SOME ISSUES WITH GRAVITINOS IN HIGH-SCALE SUSY MODELS

Based on :

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+ work in progress with Quentin Bonnefoy (DESY-TH) and
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E. Dudas – CNRS and E. Polytechnique + CERN-TH

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Outline

1) The swampland program
- Spin 3/2, potential problems

2) Gravitino sound speed in supergravity

3) Eqs. for the longitudinal gravitino, results

4) Causality and positivity bounds, a gravitino swampland conjecture

5) Conclusions

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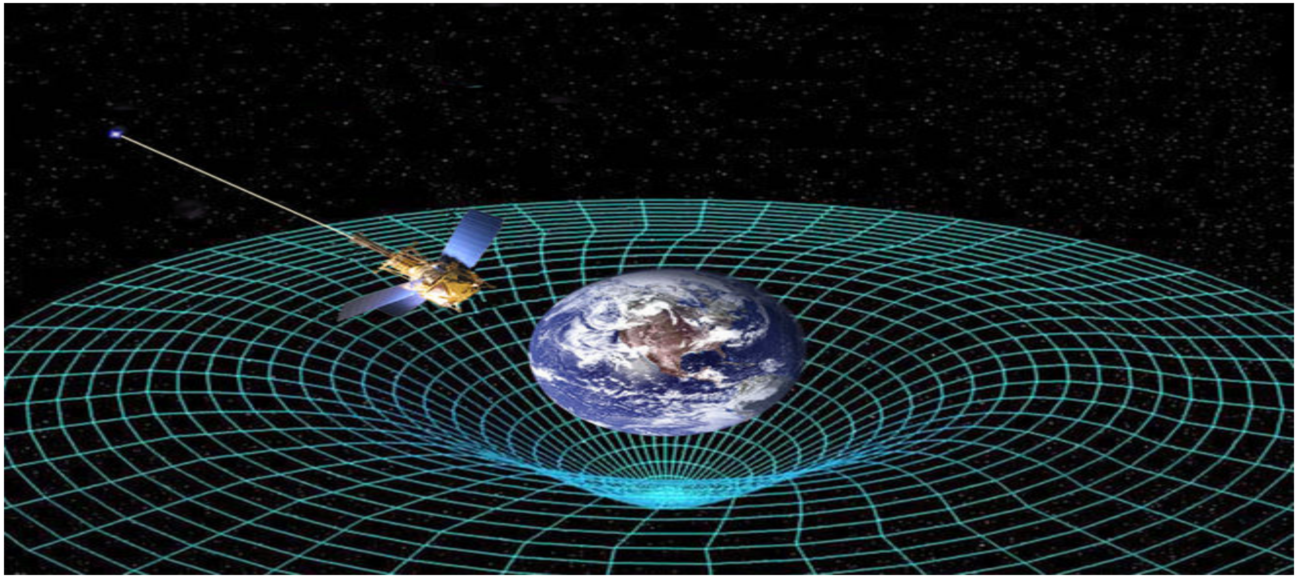
Einstein general relativity is a classical theory $g_{\mu\nu}$

Mass/energy $\longrightarrow$ spacetime geometry



Its quantization $g_{\mu\nu} = \eta_{\mu\nu} + \frac{1}{M_P} h_{\mu\nu}$ leads to UV divergences which cannot be reabsorbed in a finite number of parameters $\longrightarrow$ non-renormalizable

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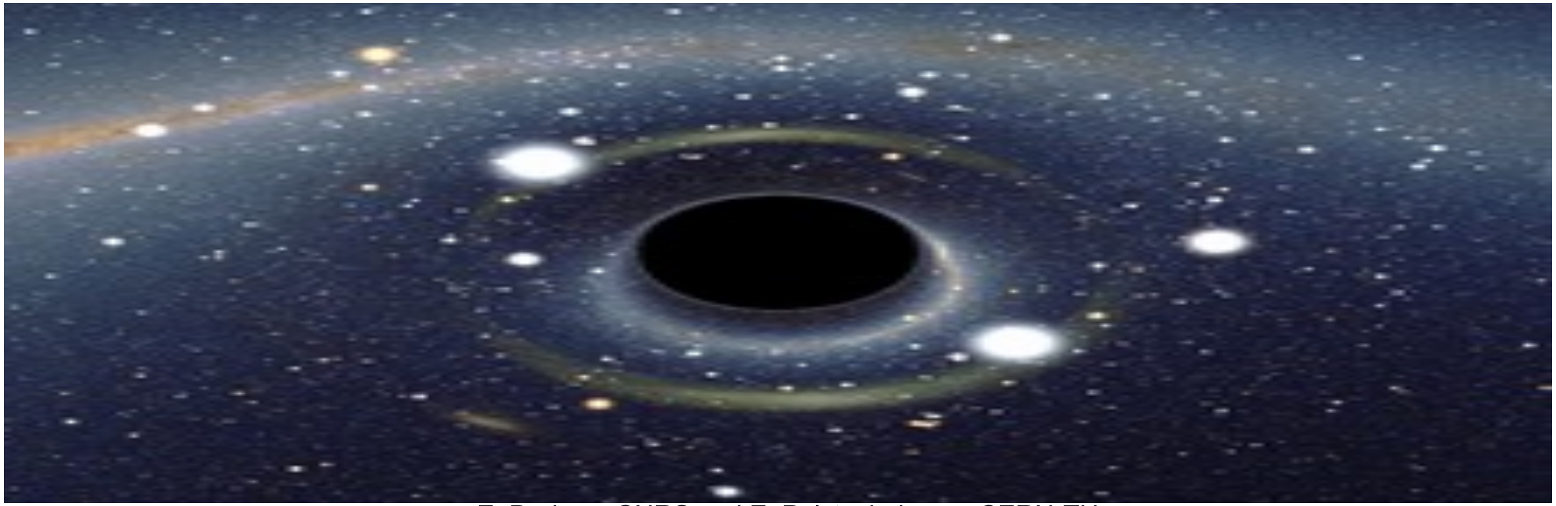
The coupling of gravitational interaction is

$$\frac{E}{M_P}$$

→ negligible quantum corrections at low energy.

At high-energies $E \sim M_P$ or in strong gravity fields, theory of quantum gravity is necessary.



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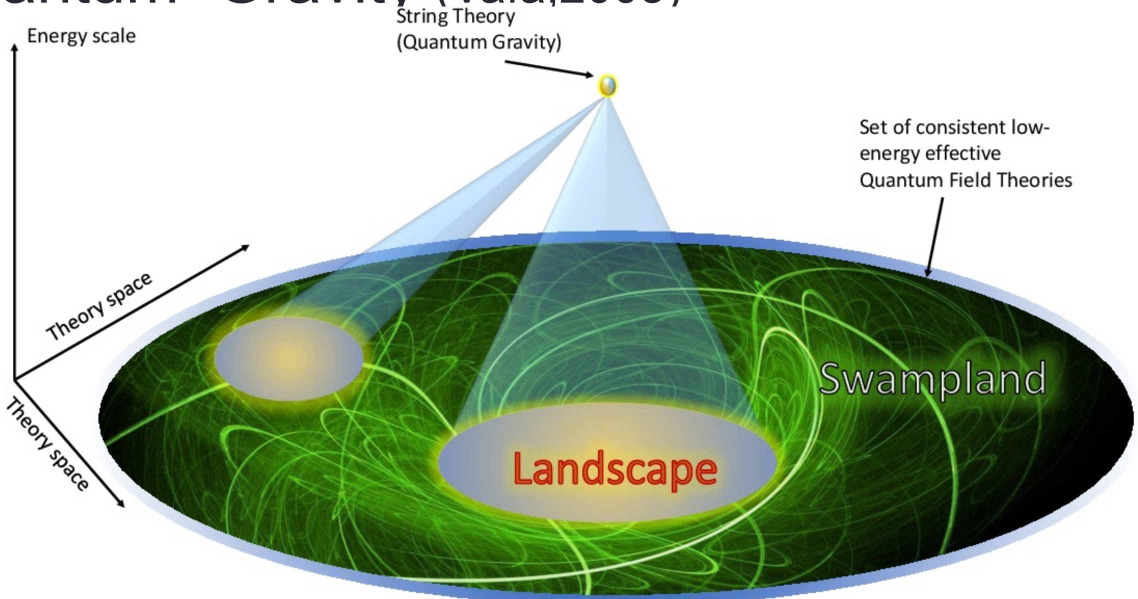
1) The swampland program

Are all consistent Quantum Field Theories obtainable from a Quantum Gravity Theory (ex. String Theory) ?

Probably NO

Swampland = the set of consistent QFT with no consistent coupling to Quantum Gravity (Vafa,2005)



(from E. Palti, « The Swampland: Introduction and Review », [arXiv:1903.06239 [hep-th]])

Final goal swampland program ?

Supplement rules of effective QFT with additional constraints, which would guide Beyond the Standard Model and cosmology constructions.

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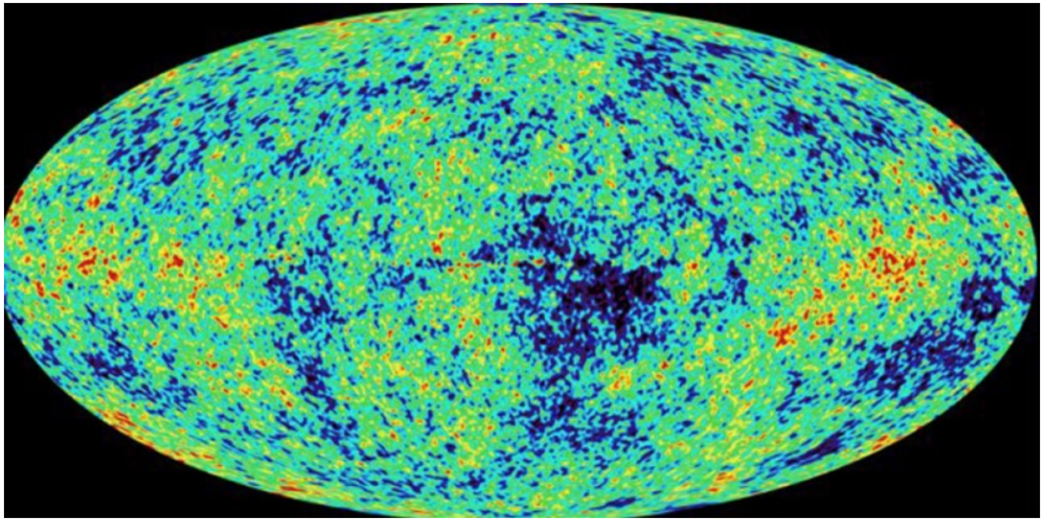
Why Supergravity for early cosmology ?



- Inflation with super-Planckian field variations needs a UV completion → String Theory
- Supersymmetry crucial ingredient in String Theory, supergravity its low-energy effective action

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SUGRA = SUSY + Gravity

Rarita-Schwinger, spin 3/2

It contains :

- gravity multiplet: Graviton $g_{\mu\nu}$, gravitino ψ_μ

- « matter » fields: (complex) Scalars , Weyl Fermions
  chiral superfields

Φ_i ϕ_i ψ_i

+ gauge multiplets, etc

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- In supergravity, the gravitino Ψ_μ becomes massive by absorbing the goldstino G

$$\Psi_\mu \begin{pmatrix} 3/2 \\ - \\ - \\ -3/2 \end{pmatrix} + G \begin{pmatrix} - \\ 1/2 \\ -1/2 \\ - \end{pmatrix} = \Psi_\mu \begin{pmatrix} 3/2 \\ 1/2 \\ -1/2 \\ -3/2 \end{pmatrix}$$

and its mass is $$m_{3/2} = e^{\frac{K}{2}} |W|$$

Kahler potential

Superpotential

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The consistency of low-energy actions for the spin 3/2 Rarita-Schwinger field has a long history :

- 1941: Rarita-Schwinger action
- 1969: Velo-Zwanziger pointed out potential acausal propagation for a charged gravitino in an e.m. background
- 1977: Deser-Zumino proved that gravitino propagation in minimal supergravity is causal
- 2001: Deser-Waldron proved that gravitino propagation in gauged supergravities is causal

  ......
- 2021 – Gravitino swampland conjecture, gravitino distance conjecture

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History of the subject strongly suggest that usual supergravities have no problems with gravitino propagation.

SUSY (linearly realized): nb. bosons = nb. fermions
SUGRA: SUSY is a gauge symmetry, contains gravity
Nonlinear SUSY/SUGRA: nb. bosons $\neq$ nb. fermions

Inflation models in standard SUGRA's have at least one complex scalar field (often several).
Recently, simple nonlinear SUSY/SUGRA models were constructed. More minimal inflationary models, fewer fields.
(Antoniadis,E.D.,Ferrara&Sagnotti; Kallosh,Linde & coll, 2014-)

Even possible to construct minimal models with only: graviton, massive gravitino and inflaton (real scalar)

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Simplest nonlinear SUSY's: constrained superfields.

Example:

- Volkov-Akulov action can be constructed in superspace (Rocek,78) introducing a constrained, nilpotent superfield

$$X^2 = 0$$

whose solution is

no fundamental scalar

Superspace fermionic coordinate

$$X = \frac{GG}{2F_X} + \sqrt{2}\,\theta\, G + \theta^2 F_X$$

auxiliary field

The full VA action is

$$\mathcal{L}_{VA} = \Big[X\,\overline{X}\Big]_D + \Big[f X + h.c.\Big]_F$$

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Analogy with the sigma model :

- O(N) linear sigma model

$$\mathcal{L} = \partial_m \phi_a \partial^m \phi_a - \lambda(\phi_a \phi_a - v^2)^2 .$$

has 1 massive (« Higgs ») and N-1 goldstone bosons,

versus the

- O(N)/ O(N-1) nonlinear sigma model ($\lambda \to \infty$ limit)

$$\mathcal{L} = \partial_m \phi_a \partial^m \phi_a$$

+ constraint $\phi_a \phi_a = v^2$, describes self-interactions of the N-1 goldstone's. O(N) symmetry is nonlinearly realized.

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2) Gravitino sound speed in supergravity (SUGRA)

The talk deals with the propagation (« speed of sound » c_s) of gravitino in SUGRA, (mostly) during inflation.

Normally $$0 < c_s \leq 1$$

Recently, two potential problematic behaviours were discussed:

- $c_s = 0$ at particular points on the inflationary trajectory

  ⇒ Large (catastrophic) production of gravitinos

- $c_s > 1$ acausal behaviour at particular points on the inflationary trajectory in specific SUGRA models

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The sound speed c_s is defined from the dispersion relation

$$\omega^2 = c_s^2 \mathbf{k}^2 + a^2 m^2$$

The transverse spin 3/2 component in a FRW background has a standard dispersion relation with $c_s = 1$

$$(\gamma^0 \partial_0 + i\gamma^i k_i + a m_{3/2}) \Psi_{3/2,\mathbf{k}} = 0$$

scale factor

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The longitudinal (goldstino) component satisfies a more involved equation

$$(\gamma^0 \partial_0 - i \gamma^i k_i \frac{\alpha_1 + \gamma^0 \alpha_2}{\alpha} + a m_{3/2}) \Psi_{1/2, \mathbf{k}} = 0$$

with $\alpha_1 , \alpha_2 , \alpha$ specific functions of scalar fields in SUGRA, with the sound speed depending generically on time

$$c_s^2 = \frac{|\alpha_1|^2 + |\alpha_2|^2}{\alpha^2}$$

$c_s < 1$ $\Longrightarrow$ Slow gravitino (Benakli,Darmé, Oz, 2014)

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A general expression for longitudinal gravitino sound speed is

pressure

time-derivative

$$c_s^2 = \frac{\left(p - 3m_{3/2}^2\right)^2}{\left(\rho + 3m_{3/2}^2\right)^2} + \frac{4\dot{m}_{3/2}^2}{\left(\rho + 3m_{3/2}^2\right)^2}$$

energy density

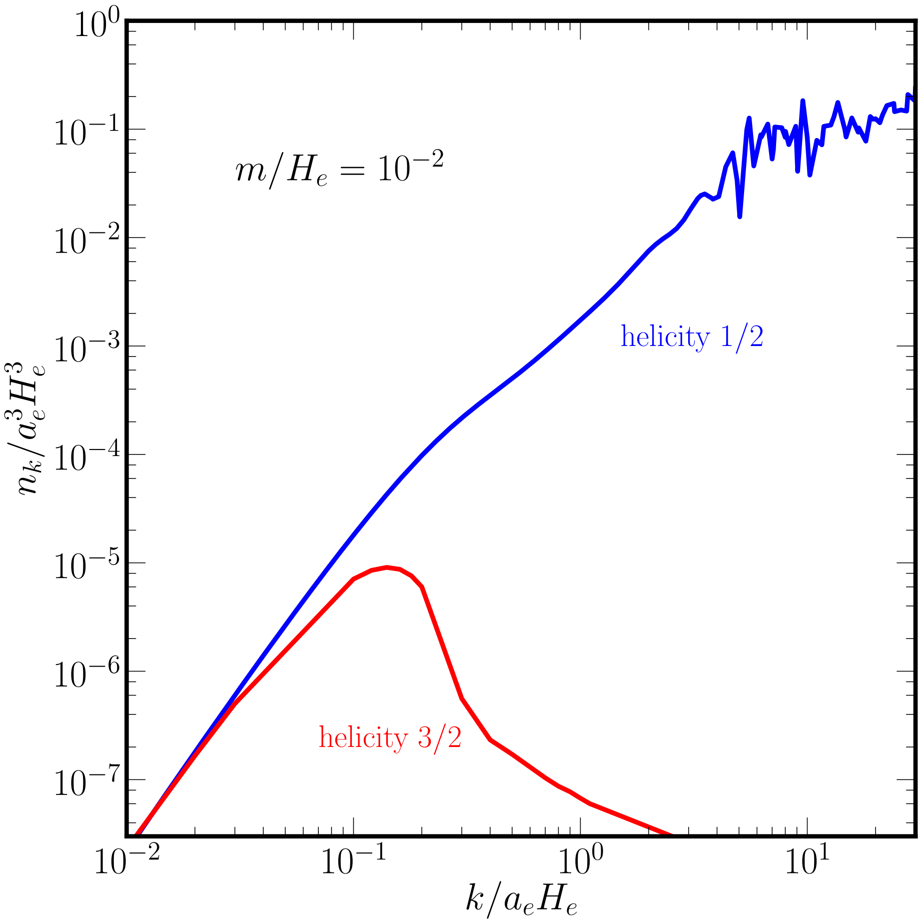
$c_s = 0$ is possible if $m_{3/2}$ is const. and $p = 3m_{3/2}^2$

In this case, there would be a catastrophic production of gravitinos during inflation

(Hasegawa, Terada et al, 2017; Kolb, Long, McDonough, 2021).

Hubble scale

The problem was argued to arise for $m_{3/2} < H$. If the problem is generic $\Longrightarrow$ potential issue for low-energy SUSY models.



(taken from Kolb et al, 2021)

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The explicit formula in SUGRA is

$$c_s^2 = 1 - \frac{4}{(|\dot{\varphi}|^2 + |F|^2)^2} \left\{ |\dot{\varphi}|^2 |F|^2 - |\dot{\varphi} \cdot F^*|^2 \right\}$$

where $F^i \equiv \mathrm{e}^{K/2} K^{ij^*} D_{j^*} W^*$ in standard SUGRA ,

$$D_i W \equiv \frac{\partial W}{\partial \varphi^i} + \frac{\partial K}{\partial \varphi^i} W$$

and we used the compact notation $|\dot{\varphi}|^2 = \dot{\varphi}^i K_{ij^*} \dot{\varphi}^{j*}$,etc

Obs: Cauchy-Schwarz inequality $\Longrightarrow$ causality $c_s \leq 1$
respected in all standard SUGRA's

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3) Eqs. for the longitudinal gravitino, results

In an expanding background, the longitudinal gravitino θ is coupled to another fermion, the inflatino

$$\Upsilon = K_{ij^*} \left(\chi^i \partial_0 \varphi^{j^*} + \chi^{j^*} \partial_0 \varphi^i \right)$$

(Kallosh,Kofman,Linde, Van Proeyen,2000; Nilles,Peloso,Sorbo, 2001)

θ and Υ are coupled via

$$\left(\gamma^0 \partial_0 + i \gamma^i k_i N + M \right) X = 0 \quad , \quad X = \begin{pmatrix} \tilde{\theta} \\ \tilde{\Upsilon} \end{pmatrix}$$

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where the « sound speed matrix »

$$N = \begin{pmatrix} -\frac{\alpha_1}{\alpha} - \gamma^0 \frac{\alpha_2}{\alpha} & -\gamma^0 \Delta \\ -\gamma^0 \Delta & -\frac{\alpha_1}{\alpha} + \gamma^0 \frac{\alpha_2}{\alpha} \end{pmatrix}$$

with $\Delta = \sqrt{1 - c_s^2}$, is now the key to the « slow gravitino » problem.

When $c_s = 0$, then $N = \begin{pmatrix} 0 & -\gamma^0 \\ -\gamma^0 & 0 \end{pmatrix}$

is nonsingular, leading to a nonvanishing sound speed for the physical eigenstates.

(DGMOPV; see also Antoniadis,Benakli and Ke, 2021)

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For the (large) majority of SUGRA models we investigated , we found no problems, $0 < c_s^i \leq 1$:

- standard SUGRA models with two chiral superfields (inflaton+SUSY breaking): general statement

- SUGRA models with nilpotent SUSY breaking field

$$S^2 = 0$$

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The only models with problems we found is with the « orthogonal constraint » for the inflaton multiplet Φ

$$S(\Phi - \overline{\Phi}) = 0 \quad \Longrightarrow$$

Only $Re\ \phi$ =inflaton is a dynamical degree of freedom.
$Im\ \phi$, the inflatino ψ_ϕ and the auxiliary field F_ϕ are determined by the constraint.

In particular F_ϕ is a bilinear in fermions and does not appear in the scalar potential : $F^\Phi \neq e^{K/2} K^{\Phi \bar{i}} D_{i^*} W^*$

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Consequences:

- There is no inflatino $\Rightarrow$ $\Upsilon = 0$, the gravitino sound speed problem $c_s = 0$ can reappear (model-dependent)

- The Cauchy-Schwarz argument for $c_s \leq 1$ not valid. We found examples with $c_s > 1$!

On the other hand, the UV origin of the orthogonal constraint is not clear (Dall'Agata,E.D.,Farakos, 2006)

$\Rightarrow$ Potential pathological behaviour reminiscent of the swampland program !

5) Causality and positivity bounds

(Quentin Bonnefoy,Gabriele Casagrande & E.D., in progress)

- The potential acausal behaviour concerns the longitudinal component of the gravitino.
- Gravitino equivalence theorem: at high-energy, gravitino longitudinal component is described by the goldstino, with enhanced couplings to matter.

Natural question: is the acausality found in SUGRA captured by the low-energy lagrangian of the goldstino coupled to matter, in the decoupling limit $M_P \to \infty$?

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Yes ! The goldstino lagrangian contains a higher-derivative operator of the form

$$\frac{1}{f^2}(1-c_s^2)(\bar{G}i\gamma^m\partial^n G)\ \partial_m\varphi\ \partial_n\varphi$$

The operator is subject to positivity constraints from dispersion relation arguments which enforce

$$c_s \leq 1$$

- This implies that the subluminality condition is independent of M_P, easy to check aposteriori
- We believe the issue arises due to the « elimination » of the auxiliary field by the orthogonal constraint, no simple physical interpretation.

- Obs: SUGRA/inflation subluminality condition valid throughout the inflationary trajectory, positivity constraints valid only in the ground state

SUGRA condition is stronger.

- Maybe causality condition of goldstino propagation in time-dependent solutions of the goldstino action is equivalent to the SUGRA constraint ?

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Interesting to contemplate a « gravitino swampland conjecture »

« In all 4d effective field theories that are low-energy limits of quantum gravity, at all points
in moduli space and for all initial conditions, the sound speed of the gravitino(s) must be non-vanishing $c_s > 0$ »
(Kolb,Long,McDonough)

a refined version

« In all 4d effective field theories that are low- energy limits of quantum gravity, at all points
in moduli space and for all initial conditions, all eigenvalues of the sound speed matrix for fermions must be non-vanishing and subluminal $0 < c_s^i \leq 1$ »

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Conclusions

- Gravitino production constraints important for phenomenological viability of SUGRA models.
- Very often, inflatino is produced, alleviate gravitino problem.
- Important to check and impose sound speed

$$0 < c_s \leq 1 \Longrightarrow \text{gravitino swampland conjecture}$$

- Most SUGRA models satisfy it, except peculiar models with orthogonal constraint (or similar).
- Subluminality constraints captured by goldstino SUSY lagrangians in $M_P \to \infty$ limit and positivity constraints, but SUGRA condition is stronger.
- General interest: consistency constraints on nonlinear SUSY/SUGRA

THANK YOU !

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