String theory for pedestrians

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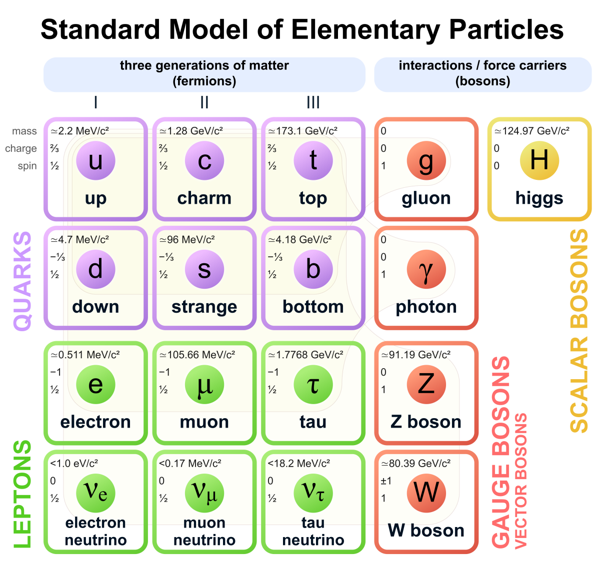
What is the world made of ?



Fundamental interactions

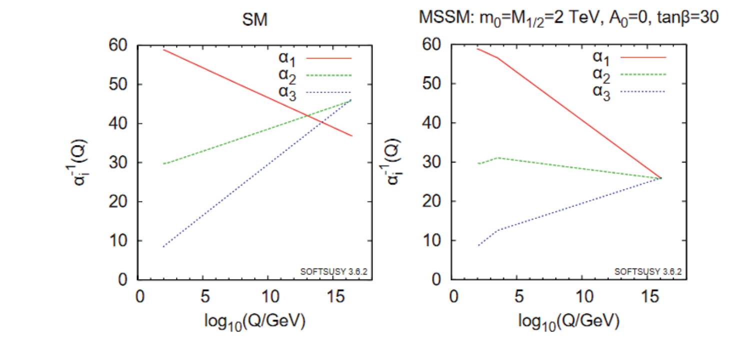
Electromagnetic interactions
Strong interactions
Weak interactions

$\Big\}$ Standard model of Particle Physics (Field Theory)

Gravity $\longrightarrow$ General Theory of Relativity

Unification of forces: Quantum Theory of Gravity

Standard Model Coupling Evolution



Running couplings in Standard Model (SM) and Minimal Supersymmetric Standard Model (MSSM).

The Standard Model

The Standard Model of particle physics has been proved remarkably successful in interpreting the results experiments. However, it is considered as a low energy effective theory as it leaves a number of unanswered questions including: charge quantization, neutrino masses, dark matter, hierarchy problem, gravity.

A popular extension of the Standard Model that provides some answers to these problems are Grand Unified Theories (GUTs) as $SU(5), SO(10)$. However, the longevity of the proton, $\tau_p > 10^{34}$ years, has become a serious challenge for GUTs. In addition GUts do not include gravity.

String theory is our best candidate for a consistent theory of quantum gravity that can incorporate gauge interactions including the Standard Model of Particle Physics.

The fundamental entities of string theory are one-dimensional objects (strings) rather than point particles.

point particle open string closed string

In the context of string theory particles corresponds to vibrations of a string.

The size and the scale of strings

Assuming string theory describes quantum gravity it must involve three fundamental constants:

$$\begin{aligned} c &: \text{ the speed of light} \\ \hbar &: \text{ Planck's constant} \\ G &: \text{ Newton's gravitational constant} \end{aligned}$$

From dimensional analysis there is a unique way to form a length

$$\ell_P = \sqrt{\frac{\hbar G}{c^3}} = 1.6 \times 10^{-33}\text{cm (Planck's length)} \ ,$$

and a mass

$$m_P = \sqrt{\frac{\hbar c}{G}} = 1.2 \times 10^{19}\text{GeV/c}^2 \text{ (Planck's mass)} \ .$$

The size and the scale of strings

Any theory that descibes quantum gravity must involve three fundamental constants:

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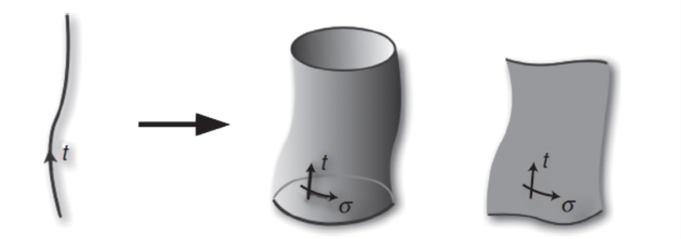
The Bosonic String

The simplest string theory is the so-called bosonic string. We can write down its action in analogy with the relativistic point particle action



world-line world-sheet

$$S_0 = -m \int ds \qquad \Rightarrow \qquad S = -T \int dA$$

$$T = \text{string tension} = \frac{1}{2\pi\alpha'}\ , \alpha' = \text{Regge slope}\ ,\ [\alpha'] = (\text{length})^2$$

The Nambu-Goto action can be expressed in terms of the string coordinates $X^\mu(t,\sigma)$, $\mu = 0, 1, \ldots, D-1$ as

$$S_{NG} = -\frac{1}{2\pi\alpha'}\int dA = -\frac{1}{2\pi\alpha'}\int d\sigma dt\sqrt{-h}\ ,$$

where $h_{ab} = \partial_a X^\mu \partial_b X_\mu$.

For the quantization of the bosonic string we use the (classically equivalent) string sigma model or Polyakov action

$$S_P = -\frac{1}{4\pi\alpha'}\int d^2\xi\sqrt{-\mathrm{det}g}\, g^{ab}\partial_a X^\mu \partial_b X^\nu \eta_{\mu\nu}\ ,\ a, b = 0, 1, \xi^0 = t, \xi^1 = \sigma$$

In the conformal gauge $g_{ab} = \eta_{ab}$ this simplifies to

$$S_P = -\frac{1}{4\pi\alpha'}\int d^2\sigma \eta^{ab}\partial_\alpha X^\mu \partial_\beta X_\mu\ .$$

The Bosonic String: Equations of motion

The equations of motion are two-dimensional wave equations

$$\left(\partial_t^2 - \partial_\sigma^2\right) X^\mu = 0 \ , \ \ \mu = 0, 1, \ldots, D-1 \, .$$

For periodic boundary conditions $X^\mu(\sigma,\tau) = X^\mu(\sigma+\pi,\tau)$ (closed strings) the solutions are right and left moving travelling waves

$$X^\mu(\sigma,\tau) = X_R^\mu(\tau-\sigma) + X_L^\mu(\tau+\sigma)$$

where

$$X_R^\mu(\tau-\sigma) = \frac{1}{2}x^\mu + \frac{1}{2}\ell_s^2 p^\mu(\tau-\sigma) + \frac{i}{2}\ell_s \sum_{n\neq 0} \frac{1}{n}\alpha_n^\mu e^{-2in(\tau-\sigma)}$$

$$X_L^\mu(\tau+\sigma) = \frac{1}{2}x^\mu + \frac{1}{2}\ell_s^2 p^\mu(\tau+\sigma) + \frac{i}{2}\ell_s \sum_{n\neq 0} \frac{1}{n}\tilde{\alpha}_n^\mu e^{-2in(\tau+\sigma)}$$

with $\ell_s = \sqrt{2\alpha'}$ and $\alpha_{-n}^\mu = (\alpha_n^\mu)^*$, $\tilde{\alpha}_{-n}^\mu = (\tilde{\alpha}_n^\mu)^*$, $\alpha_0^\mu = \tilde{\alpha}_0^\mu = \frac{1}{2}\ell_s p^\mu$.

The Bosonic String : Quantization

To quantize we introduce the canonical momenta

$$P^\mu(\sigma,\tau) = \frac{\delta S}{\delta \dot{X}_\mu} = T\dot{X}^\mu$$

and promote Poisson brackets to commutators, which leads to

$$\left[a_m^\mu, a_n^{\nu\dagger}\right] = \left[\tilde{a}_m^\mu, \tilde{a}_n^{\nu\dagger}\right] = \eta^{\mu\nu}\delta_{mn} \;\; m,n>0 \text{ with } a_m^\mu = \frac{1}{\sqrt{m}}\alpha_m^\mu$$

However, $\left[a_m^0, a_n^{0\dagger}\right] = -1$ results in negative norm states. These decouple at $D = 26$ (critical dimension).

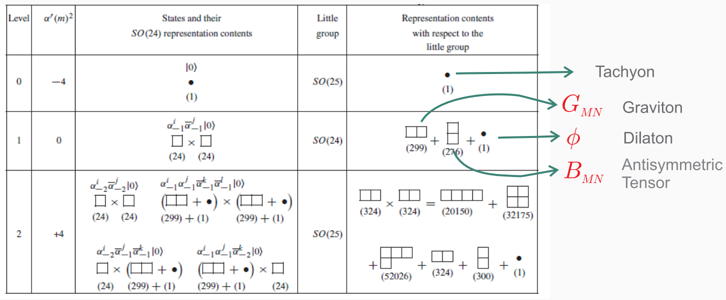
$$\text{Ground state: } a_n^\mu \left|0\right\rangle = 0 \text{ for } m > 0$$

$$\text{One particle states : } a_{m_1}^{\mu 1\,\dagger} \dots a_{m_n}^{\mu n\,\dagger} \left|0\right\rangle$$

Level	$\alpha'(m)^2$	States and their $SO(24)$ representation contents	Little group	Representation contents with respect to the little group
0	-4	$\|0\rangle$ $\bullet$ (1)	$SO(25)$	$\bullet$ (1)
1	0	$\alpha^i_{-1}\bar{\alpha}^j_{-1}\|0\rangle$ $\square \times \square$ (24) (24)	$SO(24)$	(299) + (276) + (1)
2	+4	$\alpha^i_{-2}\bar{\alpha}^j_{-2}\|0\rangle$: $\square \times \square$, (24) (24) $\alpha^i_{-1}\alpha^j_{-1}\bar{\alpha}^k_{-1}\bar{\alpha}^l_{-1}\|0\rangle$: $(\ + \bullet) \times (\ + \bullet)$, (299) + (1) (299) + (1) $\alpha^i_{-2}\bar{\alpha}^j_{-1}\bar{\alpha}^k_{-1}\|0\rangle$: $\square \times (\ + \bullet)$, (24) (299) + (1) $\alpha^i_{-1}\alpha^j_{-1}\bar{\alpha}^k_{-2}\|0\rangle$: $(\ + \bullet) \times \square$, (299) + (1) (24)	$SO(25)$	(324) × (324) = (20150) + (32175) + (52026) + (324) + (300) + (1)

The lowest mass levels of the closed bosonic string.

The Bosonic String : Spectrum



Level	$\alpha'(m)^2$	States and their $SO(24)$ representation contents	Little group	Representation contents with respect to the little group
0	-4	$\vert 0\rangle$ • (1)	$SO(25)$	• (1)
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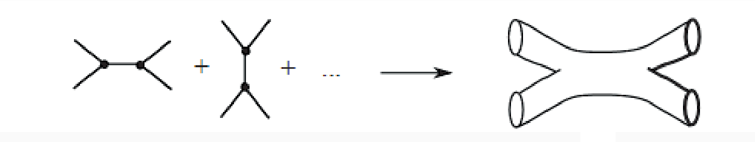
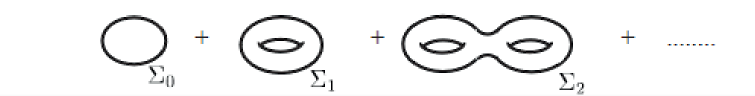
The lowest mass levels of the closed bosonic string.

The world-sheet theory we have described is clearly non-interacting, however, non-trivial interactions arise in space-time theories due to the world-sheet topologies.

Feynman Diagrams:



Loop expansion:



In the critical dimension, all string amplitudes are finite order by order in perturbation theory.

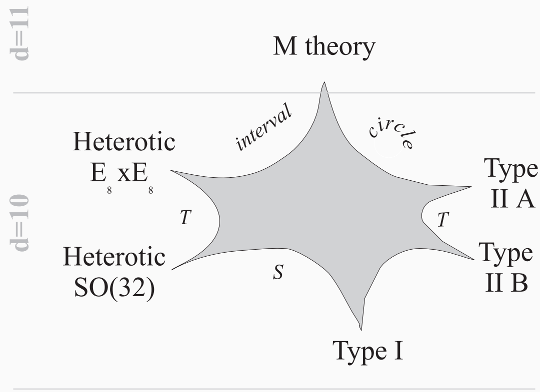
Superstrings

The simplest string theory, the bosonic string, exhibits a tachyon and has no space-time fermions in its spectrum. To avoid these problems, we introduce fermionic partners ψ^μ for the world-sheet bosonic fields X^μ preserving world-sheet supersymmetry. As the field content changes so does the critical dimension D. For the superstrings we have $D = 10$.

The appearance of both gravitational and gauge degrees of freedom in the spectrum makes string theory a candidate theory for the unification of all interactions.

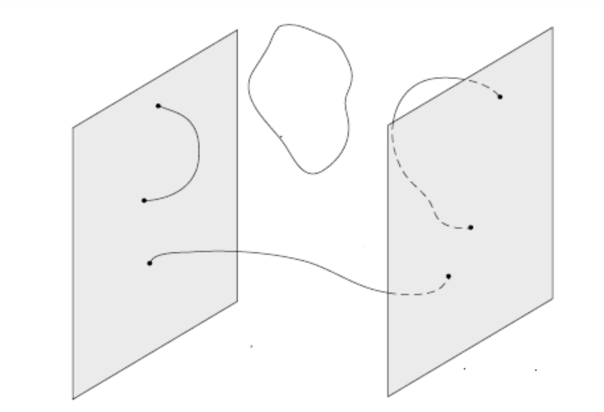
How many string theories exist?

It turns out that in ten dimensions there are five consistent string theories: Type IIA and IIB, $E_8 \times E_8$ and $SO(32)$ heterotic, and Type I theory.



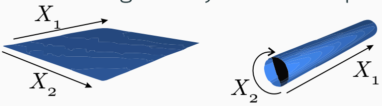
D branes

D-branes are hypersurfaces on which open strings end. They appear in the context of Type I and Type II strings.



String Phenomenology

Of course string theory requires $D = 9 + 1 = 10$ space-time dimensions. Going down to 3+1=4 dimensions requires "compactification" of the 6 spacial dimensions.
In four dimensions string theory is not unique.



String phenomenology focuses on the construction and study of phenomenological features of string derived gauge models. These include extensions of the SM or GUTs that comprise the SM. The research in this field has yielded low energy effective models with realistic characteristics. These include the $SU(3) \times SU(3) \times SU(3)$, $SU(5) \times U(1)$, Pati-Salam models.

Further reading

1. Introduction to String Theory, Kostas Skenderis, University of Amsterdam course,
https://math.berkeley.edu/ kwray/papers/string_theory.pdf,
2. String Theory andM-Theory, (K. Becker, M. Becker and J. Schwarz)
3. Introduction to String Theory, J. Polchinski
4. String Theory in a Nutshell, E. Kiritsis
5. Basic Concepts of String Theory, R. Blumenhagen, D. Lüst, S. Theisen