Niš Lectures on Cosmology 3

Mark Hindmarsh

Department of Physics & Astronomy

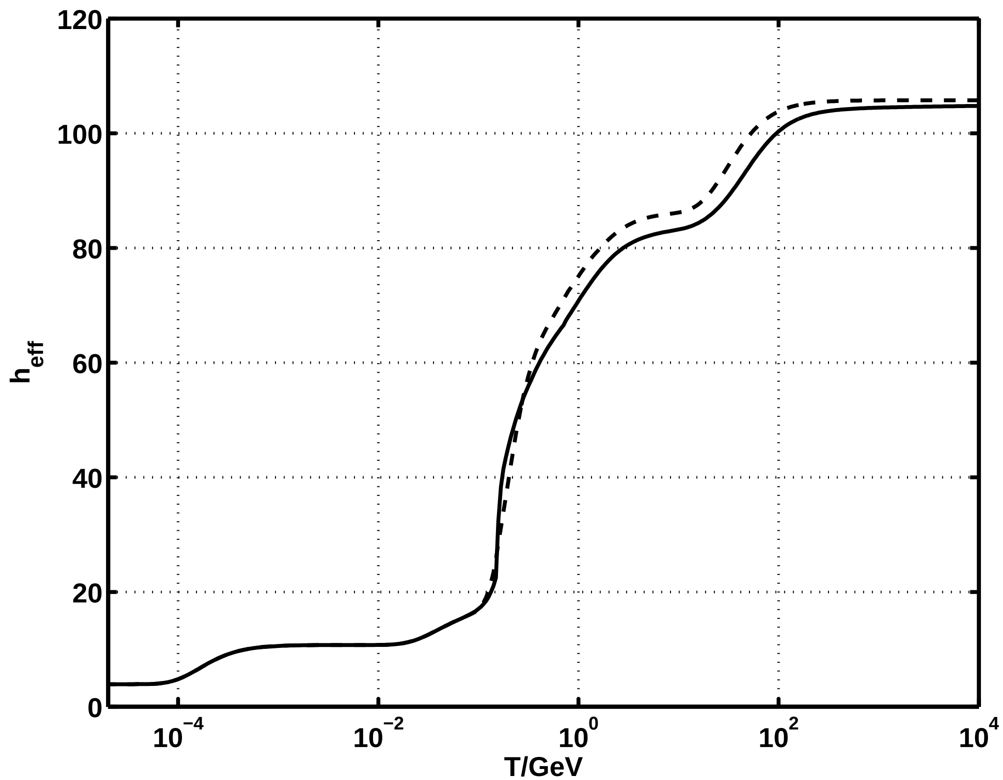
University of Sussex

m.b.hindmarsh@sussex.ac.uk

University of Sussex

Phase Transitions in the Early Universe

Olive 1981 (dashed), Hindmarsh and Philipsen 2005 (solid)



Temp.	Event
100 GeV	t non-relativistic
1 GeV	b non-relativistic
500 GeV	c, τ non-relativistic
200 MeV	QCD phase transition
30 MeV	μ non-relativistic
2 MeV	ν freeze-out
0.2 MeV	e non-relativistic
1 eV	matter = radiation
0.1 eV	photon decoupling

Free energy of an ideal gas

- Free energy density $f = \rho - Ts$ (also $f = -p$)
- To find equilibrium state we minimise free energy
- Dimensions: $f = T^4\phi(m/T)$ with $\phi(0) = -g\pi^2/90$.

Recall pressure mass m in equilibrium (no chemical potential, $\eta = \pm 1$ (FD/BE)):

$$p = \int đ^3k \frac{1}{e^{E/T} + \eta}\frac{k^2}{3E}, \qquad E = (k^2 + m^2)^{\frac{1}{2}}$$

Free energy density ($f = -kT \ln Z/V$):

$$f = -\eta T \int đ^3k \ln(1 + \eta e^{-E/T})$$

Can obtain from p by partial integration.

Free energy: exact formulae in high T expansion

Bosons:

$$f_B = -\frac{\pi^2}{90}T^4 + \frac{m^2T^2}{24} - \frac{(m^2)^{\frac{3}{2}}T}{12\pi} - \frac{m^4}{64\pi^2}\ln\left(\frac{m^2}{a_bT^2}\right) - \frac{m^4}{16\pi^{\frac{5}{2}}}\sum_\ell (-1)^\ell \frac{\zeta(2\ell+1)}{(\ell+1)!}\left(\frac{m^2}{4\pi^2T^2}\right)^\ell$$

Fermions:

$$f_F = -\frac{\pi^2}{90}\frac{7}{8}T^4 + \frac{m^2T^2}{48} + \frac{m^4}{64\pi^2}\ln\left(\frac{m^2}{a_fT^2}\right) + \frac{m^4}{16\pi^{\frac{5}{2}}}\sum_\ell (-1)^\ell \frac{\zeta(2\ell+1)}{(\ell+1)!}(1-2^{-2\ell-1})\Gamma(\ell+\frac{1}{2})\left(\frac{m^2}{4\pi^2T^2}\right)^\ell$$

$a_b = 16\pi^2\ln(\frac{3}{2} - 2\gamma_E)$, $a_f = a_b/16$, $\gamma_E = 0.5772\ldots$ (Euler's constant)

Model field theory: real scalar

$$\mathcal{L} = \frac{1}{2}\partial\phi\cdot\partial\phi - V(\phi) + J\phi$$

- $V(\phi) = \frac{1}{2}\mu^2\phi^2 + \frac{1}{4!}\lambda\phi^4$
- Symmetry: $\phi \to -\phi$ (Z_2).
- J external “source” removed at end of any calculation

$\mu^2 > 0$

- Minimum (ground state) at T=0: $\phi = 0$
- Ground state respects symmetry.

$\mu^2 < 0$

- Minimum (ground state) at T=0: $\phi = v = \sqrt{6|\mu^2|/\lambda}$
- Ground state “breaks” symmetry.

Effective potential at high T for real scalar field

Expand V around a constant value: $\phi = \bar{\phi} + \varphi$, set $J = V'(\bar{\phi})$.

$$\mathcal{L} = \frac{1}{2}\partial\phi\cdot\partial\phi - V(\bar{\phi}) - \frac{1}{2}V''(\bar{\phi})\varphi^2 - \frac{1}{3!}V'''(\bar{\phi})\varphi^3 - \frac{1}{4!}\varphi^2$$

A real scalar field theory with mass^2 $M^2(\bar{\phi}) = V''(\bar{\phi}) = \frac{1}{2}\lambda\bar{\phi}^2 + \mu^2$.

Free energy depends on $\bar{\phi}$. Equilibrium $\bar{\phi}$ from $df/d\bar{\phi} = 0$.

Another name for free energy: effective potential $V_T(\bar{\phi})$

$$\begin{aligned} V_T(\bar{\phi}) &= V_T(0) + \frac{1}{2}\mu^2\bar{\phi}^2 + \frac{1}{4!}\lambda\bar{\phi}^4 + \frac{1}{24}M^2(\bar{\phi})T^2 - \frac{(M^2(\bar{\phi}))^{\frac{3}{2}}T}{12\pi} + \cdots \\ &\simeq V_T(0) + \frac{1}{2}(\mu^2 + \frac{1}{24}\lambda T^2)\bar{\phi}^2 + \frac{1}{4!}\lambda\bar{\phi}^4 \end{aligned}$$

Can neglect higher order terms where $M^2(\phi)/T^2 \ll 1$.

$\mu^2 > 0$: Equilibrium still at $\bar{\phi} = 0$, with thermal mass $m^2(T) = \mu^2 + \frac{1}{12}\lambda T^2$

Effective potential for scalar field with gauge fields and fermions

Let scalar field give masses to scalars ($M_S(\bar\phi)$), vectors ($M_V(\bar\phi)$) and (Dirac) fermions ($M_F(\bar\phi)$)

$$
\begin{aligned}
V_T(\bar\phi) \quad = \quad & V_T(0) + \frac{1}{2}\mu^2\bar\phi^2 + \frac{1}{4!}\lambda\bar\phi^4 \\
& + \frac{T^2}{24}\left(\sum_S M_S^2(\bar\phi) + 3\sum_V M_V^2(\bar\phi) + 2\sum_F M_F^2(\bar\phi)\right) \\
& - \frac{T}{12\pi}\left(\sum_S (M_S^2(\bar\phi))^{\frac{3}{2}} + 3\sum_V (M_V^2(\bar\phi))^{\frac{3}{2}}\right) + \cdots
\end{aligned}
$$

Again, can neglect higher order terms where $M^2(\phi)/T^2 \ll 1$.

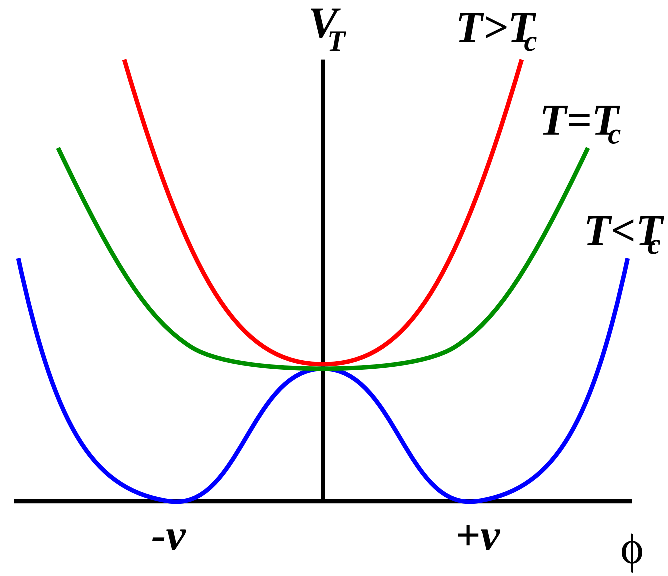
Symmetry restoration at high T

Suppose $\mu^2 < 0$ and $M(\bar\phi)/T \ll 1$.

$$\Delta V_T = \frac{1}{2}(-|\mu|^2 + \frac{1}{24}\lambda T^2)\bar\phi^2 + \frac{1}{4!}\lambda\bar\phi^4$$

Equilibrium at

$$\begin{aligned}\bar\phi^2 &= 6(|\mu^2| - \frac{1}{24}\lambda T^2)/\lambda \\ &= v^2(1 - T^2/T_c^2)\end{aligned}$$



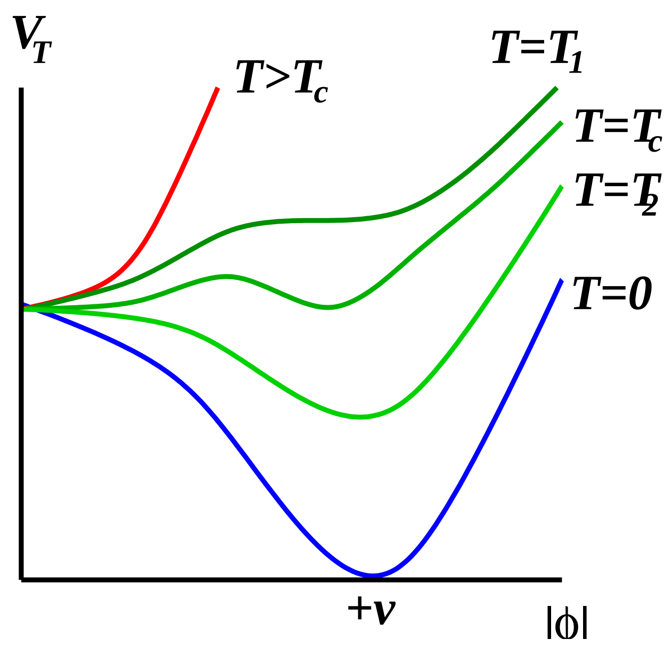
- Critical temperature $T_c^2 = 24|\mu^2|/\lambda$
- Above T_c equilibrium at $\bar\phi = 0$
- Second-order phase transition

  discontinuity in specific heat, divergence in correlation length $\xi = 1/m(T)$

First order phase transition

Now consider multiple fields:

$$\Delta V_T \simeq \frac{\gamma}{2}(T^2 - T_2^2)|\bar{\phi}|^2 - \delta T|\bar{\phi}|^2 + \frac{1}{4!}\lambda|\bar{\phi}|^4$$



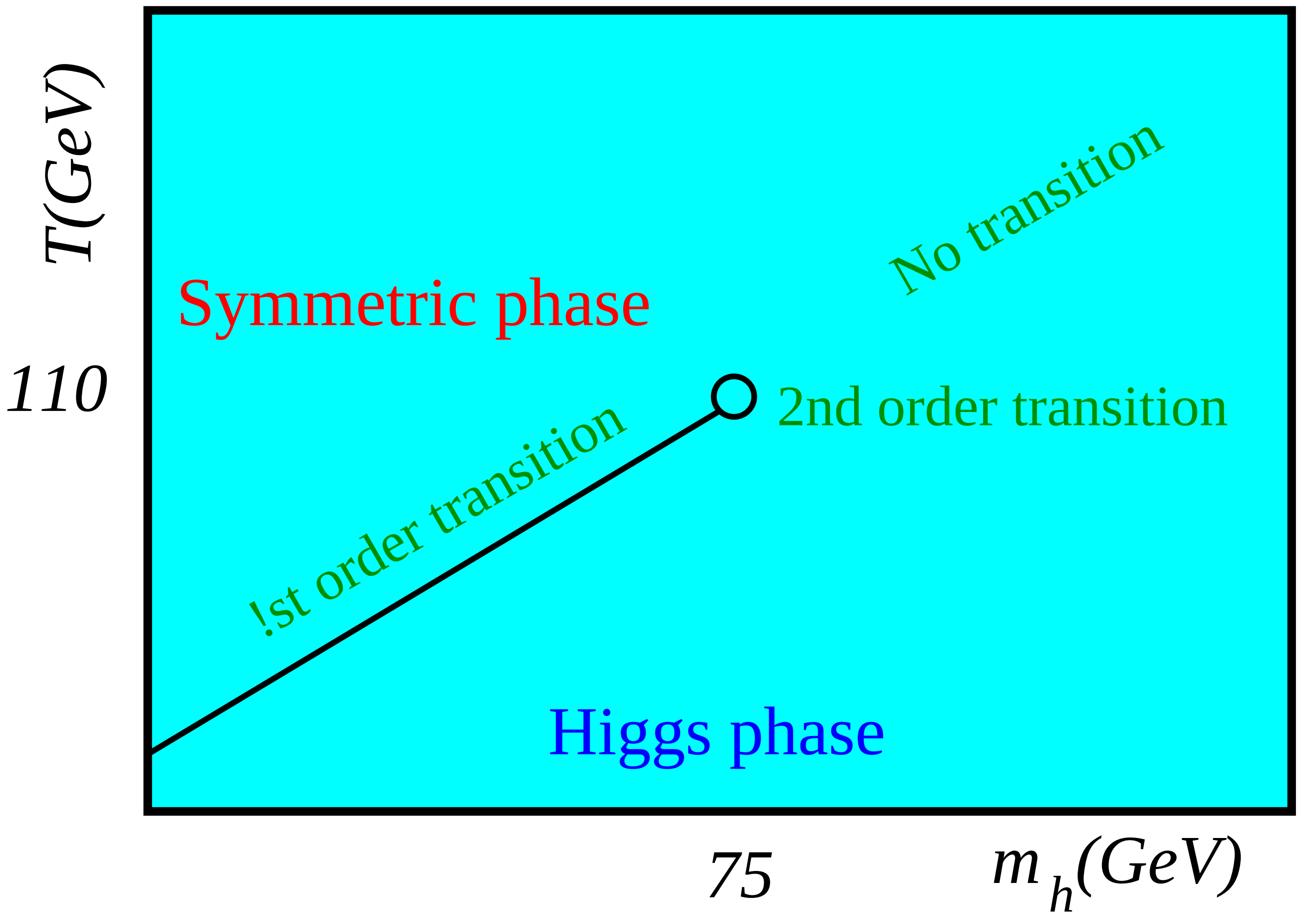
- Second minimum develops at T_1
- Critical temperature T_c:

  free energies are equal.
- System can supercool below T_c.
- First order transition

  discontinuity in free energy

Phase transitions in gauge theories

Standard Model phase diagram (Kajantie et al 1996):



Baryogenesis

WMAP (Nucleosynthesis): $n_B/s \simeq 6.1^{+0.3}_{-0.2} \times 10^{-10}$

Why is the baryon number density of the Universe non-zero?

Sakharov conditions for generating baryon number from $n_B = 0$:

B violation Obvious!

C and CP violation Otherwise B-violating interactions will produce B and $\bar{B}$ excesses at the same rate (B is odd under both C and CP).

non-equilibrium In equilibrium entropy maximised when chemical potential vanishes.

Electroweak phase transition & baryogenesis

Sakharov conditions for baryogenesis:

B violation Electroweak theory has *unstable* topological defects – sphalerons (S).

Formation and decay of S results in change in $B + L$ of LH fermions.

C and CP violation C violation automatic in SM. CP violation needs more than CKM.

non-equilibrium Supercooling at 1st order phase transition?

Electroweak baryogenesis: current

Minimal Supersymmetric Standard Model:

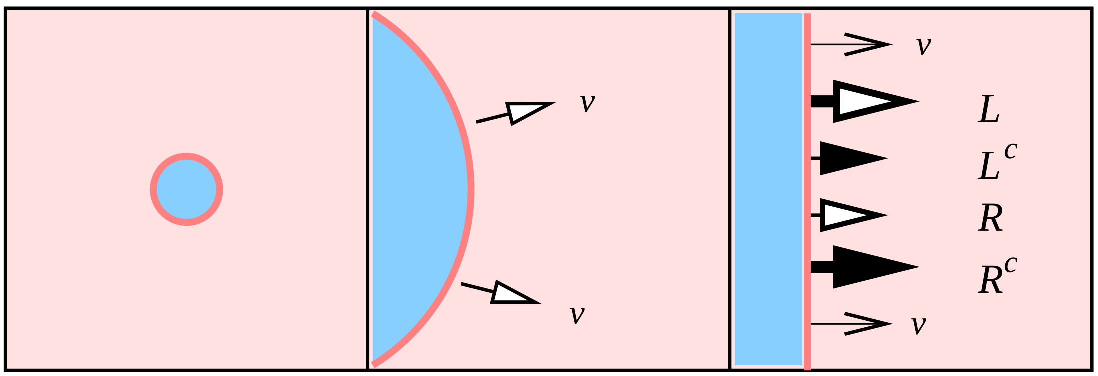
- LEP Higgs mass bounds (just) allow 1st order transition ($m_{\tilde{t}_L} \sim 2\mathrm{TeV}$):[a]

$$85\ \mathrm{GeV} < m_h < 120\ \mathrm{GeV}, \qquad 120\ \mathrm{GeV} < m_{\tilde{t}_R} < 170\ \mathrm{GeV}$$

- CP violation from $\tilde{w}^{\pm}$, $\tilde{h}^{\pm}$, $\tilde{t}_R$, $\tilde{t}_L$ and neutralino mass matrices.[b]

Mechanism:[c]

- CP-violating bubble wall: asymmetry in reflection of fermions
- Sphalerons convert chiral asymmetry in front of wall into baryon asymmetry.



[a] Espinosa, de Carlos 1996; Laine, Rummukainen 1998; Cline, Moore 1998

[b] Huet, Nelson 1996; Carena et al 1997; Cline, Kainulainen 2000

[c] Cohen, Kaplan, Nelson 1991

Formation of topological defects: domain walls

- ϕ^4 field theory has static solution $\phi = v\tanh(\mu z/\sqrt{2})$
- Energy density $T_{00} \propto v^4\mathrm{sech}^4(\mu z/\sqrt{2})$: concentrated on sheet at $z = 0$.
- This is a Domain Wall
- As Universe cools through T_c, breaks up into domains with $\phi = \pm v(T)$.
- Cannot choose same minimum everywhere[a]
- Domain walls must form around surfaces $\phi = 0$
- evolution of walls is self-similar: Area $\propto t^{-\alpha}$, with $\alpha \simeq 1$.[b]

[a] Kibble (1976)

[b] Press, Ryden, Spergel, ApJ (1990), Garagounis and Hindmarsh, Phys Rev D (2003)

Another model field theory

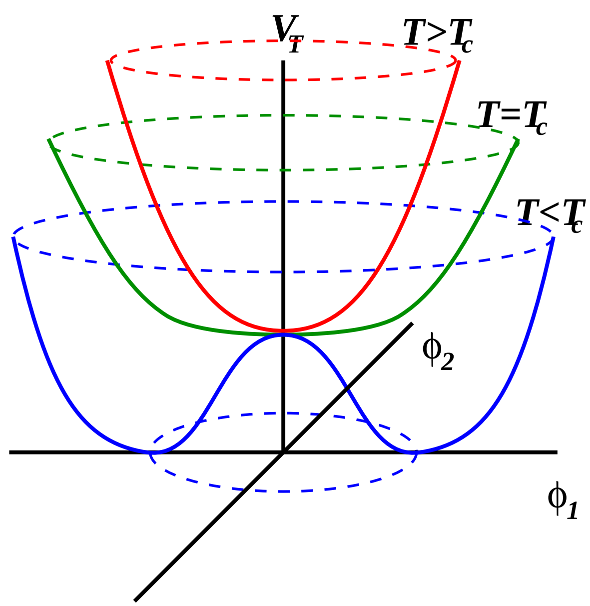
Abelian Higgs model: complex scalar field $\phi(x)$, vector field $A_\mu(x)$.

$$\mathcal{L}_{\rm eff} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + |D\phi|^2 - V_T(\phi),$$

$$V_T(\phi) \simeq V_0 + m^2(T)|\phi^2| + \frac{1}{4}\lambda(T)|\phi|^4$$

where $m^2(T) = \frac{1}{12}(\lambda + 3e^2)T^2 - |\mu^2|$.

The potential energy function $V_T(\phi)$ changes shape at the critical temperature T_c.



Abelian Higgs model phase transition

- Phase transition in finite time makes cosmic strings[a]
- Evolution of strings is also self-similar[b]
- Length density $(L/V) \propto t^{-\beta}$, with $\beta \simeq 2$

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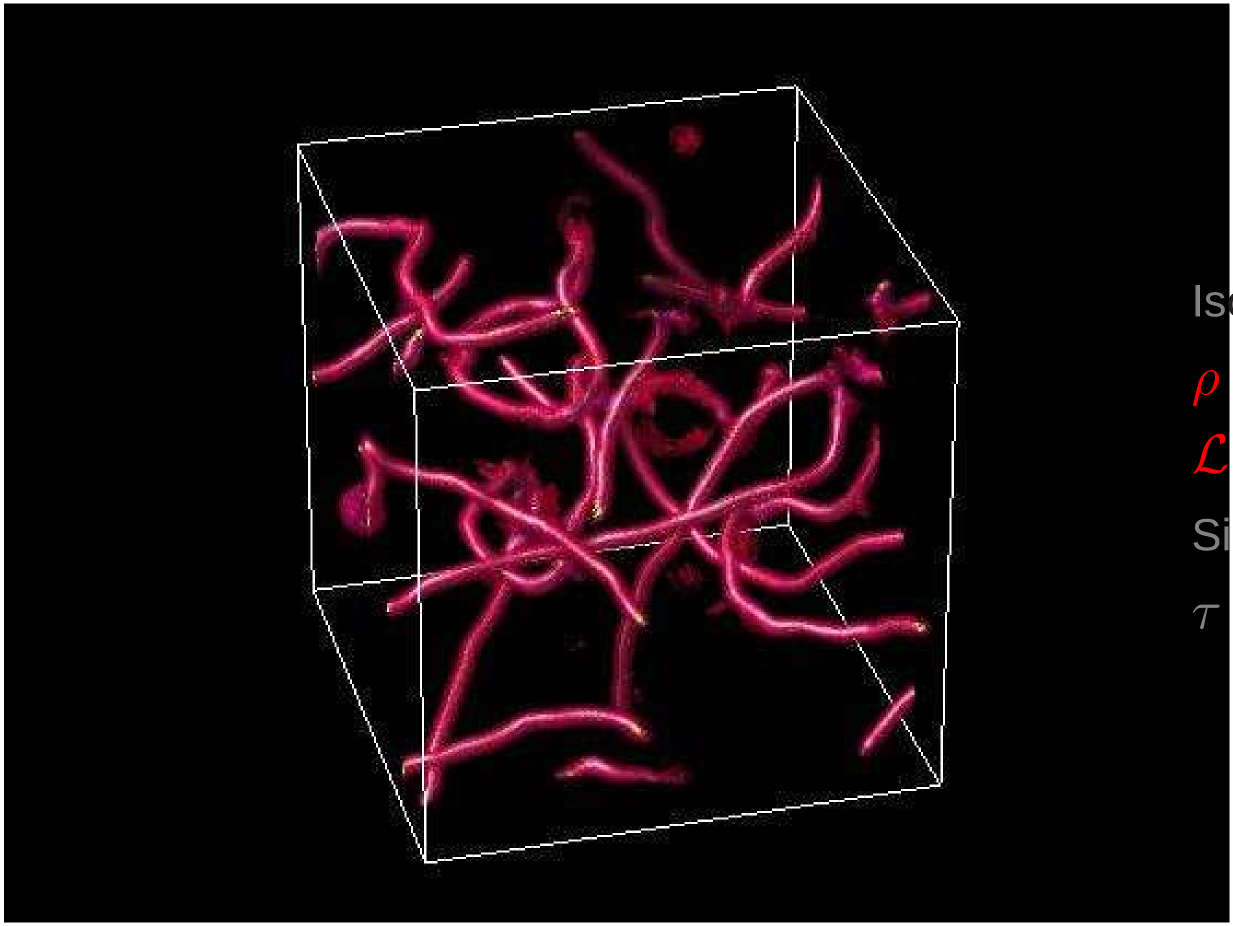
[a] Hindmarsh & Rajantie Phys Rev D (2002); Rajantie, Int. J. Mod. Phys. A (2002).

[b] Vincent, Antunes, Hindmarsh PRL (1998)

Visualising Abelian Higgs model simulations



Isosurfaces:

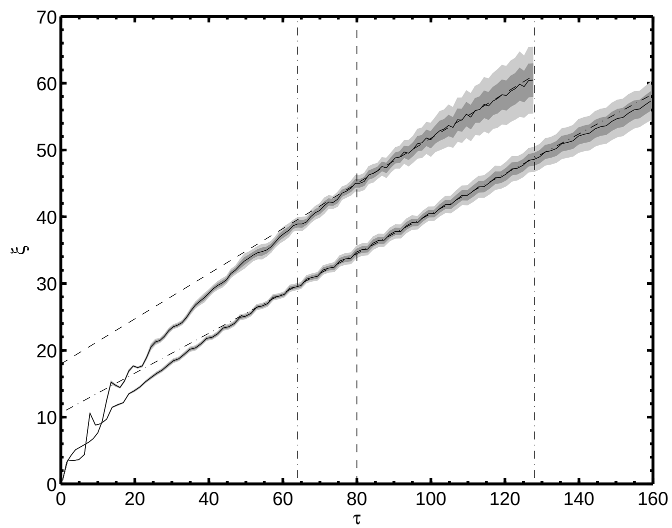
ρ (pink)

$\mathcal{L}$ (yellow)

Size: 192^3

$\tau = L/2$,

Abelian Higgs model simulations: string length scale



Radiation era,

Scaling: $L/V \propto t^{-2}$

Network scale: $\xi = \sqrt{(V/L)}$

Hence $\xi \propto t$

Energy density $\rho_s = \mu L/V = \mu/\xi^2 \sim t^{-2}$

Density parameter $\Omega_s = \rho_s/\rho_c \sim G\mu$ - **constant**.

Cosmology with cosmic strings

Q: Are there cosmic strings in the Universe?

A: Maybe ...

- string density parameter $\Omega_{\rm s} \simeq G\mu(Lt^2/V)$ constant
- produce perturbations $\delta T/T \simeq 10^{-5}$ if from GUT ($G\mu \sim 10^{-6}$)
- 10% of fluctuations from strings?[a]

But ... cosmic rays?

But ... gravitational waves?

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[a] Bevis et al 2007