = \langle A(\phi_1)B(\phi_3)\rangle + \langle A(\phi_1')B(\phi_3)\rangle + \langle A(\phi_1)B(\phi_3')\rangle - \langle A(\phi_1')B(\phi_3')\rangle$$

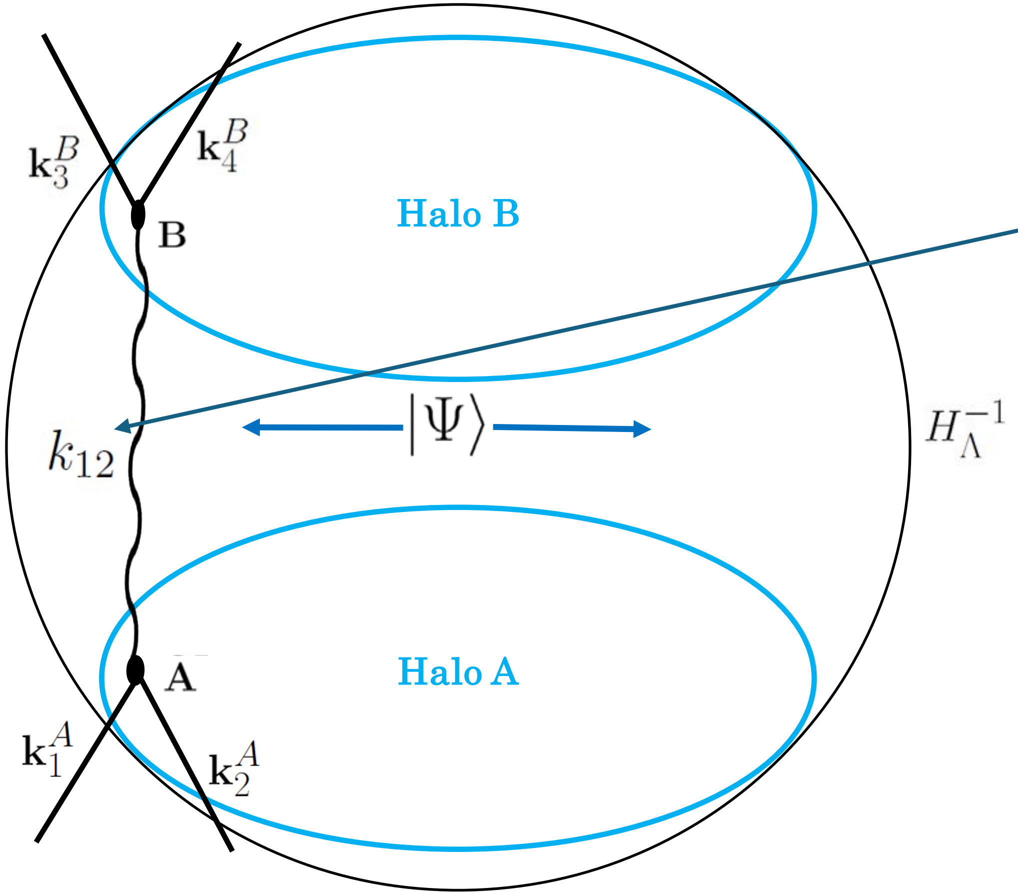
$$\propto \left\langle \langle \zeta_{\mathbf{k}_1}\zeta_{\mathbf{k}_2}\rangle^{A} \langle \zeta_{\mathbf{k}_3}\zeta_{\mathbf{k}_4}\rangle^{B} \right\rangle^{\mathrm{GE}} + \left\langle \langle \zeta_{\mathbf{k}_1}\zeta_{\mathbf{k}_2}\rangle^{A'} \langle \zeta_{\mathbf{k}_3}\zeta_{\mathbf{k}_4}\rangle^{B} \right\rangle^{\mathrm{GE}} +$$

$$+ \left\langle \langle \zeta_{\mathbf{k}_1}\zeta_{\mathbf{k}_2}\rangle^{A} \langle \zeta_{\mathbf{k}_3}\zeta_{\mathbf{k}_4}\rangle^{B'} \right\rangle^{\mathrm{GE}} - \left\langle \langle \zeta_{\mathbf{k}_1}\zeta_{\mathbf{k}_2}\rangle^{A'} \langle \zeta_{\mathbf{k}_3}\zeta_{\mathbf{k}_4}\rangle^{B'} \right\rangle^{\mathrm{GE}}$$

Interpretation?

Bell observable S^{QM}

$$S^{QM} = \langle A(\phi_1)B(\phi_3) \rangle + \langle A(\phi_1')B(\phi_3) \rangle + \langle A(\phi_1)B(\phi_3') \rangle - \langle A(\phi_1')B(\phi_3') \rangle$$

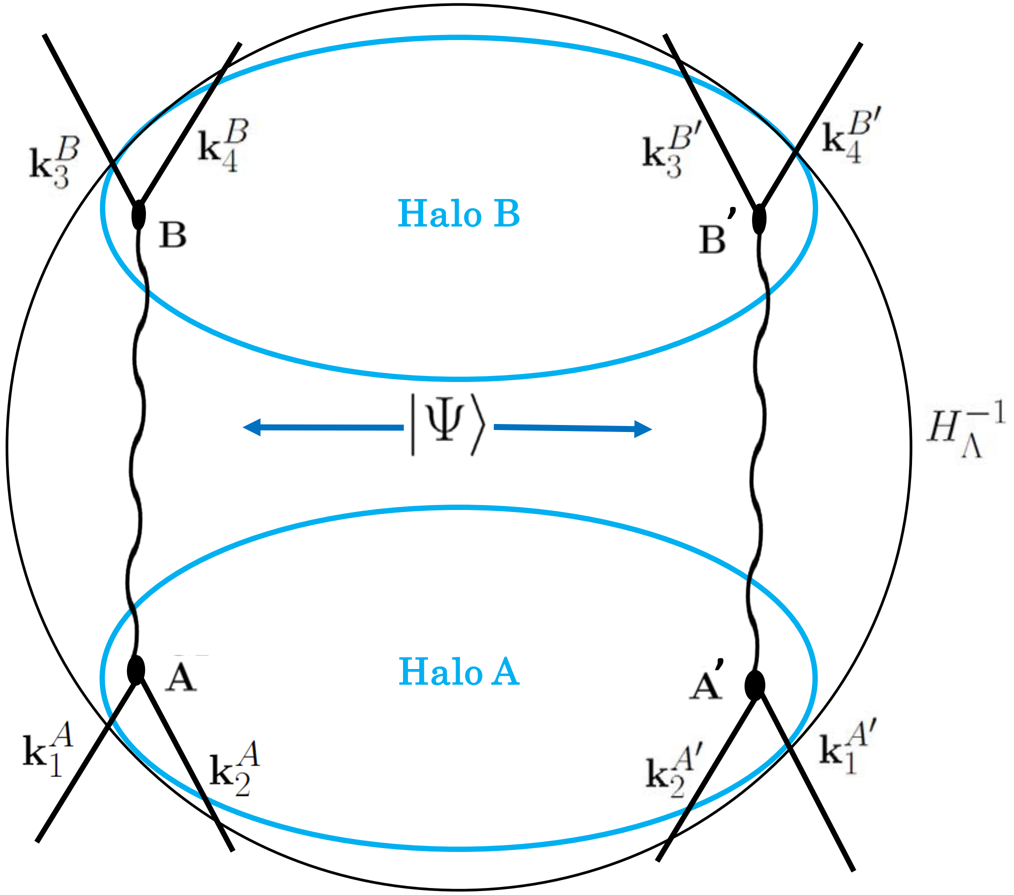


$$k_{12} \ll |\mathbf{k}_1| \approx |\mathbf{k}_2|$$

The momentum of the exchanged graviton will set the distance (inverse) between the 2 halos A and B

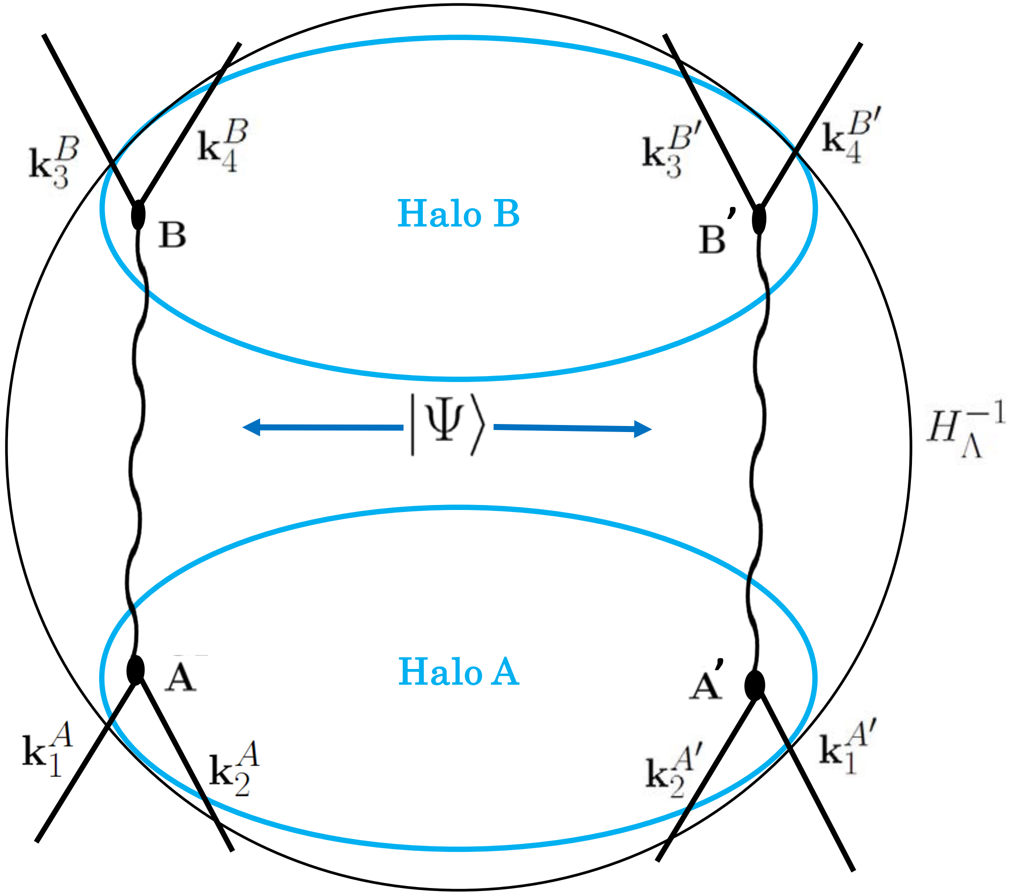
Bell observable S^{QM}

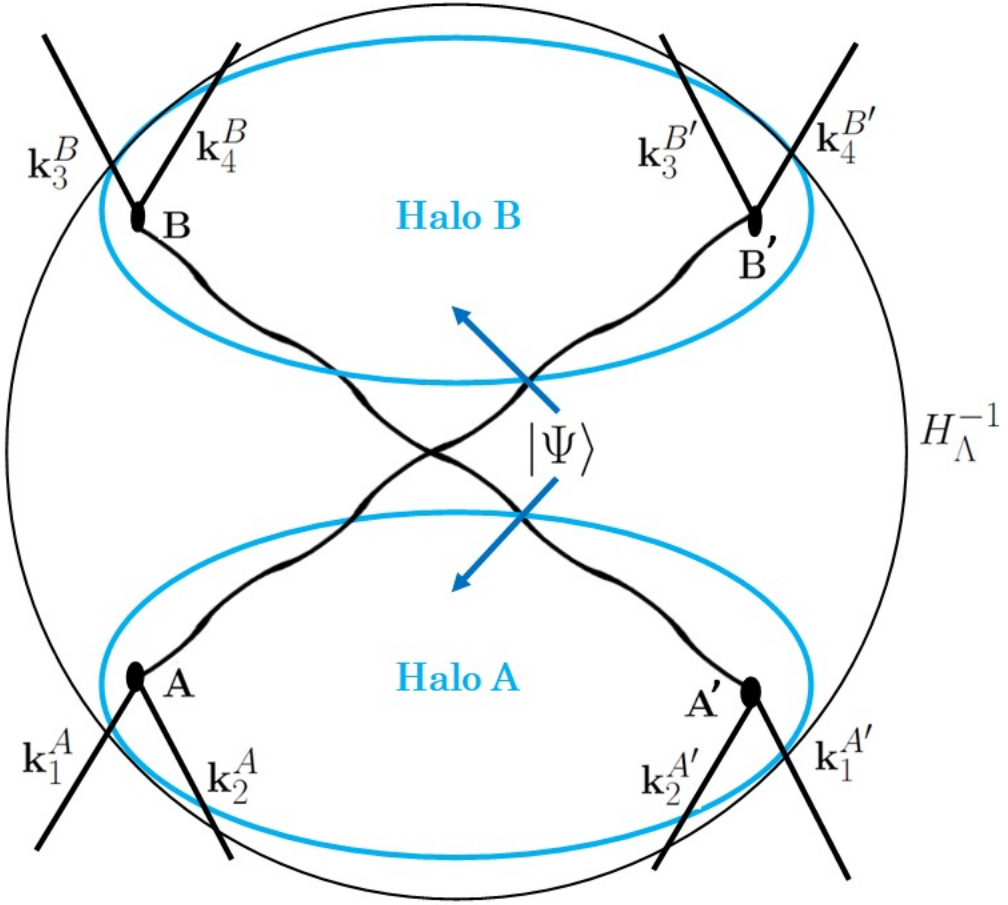
$$S^{QM} = \langle A(\phi_1)B(\phi_3) \rangle + \langle A(\phi_1{}')B(\phi_3) \rangle + \langle A(\phi_1)B(\phi_3{}') \rangle - \langle A(\phi_1{}')B(\phi_3{}') \rangle$$



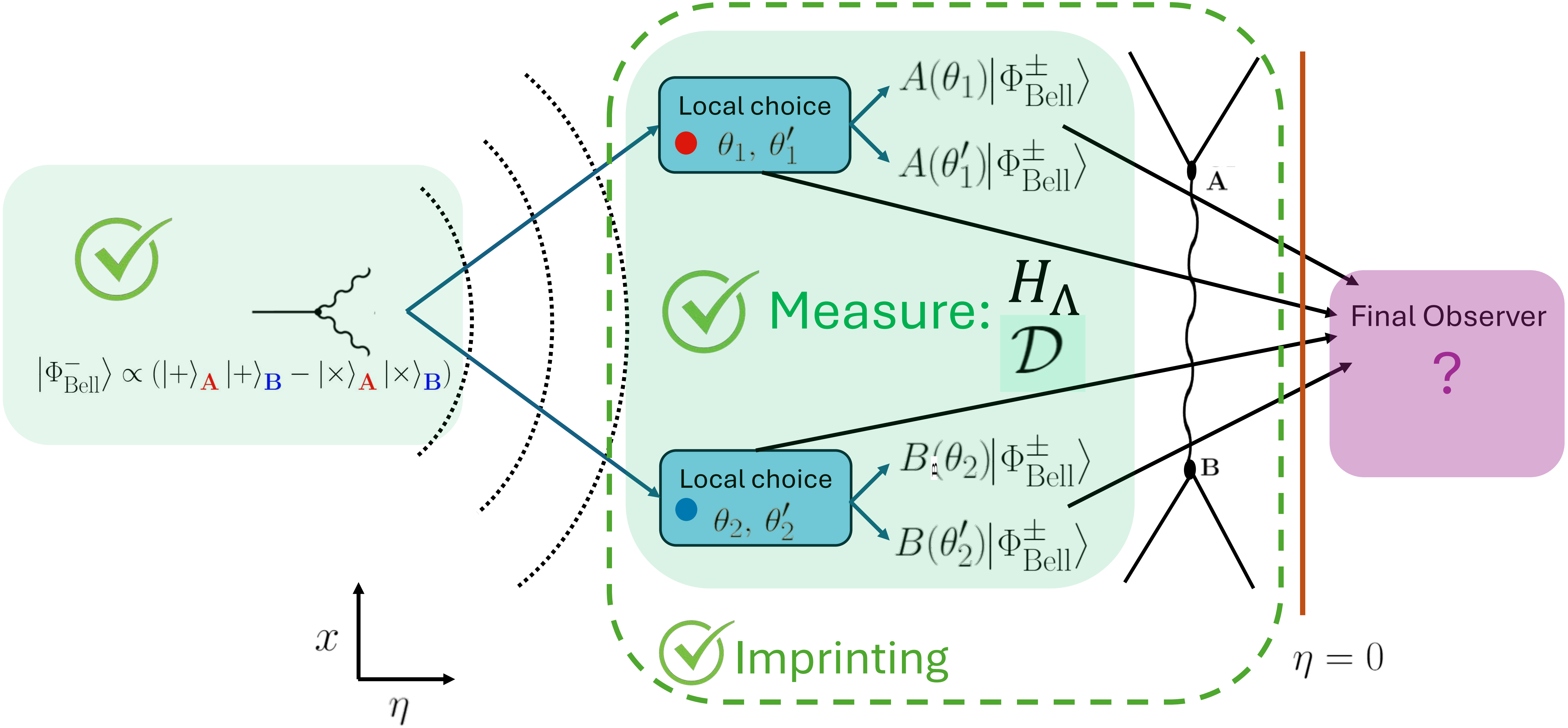
Bell observable S^{QM}

$$S^{QM} = \langle A(\phi_1)B(\phi_3)\rangle + \langle A(\phi_1{}')B(\phi_3)\rangle + \langle A(\phi_1)B(\phi_3{}')\rangle - \langle A(\phi_1{}')B(\phi_3{}')\rangle$$



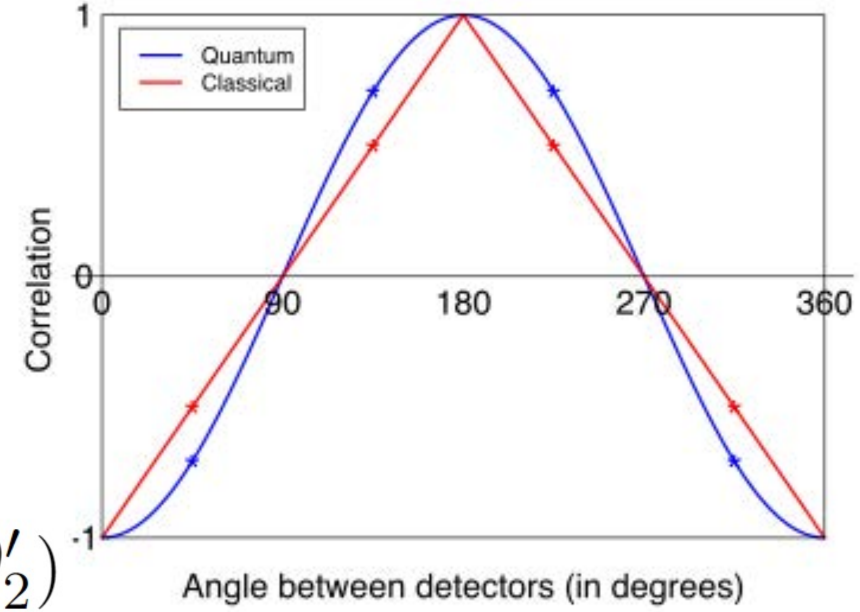


Elements of a Bell experiment



Bell Experiment and Inequalities

$$C(\theta_1, \theta_2) = \langle A(\theta_1)\, B(\theta_2) \rangle$$



$$S = C(\theta_1, \theta_2) + C(\theta_1', \theta_2) + C(\theta_1, \theta_2') - C(\theta_1', \theta_2')$$

$$S^{\mathrm{QM}} \leq 2\sqrt{2}$$

$$S^{\text{local hidden variables}} \leq 2$$

Hamiltonian for creation of entangled state

$$H_{\rm int}(\eta) = -\frac{M_p^3\epsilon_1 a^2}{8}\int d^3\mathbf{x}\, a\epsilon_1\zeta(\eta,\mathbf{x}) \otimes \partial_l\gamma_{ij}(\eta,\mathbf{x})\partial_l\gamma_{ij}(\eta,\mathbf{x})$$

$$\begin{aligned}
H_T^{(3)}(\mathbf{k}=-\mathbf{q}) &\sim \sum_{P,P'=+,\times}\int \frac{d^3\mathbf{k}\,d^3\mathbf{q}}{(2\pi)^3}\,(\mathbf{k}\cdot\mathbf{q})\,e^{i(\mathbf{k}+\mathbf{q})\cdot\mathbf{x}}\,\epsilon_{ij}^P(\mathbf{k})\epsilon_{ij}^{P'}(\mathbf{q})\,\left\{\text{combination of } b\,,b^\dagger\right\}\delta(\mathbf{k}+\mathbf{q}) \\
&\sim \int \frac{d^3\mathbf{k}}{(2\pi)^3}\,k^2\,e^{2i\mathbf{k}\cdot\mathbf{x}}\cdot\left\{\left[\hat{b}^+_{\mathbf{k}}\hat{b}^+_{-\mathbf{k}} - \hat{b}^\times_{\mathbf{k}}\hat{b}^\times_{-\mathbf{k}}\right](u_k)^2 + \right. \\
&\qquad + \left[\left(\hat{b}^+_{\mathbf{k}}\right)^\dagger\hat{b}^+_{\mathbf{k}} + \left(\hat{b}^\times_{\mathbf{k}}\right)^\dagger\hat{b}^\times_{\mathbf{k}} + \left(\hat{b}^+_{-\mathbf{k}}\right)^\dagger\hat{b}^+_{-\mathbf{k}} + \left(\hat{b}^\times_{-\mathbf{k}}\right)^\dagger\hat{b}^\times_{-\mathbf{k}}\right]|u_k|^2 + \\
&\qquad \left. + \left[\left(\hat{b}^+_{-\mathbf{k}}\right)^\dagger\left(\hat{b}^+_{\mathbf{k}}\right)^\dagger - \left(\hat{b}^\times_{-\mathbf{k}}\right)^\dagger\left(\hat{b}^\times_{\mathbf{k}}\right)^\dagger\right](u_k^*)^2\right\}
\end{aligned}$$

$$H_{\rm int}^{(3)} \supset \int \frac{d^3\mathbf{x}\,d^3\mathbf{k}}{(2\pi)^{3/2}}\,\frac{e^{2i\mathbf{k}\cdot\mathbf{x}}}{(2\pi)^3}\,k^2\,\hat{a}_{\mathbf{0}}\otimes\left[\left(\hat{b}^+_{-\mathbf{k}}\right)^\dagger\left(\hat{b}^+_{\mathbf{k}}\right)^\dagger - \left(\hat{b}^\times_{-\mathbf{k}}\right)^\dagger\left(\hat{b}^\times_{\mathbf{k}}\right)^\dagger\right](u_k^*)^2$$

Creation of entangled state – Schrödinger'sº Equation

- Parametric approximation: $\hat{a}_{\mathbf{p}} \approx \alpha e^{-i\omega_P \Delta t}$
- Interaction picture: $H_S = H_{0,S} + H_{1,S}$
- Expanding the evolution operator:

$$H_{0,I} = H_{0,S} = \alpha e^{-i\omega_P} \sum_{P=+,\times} \left[\left(\hat{b}^P_{\mathbf{k}}\right)^\dagger \hat{b}^P_{\mathbf{k}} + \left(\hat{b}^P_{-\mathbf{k}}\right)^\dagger \hat{b}^P_{-\mathbf{k}} \right] \frac{1}{k} \ ,$$

$$H_{1,I} = e^{i\omega\Delta t} \cdot H_{1,S} \cdot e^{-i\omega\Delta t}$$

$$= \alpha \left[\left(\hat{b}^+_{-\mathbf{k}}\right)^\dagger \left(\hat{b}^+_{\mathbf{k}}\right)^\dagger - \left(\hat{b}^\times_{-\mathbf{k}}\right)^\dagger \left(\hat{b}^\times_{\mathbf{k}}\right)^\dagger \right] e^{-i(\omega_P - 2\omega_k)\Delta t} \left[\frac{1}{k} e^{-2ik\eta} \right]$$

$$|\Psi(t)\rangle = |\alpha\rangle \otimes \left[\left(1 - \frac{\alpha^2 \Delta t^2}{k^2 \hbar^2} |0\rangle_{1,+} |0\rangle_{1,\times} \otimes |0\rangle_{2,+} |0\rangle_{2,\times} \right) - \right.$$
$$\left. - \frac{i\alpha\Delta t}{k\hbar} \left(|1\rangle_{+,1} |0\rangle_{\times,1} \otimes |1\rangle_{+,2} |0\rangle_{\times,2} - |0\rangle_{+,1} |1\rangle_{\times,1} \otimes |0\rangle_{+,2} |1\rangle_{\times,2} \right) \right]$$

$$= |\alpha\rangle \otimes \left[\left(1 - \frac{\alpha^2 \Delta t^2}{k^2 \hbar^2} |0\rangle^\gamma \right) - \frac{i\alpha\Delta t}{k\hbar} \left(|+\rangle_1 |+\rangle_2 - |\times\rangle_1 |\times\rangle_2 \right) \right]$$

Decoherence by Killing horizon

$$\mathcal{D} = 1 - e^{-\frac{1}{2}\langle N \rangle}$$

$\langle N \rangle \equiv$ Average # of soft gravitons (gravitational field) radiated through H^+

- For Killing horizons: $\langle N \rangle \sim \dfrac{m^2 d^4}{R_H^5} T$

- Time of decoherence compared to time scale of GE:

$$T_D \sim \frac{R_H^5}{Gkm^2 d^4} = \frac{R_H^5}{d^4}\frac{M_{\rm Pl}^2}{m^2}$$

$$T_D \sim t_{\rm GE} \sim \frac{1}{M_{\rm Pl}}$$

$$k_D \sim \left(\frac{m^2}{M_{\rm Pl}\, R_H^5}\right)^{1/4} \sim \left(\frac{m^2}{M_{\rm Pl}}\right)^{1/4} H_\Lambda^{5/4}$$

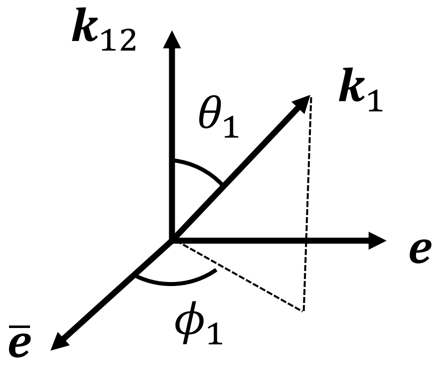
More on GE and Observable definition

$$A(\theta) = \epsilon_{ij}^{P_A}(\phi_1^A) k_1^i k_2^j \quad ; \quad B(\phi) = \epsilon_{lm}^{P_B}(\phi_3^B) k_3^l k_4^m \quad ; \quad P_A = P_B = P_{A'} = P_{B'} \equiv P$$

$$\epsilon_{ij}^{P}(\mathbf{k}_{ab}) k_a^i k_b^j = \mathcal{N} \cos(2\phi_a - \delta_P) \quad ; \quad \text{where} \quad \delta_P = \begin{cases} 0 & \text{if } P = + \\ \frac{\pi}{2} & \text{if } P = \times \end{cases}$$

$$k_{1,3} \sin\theta_{1,3} = k_{2,4} \sin\theta_{2,4}$$

$$\phi_{2,4} = \phi_{1,3} + \pi$$



More on Bell Inequiality Violation

$$|\Phi^-_{\text{Bell}}\rangle \propto (|+\rangle_{\mathbf{A}}|+\rangle_{\mathbf{B}} - |\times\rangle_{\mathbf{A}}|\times\rangle_{\mathbf{B}})$$

$$\boxed{\begin{aligned} |+\rangle_1 &= \cos(2\theta)|\theta\rangle_1 - \sin(2\theta)|\theta^\perp\rangle_1 \\ |\times\rangle_1 &= \sin(2\theta)|\theta\rangle_1 + \cos(2\theta)|\theta^\perp\rangle_1 \\ |+\rangle_2 &= \cos(2\phi)|\phi\rangle_2 - \sin(2\phi)|\phi^\perp\rangle_2 \\ |\times\rangle_2 &= \sin(2\phi)|\phi\rangle_2 + \cos(2\phi)|\phi^\perp\rangle_2 \end{aligned}} + \boxed{\begin{array}{l} A(\theta) = +1 \text{ for state } |\theta\rangle \\ A(\theta) = -1 \text{ for state } |\theta^\perp\rangle \\ (\text{idem for } B(\phi) = \pm 1), \end{array}} \Rightarrow C(\theta,\phi) = \cos[4(\theta+\phi)]$$

$$\theta^\perp = \theta + \pi/4$$

$$\tilde{A}(\theta) = \frac{A(\phi_1^A)}{\mathcal{N}} = \cos(4\theta - \delta_P)$$

$$\phi_1^A = 2\theta \text{ and } \phi_3^B = 2\phi$$

$$\phi_1^A = \phi_3^B = 0 \text{ and } \phi_1^{A'} = \phi_3^{B'} = \pi/16,$$

$$S^{\text{QM}} > 2 = \max\left(S^{\text{local hidden variables}}\right)$$

More specifications of S^{QM}

$$\lim_{\substack{k_{12}\to 0\\ k_{34}\to 0}} \langle \zeta_{\mathbf{k}_1}\zeta_{\mathbf{k}_2}\zeta_{\mathbf{k}_3}\zeta_{\mathbf{k}_4}\rangle^{\mathrm{GE}} = \langle\langle \zeta_{\mathbf{k}_1}\zeta_{\mathbf{k}_2}\rangle \langle \zeta_{\mathbf{k}_3}\zeta_{\mathbf{k}_4}\rangle\rangle^{\mathrm{GE}}$$

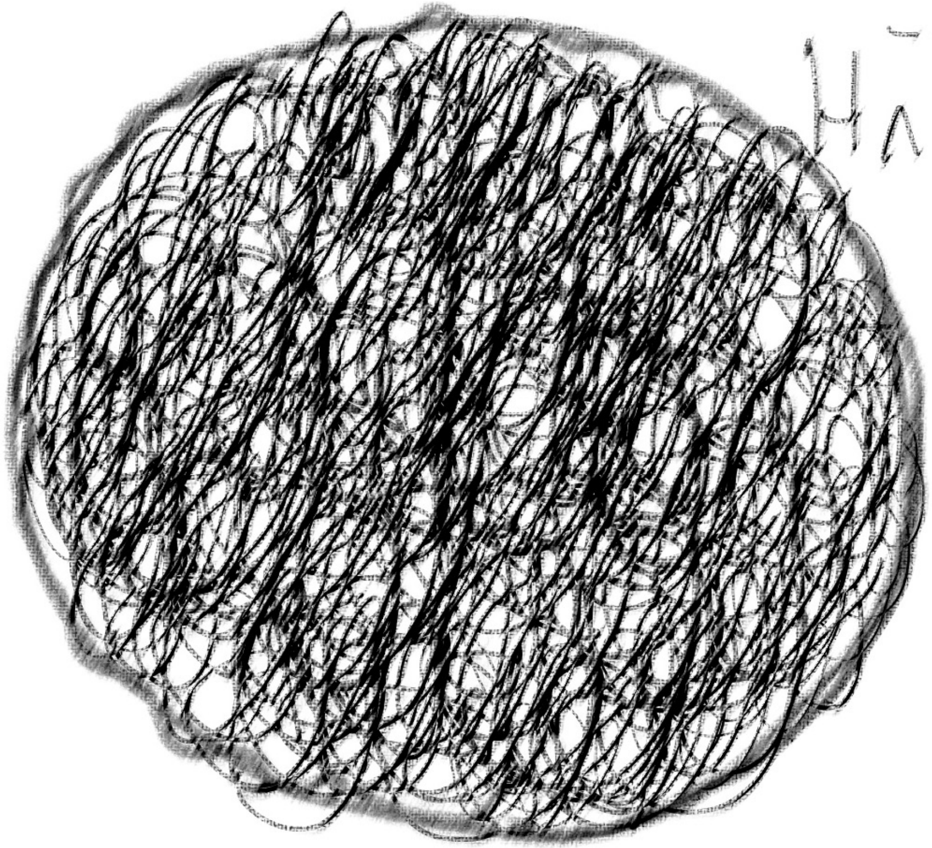
$$S^{\mathrm{QM}} \propto \langle \zeta^A_{\mathbf{k}_1}\zeta^A_{\mathbf{k}_2}\zeta^B_{\mathbf{k}_3}\zeta^B_{\mathbf{k}_4}\rangle^{\mathrm{GE}} + \left\langle \zeta^{A'}_{\mathbf{k}_1}\zeta^{A'}_{\mathbf{k}_2}\zeta^B_{\mathbf{k}_3}\zeta^B_{\mathbf{k}_4}\right\rangle^{\mathrm{GE}} + \left\langle \zeta^A_{\mathbf{k}_1}\zeta^A_{\mathbf{k}_2}\zeta^{B'}_{\mathbf{k}_3}\zeta^{B'}_{\mathbf{k}_4}\right\rangle^{\mathrm{GE}} - \left\langle \zeta^{A'}_{\mathbf{k}_1}\zeta^{A'}_{\mathbf{k}_2}\zeta^{B'}_{\mathbf{k}_3}\zeta^{B'}_{\mathbf{k}_4}\right\rangle^{\mathrm{GE}}$$

$$= \left\langle \langle \zeta_{\mathbf{k}_1}\zeta_{\mathbf{k}_2}\rangle^A \langle \zeta_{\mathbf{k}_3}\zeta_{\mathbf{k}_4}\rangle^B \right\rangle^{\mathrm{GE}} + \left\langle \langle \zeta_{\mathbf{k}_1}\zeta_{\mathbf{k}_2}\rangle^{A'} \langle \zeta_{\mathbf{k}_3}\zeta_{\mathbf{k}_4}\rangle^B \right\rangle^{\mathrm{GE}} +$$

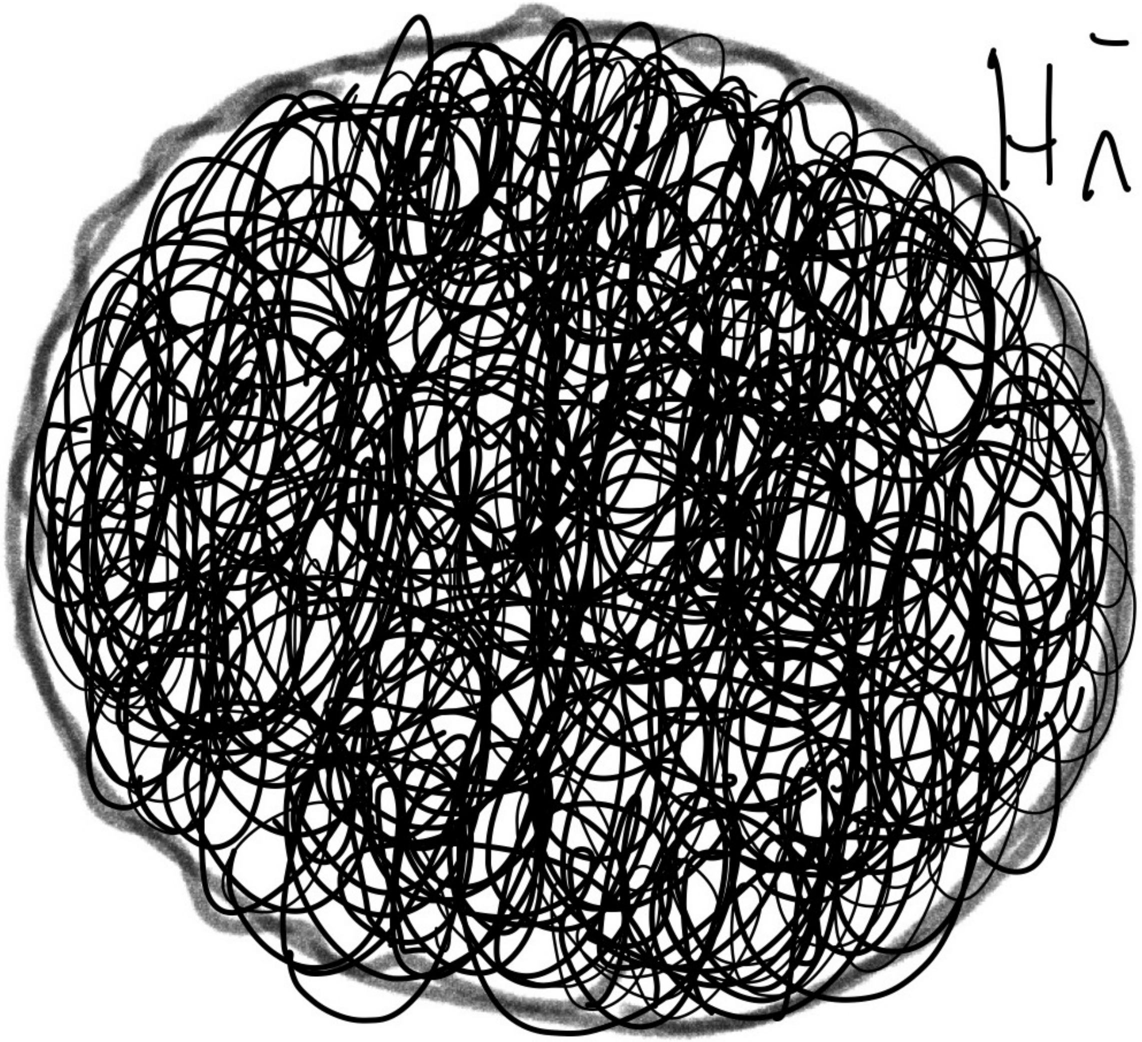
$$+ \left\langle \langle \zeta_{\mathbf{k}_1}\zeta_{\mathbf{k}_2}\rangle^A \langle \zeta_{\mathbf{k}_3}\zeta_{\mathbf{k}_4}\rangle^{B'} \right\rangle^{\mathrm{GE}} - \left\langle \langle \zeta_{\mathbf{k}_1}\zeta_{\mathbf{k}_2}\rangle^{A'} \langle \zeta_{\mathbf{k}_3}\zeta_{\mathbf{k}_4}\rangle^{B'} \right\rangle^{\mathrm{GE}}$$

BACKUPS

Second PART



H_Λ^{-1}



H_{Λ}^{-1}