Niš Lectures on Cosmology 1

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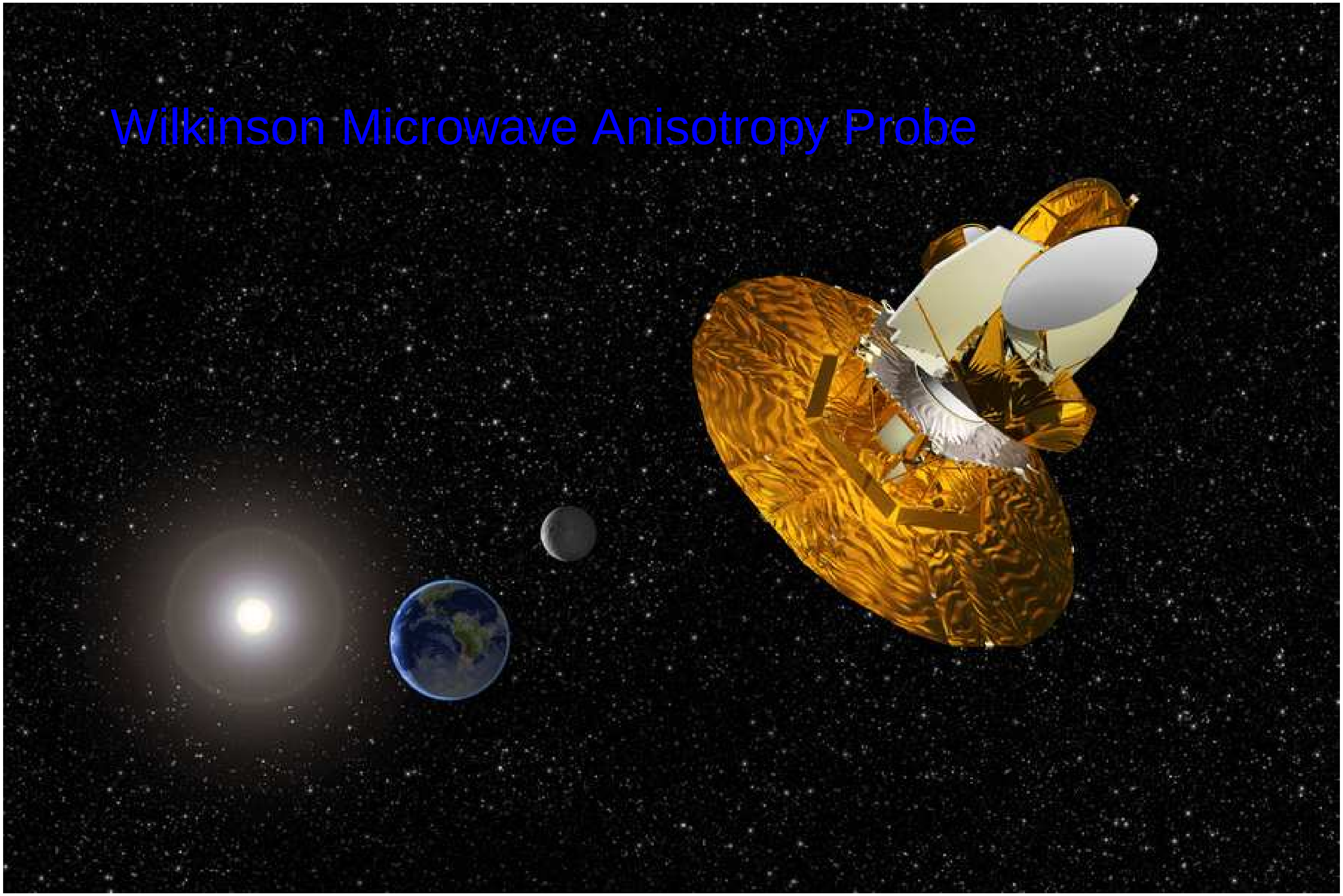
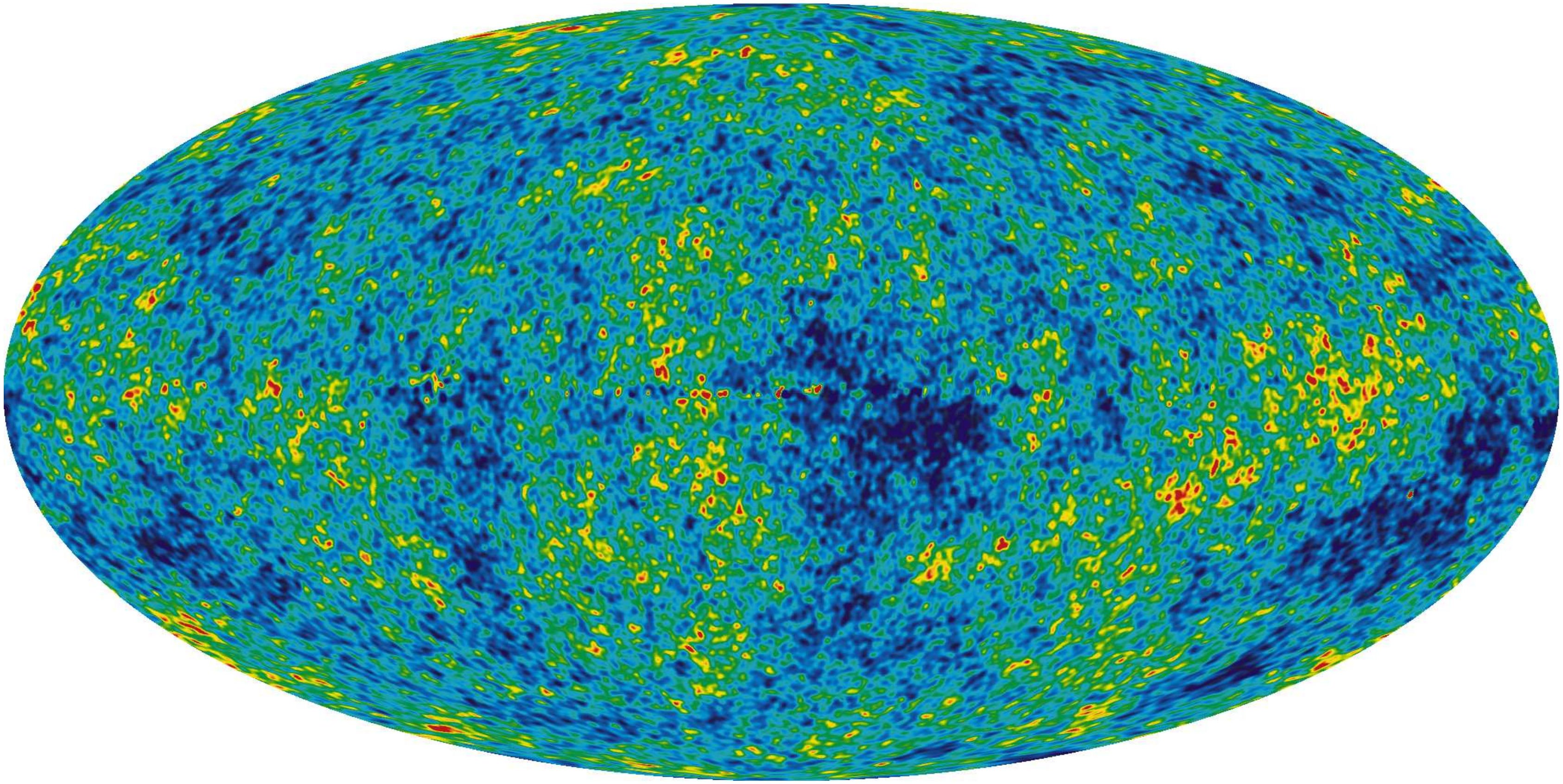
Wilkinson Microwave Anisotropy Probe



WMAP 3 Year Data



$-200 < \Delta T < 200$ μK

Modern Cosmology: Big Questions

- **What is the Universe made of?**
  - and how do we account for the proportions of the contents?
- **Why is the Universe accelerating today?**
- **What made the density perturbations?**
- **Are there any primordial gravitational waves?**
- **How did the Universe begin?**
- **Why 3+1 dimensions?**

Topics & references

- Standard Cosmology
- Dark Energy
- Dark Matter
- Inflation
- Phase transitions
- Baryogenesis

Books

- Kolb & Turner, *The Early Universe* (1990)
- Liddle & Lyth, *Cosmological Inflation & Large-Scale Structure* (2000)
- Bailin & Love, *Cosmology in Gauge Field Theory & String Theory* (2004)

Reviews

- R. Durrer, “Cosmological perturbation theory,” arXiv:astro-ph/0402129.

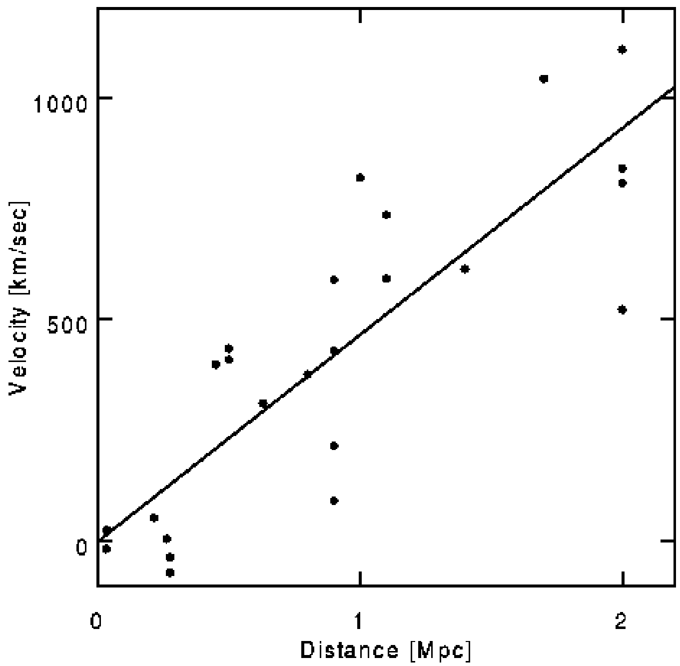
The Standard Cosmology

Theoretical Ingredients

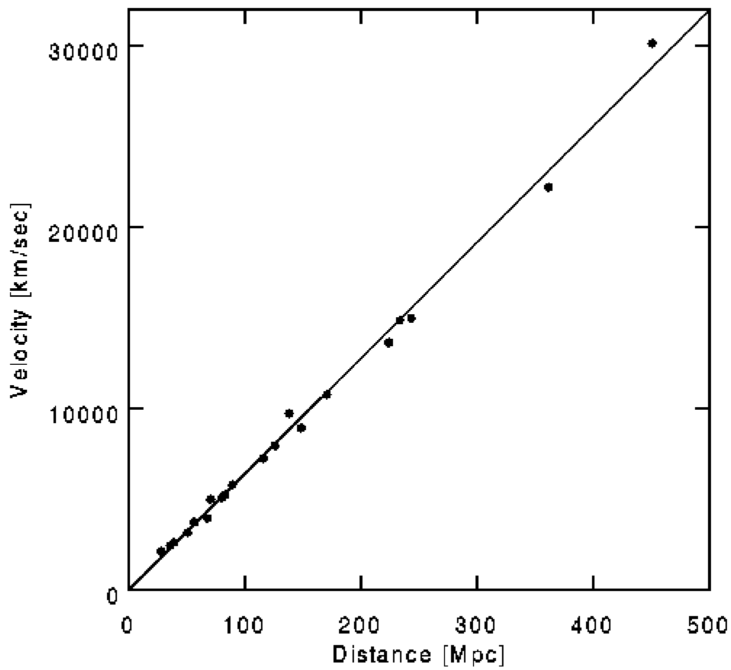
- General relativity: Friedmann equation
- Relativistic (mostly equilibrium) thermodynamics

Expansion

- Galaxies are (on average) receding: $v = Hr$
- Velocities inferred from redshift: $z = (\lambda_{\mathrm{obs}} - \lambda_{\mathrm{emit}})/\lambda_{\mathrm{emit}}$
- Distances inferred by a variety of phenomenological methods



Hubble (1926)

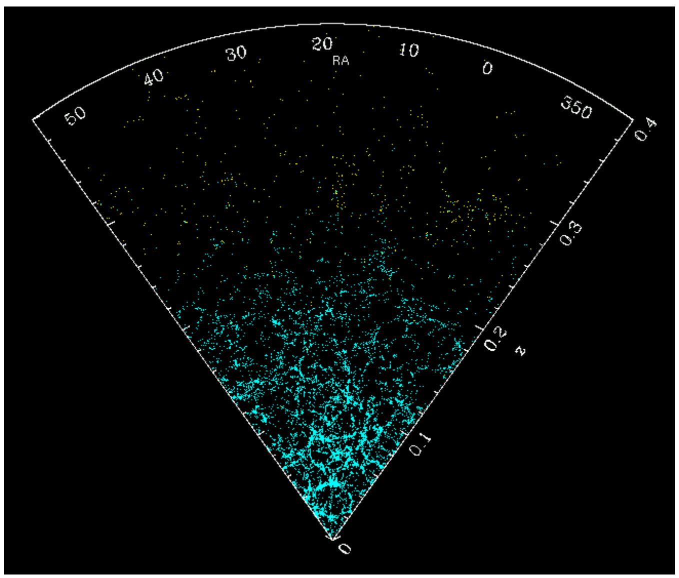


Riess et al. (1996)

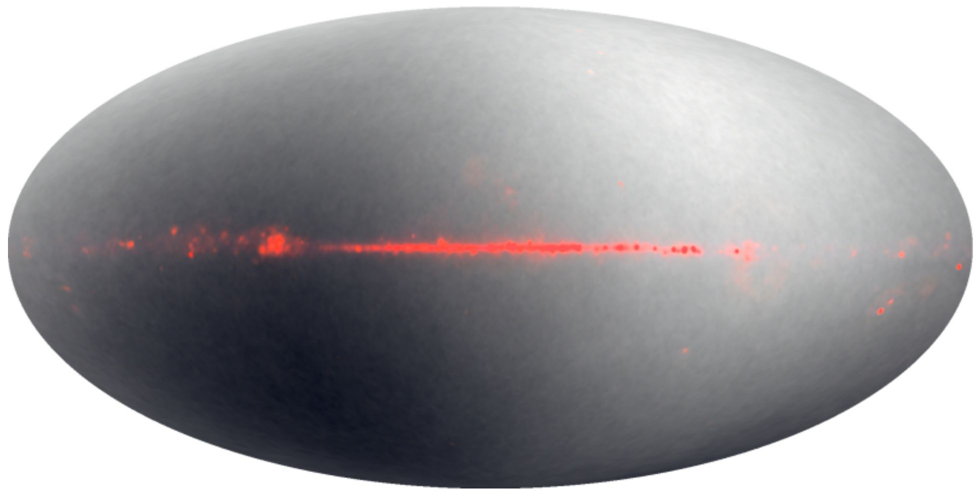
(Figures from Ned Wright's Cosmology pages)

Homogeneity and isotropy

- On the largest scales galaxies are uniformly distributed
- The temperature of CMB is almost the same in every direction



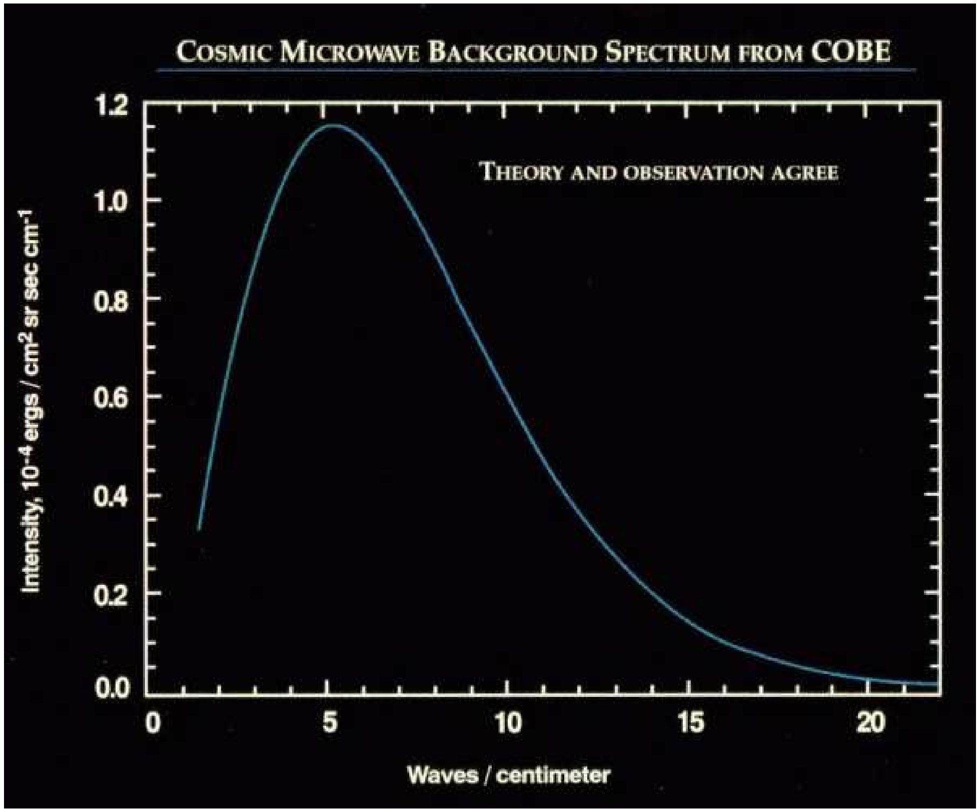
Sloan Digital Sky Survey (2003)



WMAP (2003)

Cosmic Microwave Background (CMB)

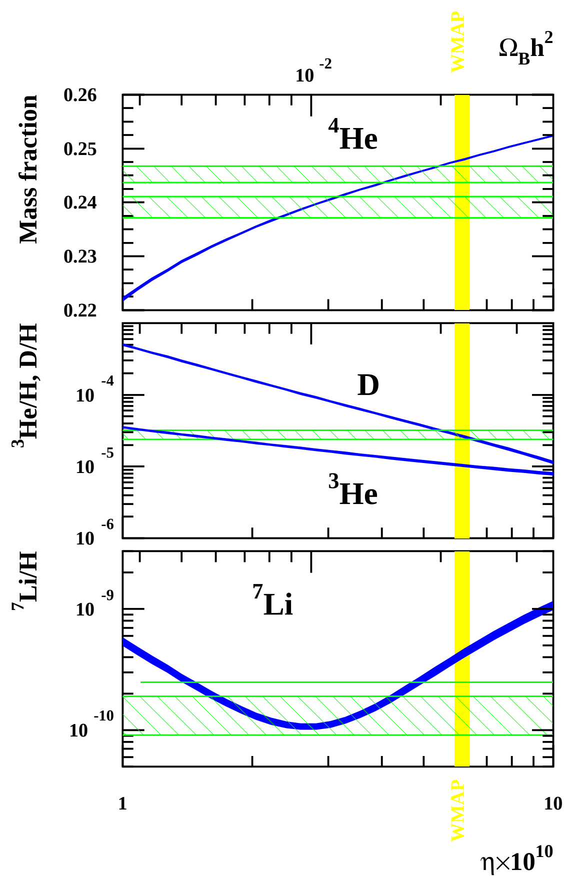
- The CMB is a perfect blackbody (as accurately as can be measured)
- The temperature of CMB is $T_{\rm CMB} = 2.728 \pm 0.004$ K.[a]



[a] Mather et al. (1994)

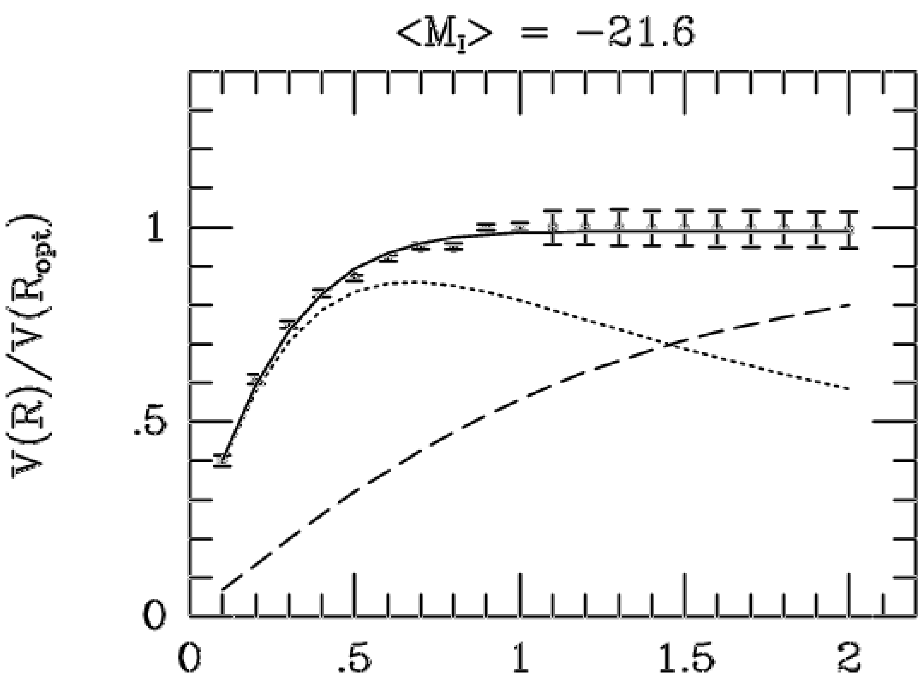
Light element abundances: Big Bang Nucleosynthesis

- The Universe contains approximately 25% ^{4}He by mass
- Also trace quantities of D, ^{3}He, and ^{7}Li.
- The abundances of these elements can be explained by nuclear reactions at $1\,\mathrm{s} < t < 100\,\mathrm{s}$.



Dark Matter

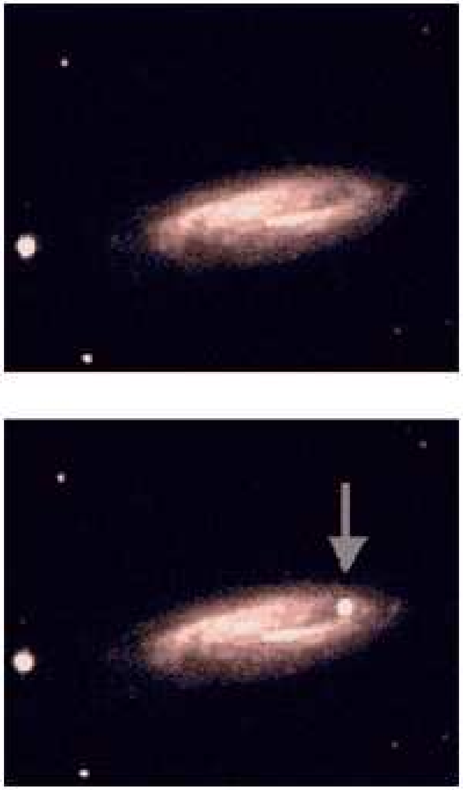
- Peculiar dynamics of stars in galaxies and galaxies in clusters
- Solution: About 25% of the Universe is in the form of unknown neutral particles



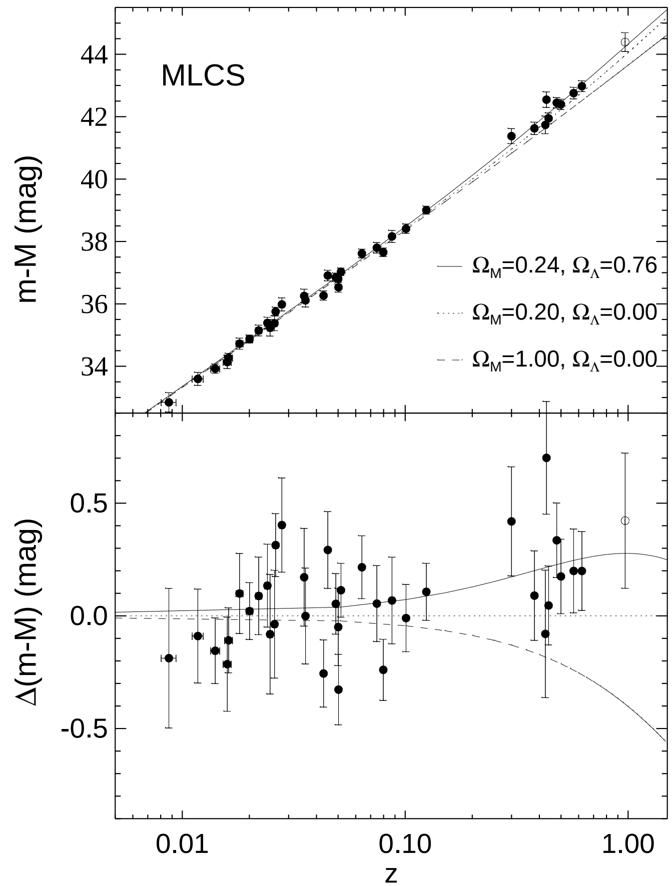
Persic, Salucci, Stel (1996)

Cosmic Acceleration

- 1998: The Universe is accelerating: $\ddot{a} > 0$[a]: is there a cosmological constant?



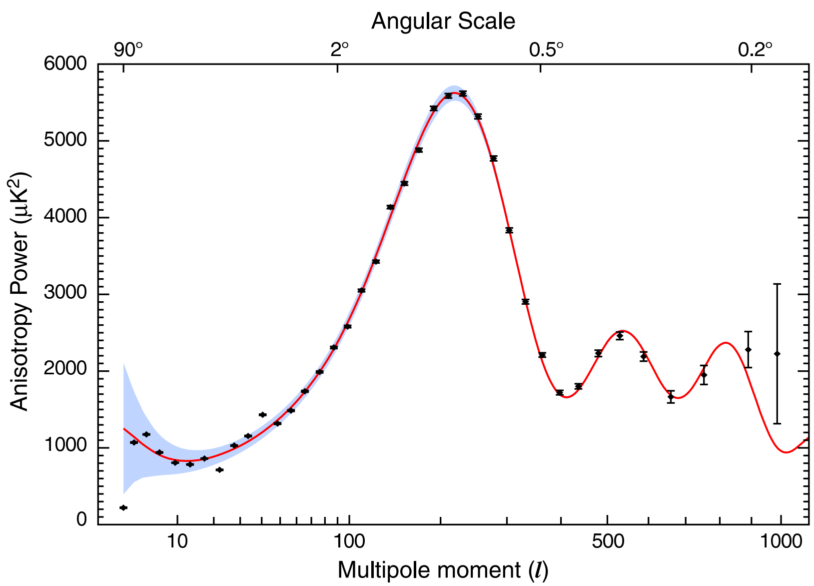
Type 1a Supernova



[a] Riess et al. (1998), Perlmutter et al. (1998)

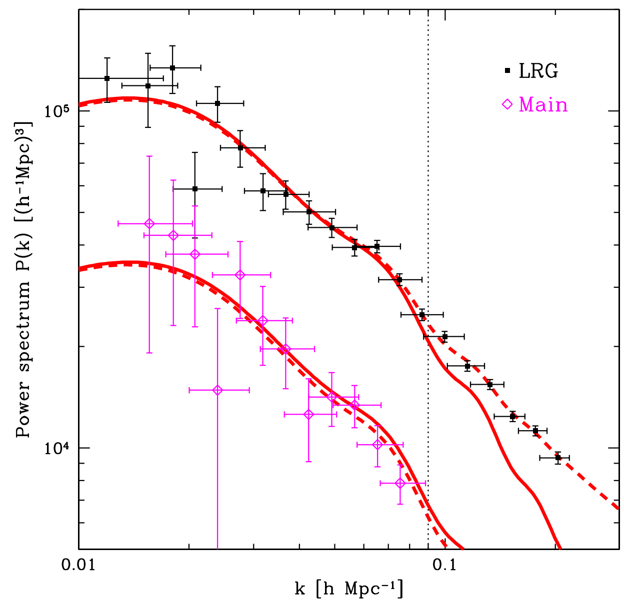
CMB perturbations and large scale structure

- Simple model for CMB angular power spectrum & 3D galaxy power spectrum:
- $(\delta\rho_i/\rho_i)_{\mathbf{k}} = A(k/k_0)^{n_s-1}$ for all species i. (Scalar) spectral index $n_s \simeq 1$.



WMAP 3 angular power spectrum[a]

[a] Hinshaw et al. (2006)



Sloan Digital Sky Survey (2006)[a]

[a] Tegmark et al. (2006), Perceval et al (2007)

What’s not included here

- Ultra-High Energy Cosmic Rays
- Cosmic Neutrinos
- Gravitational waves
- Leptogenesis
- Primordial magnetic fields
- Cosmic (super)strings
- Braneworlds
- Extra dimensions
- Weak lensing
- Non-gaussianity

See plenary talks at

`http://www.cosmo07.info/`

Follow timetable link.

Friedmann equation

Robertson-Walker metric: $ds^2 = g_{\mu\nu}dx^\mu dx^\nu = -dt^2 + a^2(t)d\mathbf{x}^2$

Spatial sections: $d\mathbf{x}^2 = (1 - Kr^2)^{-1}dr^2 + r^2 d\Omega^2$

Describes expanding space of constant curvature K.

General relativity: $R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G T_{\mu\nu} - \Lambda g_{\mu\nu}$
Homogeneity: $T^\mu{}_\nu = \mathrm{diag}(-\rho, p\delta^i_j)$

$(\mu,\nu) = (0,0)$ gives Friedmann equation: $\left(\frac{\dot a}{a}\right)^2 + \frac{K}{a^2} = \frac{8\pi G}{3}\rho + \frac{1}{3}\Lambda$

NB $\mathbf{r} = a(t)\mathbf{x}$, $\dot{\mathbf{r}} = \mathbf{v} = (\dot a/a)\mathbf{r} \equiv H\mathbf{r}$ defines Hubble parameter H.

NB Reduced Planck mass m_P: $m_P^2 = 1/8\pi G$, Planck mass M_P: $M_P^2 = 1/G$

$$H^2 + \frac{K}{a^2} = \frac{1}{3m_P^2}\rho + \frac{1}{3}\Lambda$$

Critical density

(Friedmann)/H^2: $\frac{K}{a^2H^2} = \frac{\rho}{3m_P^2 H^2} - 1 = \frac{\rho}{\rho_c} - 1 = \Omega - 1$

Define critical density $\rho_c = 3m_P^2 H^2$.

Define density parameter $\Omega = \rho/\rho_c$ —-

$$\begin{cases} \Omega > 1 & \text{closed} \\ \Omega = 1 & \text{flat} \\ \Omega < 1 & \text{open} \end{cases}$$

Some numbers:

$$\begin{aligned} H &= 100h \quad \text{km s}^{-1}\ \text{Mpc}^{-1}, \\ h &= 0.72 \pm 0.08 \quad \text{(Hubble Key Project 2001)} \\ \rho_c &= (1.88 \times 10^{-29})h^2 \quad \text{g cm}^{-3} \\ &= (1.05 \times 10^4)h^2 \quad \text{eV cm}^{-3} \\ &\sim (5 \times 10^{-47}) \quad \text{GeV}^4 \sim (10^{-123})M_P^4 \end{aligned}$$

Solving the Friedmann equation

Energy conservation

In GR energy & momentum is covariantly conserved: $\nabla_\mu T^{\mu\nu} = 0$

$$\dot{\bar{\rho}} + 3\frac{\dot{a}}{a}(\bar{\rho} + \bar{p}) = 0$$

Given equation of state $\bar{p} = w\bar{\rho}$

$$\rho \propto a^{-3(1+w)}$$

we can solve Friedmann equation:

$$\rho \propto a^{-p} \longrightarrow \left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3m_P^2}\rho_1\left(\frac{a_1}{a}\right)^p \longrightarrow \boxed{a = a_1(t/t_1)^{2/p}}$$

Simple solutions to the Friedmann equation

Species		w_i	$\bar{\rho}(t)_i$	$a(t)$
Radiation	Relativistic ($T > m$)	$\frac{1}{3}$	a^{-4}	$t^{1/2}$
Matter	Non-relativistic ($T \lesssim m/3$)	0	a^{-3}	$t^{2/3}$
Curvature	Effective contribution[a]	$-\frac{1}{3}$	a^{-2}	t
Cosm. Constant	Effective contribution[b]	-1	a^{0}	$\exp(Ht)$

[a] Define $\rho_K = 3m_P^2 K/a^2$

[b] Define $\rho_\Lambda = m_P^2 \Lambda$

NB $H = \sqrt{\Lambda/3} = \sqrt{\rho_\Lambda/3m_P^2}$

Cosmological constant = inflation: Accelerated expansion ($\ddot{a}/a > 0$).

Deceleration parameter: $q = -\left(\frac{a}{\dot{a}}\right)^2 \frac{\ddot{a}}{a}$

Contents of the Universe today

Species i	Ω_i	*Source*	*Comment*
Photons γ	5×10^{-5}	COBE	$T_\gamma = 2.728 \pm 0.004$ K
Neutrinos	< 0.015	WMAP/SDSS	Neutrino mass (total)?
Baryons	0.044	WMAP/SDSS	Why not $\Omega_b = 0$? What is Ω_l?
Dark matter	0.24	WMAP/SDSS	WIMPs? Axions? Gravitinos?
Dark energy	0.73	WMAP/SNe1a	???
Total	1.02 ± 0.02	WMAP/SDSS/SNe1a	Consistent with flat

NB Parameter estimation is model-dependent

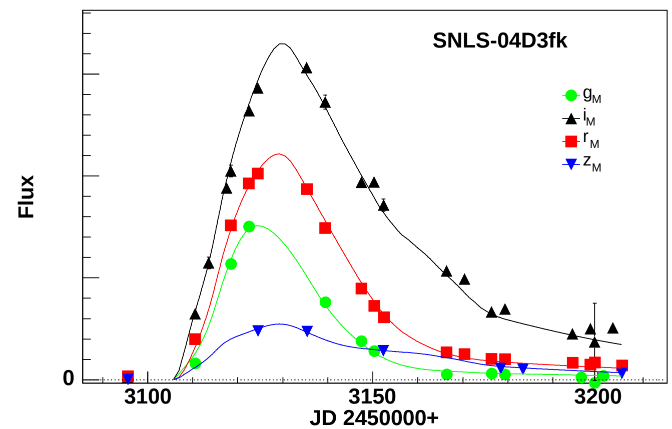
Power Law ΛCDM: the Standard Cosmological Model

WMAP3 only best fit parameters ($K = 0$ by assumption)

Parameter	*Value*	*Comment*
$100\Omega_b h^2$	$2.233^{+0.072}_{-0.091}$	Baryon density
$\Omega_m h^2$	$0.1268^{+0.0072}_{-0.0095}$	Total matter density
h	$0.734^{+0.028}_{-0.038}$	Hubble parameter/100 km/s/Mpc
A	$0.801^{+0.043}_{-0.054}$	Perturbation amplitude ($\|\Delta_{\mathcal{R}}\|^2 = 2.95 \times 10^{-9} A$)
n_s	$0.951^{+0.015}_{-0.019}$	Scalar spectral index.
τ	$0.088^{+0.028}_{-0.034}$	Optical depth to last scattering (polarisation)

The Accelerating Universe: Type 1a Supernovae

- White dwarf (WD) + companion.
- Chandrasekhar:
  $M_{WD} < 1.44 M_\odot$
- Explosion when $M_{WD} = 1.44 M_\odot$

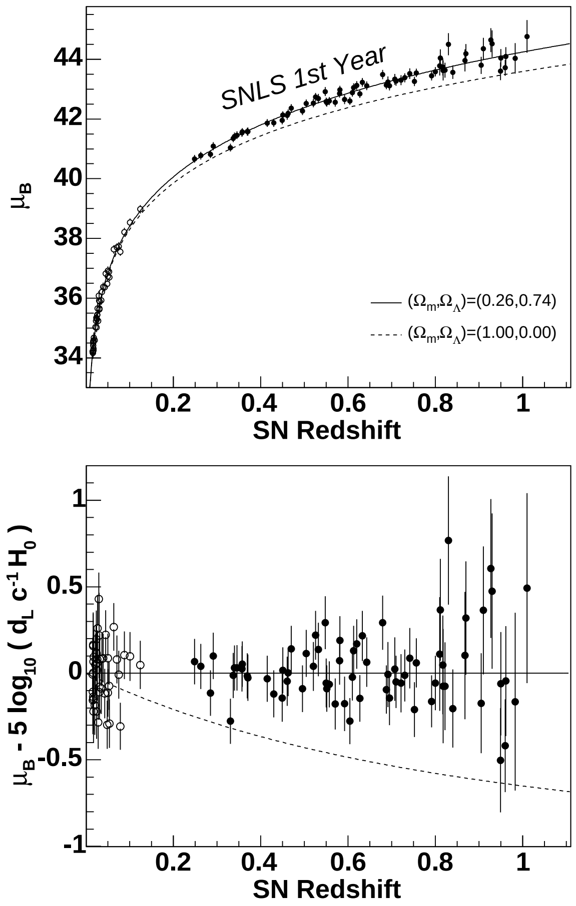


Light curves fitted by 4 parameters:

- m_B^*: Peak luminosity
- t_*: Peak time
- s: A “stretch” parameter
- c: A colour adjustment

Estimate of absolute magnitude μ_B

Accelerating Universe: is it a Cosmological Constant?



SNLS collaboration (2005)

- Absolute magnitude:

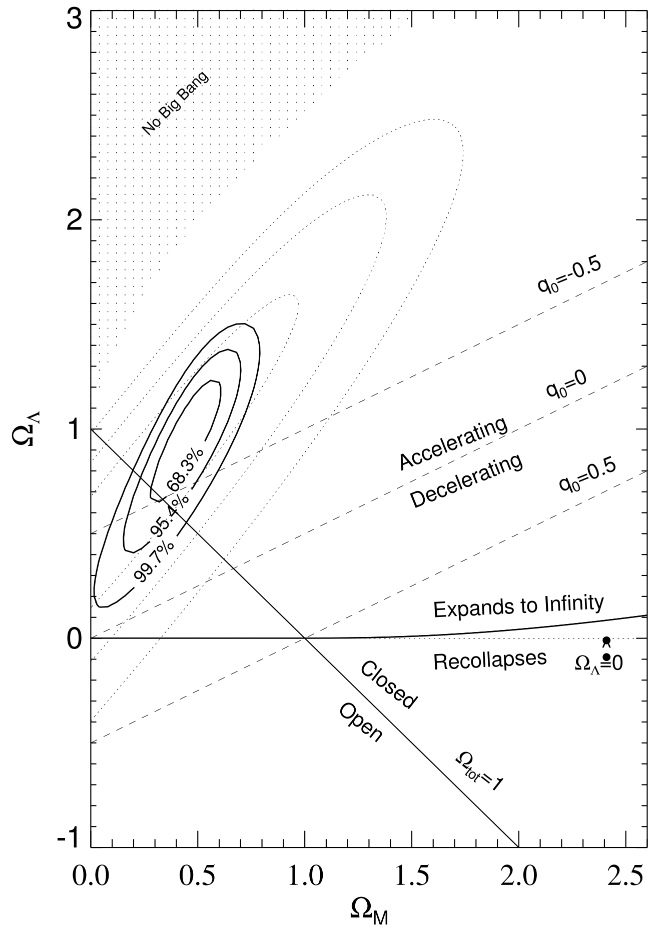
$$\mu_B = m_B^* - M + \alpha(s-1) + \beta c$$

- Theory: $\mu_B = 5\log_{10}(d_L(\theta,z)/10\,\text{pc})$
- Luminosity distance $d_L(\theta,z)$
- Cosmological: $\theta = \{\Omega_\Lambda, \Omega_K, \Omega_m\}$
- Fit $d_L(\theta,z)$ to data: minimise

$$\chi^2(\theta,\alpha,\beta,M) = \sum_{SNe} \frac{\left(\mu_B - 5\log_{10}\left(\frac{d_L(\theta,z)}{10\,\text{pc}}\right)\right)^2}{\sigma^2}$$

- Estimates of $\{\Omega_\Lambda, \Omega_K, \Omega_m\}$

Accelerating Universe: cosmological parameters



Riess et al. (2004)

Conventions

- Natural Units: $\hbar = 1$, $c = 1$, $k_B = 1$
- Natural Unit converter:

Quantity	Nat. U.	S.I. Conversion	
Energy:	GeV	1.6022×10^{-10}	Joule
Temperature:	GeV	1.1605×10^{13}	K
Mass:	GeV	1.7827×10^{-27}	kg
Length:	GeV^{-1}	1.9733×10^{-16}	m
Time:	GeV^{-1}	6.5822×10^{-25}	s

- Planck Mass (Energy): $M_P = \sqrt{\hbar c^5/G} = 1.2211 \times 10^{19}$ GeV
- Reduced Planck Mass $m_P = \sqrt{\hbar c^5/8\pi G} = 2.436 \times 10^{18}$ GeV