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Reduction of Couplings in Unified Theories: From Finiteness to Fuzzy Extra Dimensions

Kladovo '07

SM very successful

BUT with >20 free parameters

ad hoc Higgs sector

ad hoc Yukawa couplings

Best candidate for Physics Beyond SM

MSSM with >100! free parameters.

- cures problem of quadratic divergencies of the SM (hierarchy problem)
- restricts the Higgs sector

- SM with two-Higgs doublets

$$V = m_1^2 |H_1|^2 + m_2^2 |H_2|^2 + (m_3^2 H_1 H_2 + h.c.)$$
$$+ \frac{1}{2}\lambda_1 (H_1^+ H_1)^2 + \frac{1}{2}\lambda_2 (H_2^+ H_2)^2$$
$$+ \lambda_3 (H_1^+ H_1)(H_2^+ H_2) + \lambda_4 (H_1 H_2)(H_1^+ H_2^+)$$
$$+ \left\{ \frac{1}{2}\lambda_5 (H_1 H_2)^2 + \left[\lambda_6 (H_1^+ H_1) + \lambda_7 (H_1^+ H_2^+)\right](H_1 H_2) + h.c. \right\}$$

Supersymmetry provides tree level relations among couplings

$$\lambda_1 = \lambda_2 = \frac{1}{4}(g^2 + g'^2)$$
$$\lambda_3 = \frac{1}{4}(g^2 - g'^2), \quad \lambda_4 = -\frac{1}{4} g^2$$
$$\lambda_5 = \lambda_6 = \lambda_7 = 0$$

With $v_1 = \langle \mathrm{Re}\, H_1^0 \rangle$, $v_2 = \langle \mathrm{Re}\, H_2^0 \rangle$

and $v_1^2 + v_2^2 = (246\ \mathrm{GeV})^2$, $\frac{v_2}{v_1} \equiv \tan\beta$

$\Longrightarrow h^0, H^0, H^\pm, A^0$

At tree level

$$M^2_{h^\circ, H^\circ} = \frac{1}{2}\left\{M_A^2 + M_Z^2 \mp \left[(M_A^2+M_Z^2)^2 - 4M_A^2 M_Z^2 \cos^2 2\beta\right]^{1/2}\right\}$$

$$M^2_{H^\pm} = M_W^2 + M_A^2$$

$$\Rightarrow \begin{cases} M_{h^\circ} < M_Z \left|\cos 2\beta\right| \\ M_{H^\circ} > M_Z \\ M_{H^\pm} > M_W \end{cases}$$

Radiative corrections

$$M^2_{h^\circ} \simeq M_Z^2 \cos^2 2\beta + \frac{3g^2 m_t^4}{16\pi^2 M_W^2} \log \frac{\tilde{m}^2_{t_1} \tilde{m}^2_{t_2}}{m_t^4}$$

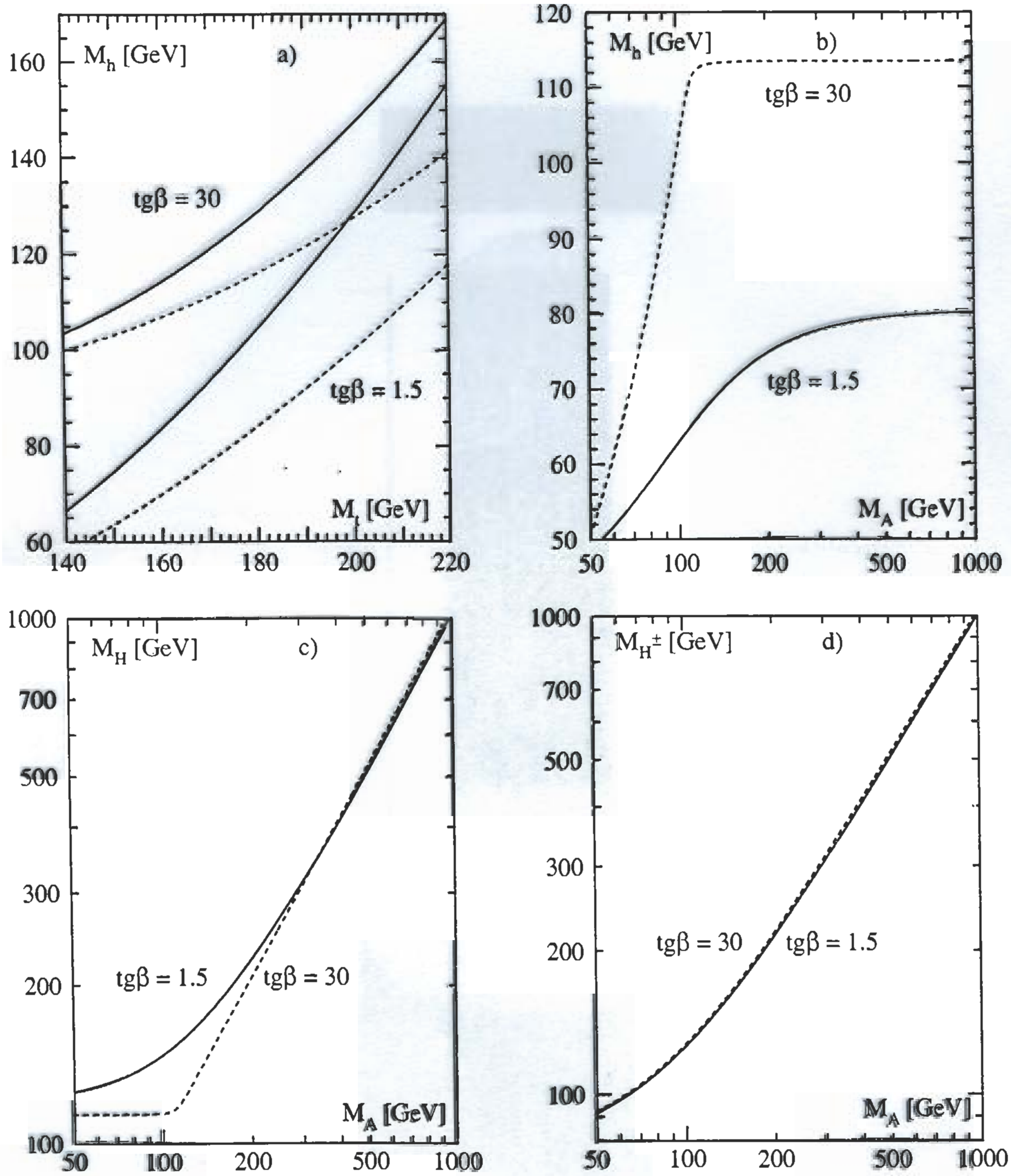

Figure 18: *(a) The upper limit on the light scalar Higgs pole mass in the MSSM as a function of the top quark mass for two values of $tg\beta = 1.5, 30$; the common squark mass has been chosen as $M_S = 1$ TeV. The full lines correspond to the case of maximal mixing $[A_t = \sqrt{6} M_S,\ A_b = \mu = 0]$ and the dashed lines to vanishing mixing. The pole masses of the other Higgs bosons, $H, A, H^{\pm}$, are shown as a function of the pseudoscalar mass in (b–d) for two values of $tg\beta = 1.5, 30$, vanishing mixing and $M_t = 175$ GeV.*

Quantum
Reduction of Couplings

Consider a GUT with

g – gauge coupling

g_i – other couplings (Yukawas, self-couplings)

Any relation among the couplings

$$\Phi(g, g_1, \ldots) = \text{const}$$

which is RGI should satisfy

$$\frac{d}{dt}\Phi = 0 \quad , \quad t = \ln\mu$$

$$\frac{d}{dt}\Phi = \frac{\partial\Phi}{\partial g}\underbrace{\frac{dg}{dt}}_{\beta_g} + \sum_i \frac{\partial\Phi}{\partial g_i}\underbrace{\frac{dg_i}{dt}}_{\beta_i} = 0$$

which is equivalent to

$$\frac{dg}{\beta_g} = \frac{dg_1}{\beta_1} = \frac{dg_2}{\beta_2} = \cdots$$

characteristic system

- Finite Unified Theories (from Quantum Reduction of Couplings)
- Higher Dimensional Unified Theories and Coset Space Dimensional Reduction (Classical Reduction of Couplings)
- Fuzzy Extra Dimensions and Renormalizable Unified Theories

$$\Rightarrow \quad \beta_g \frac{dg_i}{dg} = \beta_i$$

Reduction eqs
Oehme
Zimmermann

Demand power series solution to the REs

$$g_i = \sum_{n=0} \rho_i^{(n+1)} g^{2n+1}$$

Remarkably, uniqueness of these solutions can be decided already at 1-loop!

Assume

$$\beta_i = \frac{1}{16\pi^2}\left[\sum_{j,k,l} \beta_i^{(1)jkl} g_j g_k g_l + \sum_{j \neq g} \beta_i^{(1)j} g_i g^2\right] + \dots$$

$$\beta_g = \frac{1}{16\pi^2} \beta_g^{(1)} g^3 + \cdots$$

Assume $\rho_i^{(n)}$, $n \leq r$ have been uniquely determined

To obtain $\rho_i^{(r+1)}$, insert g_i in REs and collect terms of $O\left(g^{2r+1}\right)$

$$\rightsquigarrow \sum_{\ell \neq g} M(r)_i^{\ell} \rho_\ell^{(r+1)} = \text{lower order quantities Known by assumption}$$

where

$$M(r)_i^{\ell} = 3 \sum_{j,k \neq g} b_i^{(1)jk\ell} \rho_j^{(1)} \rho_k^{(1)} + b_i^{(1)\ell} - (2r+1) b_g^{(1)} \delta_i^{\ell}$$

$$0 = \sum_{j,k,\ell \neq g} b_i^{(1)jk\ell} \rho_j^{(1)} \rho_k^{(1)} \rho_\ell^{(1)} + \sum_{\ell \neq g} b_i^{(1)\ell} \rho_\ell^{(1)} - b_g^{(1)} \rho_i^{(1)}$$

$\Rightarrow$ for a given set of $\rho_i^{(1)}$, the $\rho_i^{(n)}$ for all $n > 1$ can be uniquely determined if

$$\det M(n)_i^{\ell} \neq 0$$

for all n

Kubo et.al.

Consider an $SU(N)$ (non-susy) theory with

$\phi^i(N)$, $\hat{\phi}_i(\bar{N})$ - complex scalars;

$\psi^i(N)$, $\hat{\psi}_i(\bar{N})$ - Weyl spinors

$\lambda^a \; (a=1,\dots,N^2-1)$ - "

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} + i\sqrt{2}\left[g_Y \bar{\psi} \lambda^a T^a \phi - \hat{g}_Y \bar{\hat{\psi}} \lambda^a T^a \hat{\phi} + \text{h.c.}\right] - V(\phi, \hat{\phi}),$$

$$V(\phi, \hat{\phi}) = \frac{1}{4}\lambda_1 (\phi^i \phi_i^*)^2 + \frac{1}{4}\lambda_2 (\hat{\phi}_i \hat{\phi}^{*i})^2 + \lambda_3 (\phi^i \phi_i^*)(\hat{\phi}_j \hat{\phi}^{*j}) + \lambda_4 (\phi^i \phi_j^*)(\hat{\phi}_i \hat{\phi}^{*j})$$

Searching for power series solution of the R.E.s we find

$$g_Y = \hat{g}_Y = g; \; \lambda_1 = \lambda_2 = \frac{N-1}{N} g^2; \; \lambda_3 = \frac{1}{2N} g^2; \; \lambda_4 = -\frac{1}{2} g^2$$

i.e. SUSY

$N=1$ gauge theories

Consider a chiral, anomaly free $N=1$ globally supersymmetric gauge th. based on a group G with gauge coupling g.

Superpotential

$$W = \frac{1}{2} m_{ij} \phi^i \phi^j + \frac{1}{6} C_{ijk} \phi^i \phi^j \phi^k$$

m_{ij}, C_{ijk} – gauge invariant tensors

ϕ^i – matter fields transforming as an ir. rep. R_i of G.

Renormalization constants associated with W

$$\phi^{0i} = (Z^i_j)^{1/2} \phi^j \,, \quad m^0_{ij} = Z^{i'j'}_{ij} m_{i'j'} \,, \quad C^0_{ijk} = Z^{i'j'k'}_{ijk} C_{i'j'k'}$$

$N=1$ non-renormalization thm ensures absence of mass and cubic-int-term infinities

$$Z^{ijk}_{i'j'k'} Z^{1/2\, i'}_{i''} Z^{1/2\, j'}_{j''} Z^{1/2\, k'}_{k''} = \delta^i_{(i''} \delta^j_{j''} \delta^k_{k'')}$$

$$Z^{ij}_{i'j'} Z^{1/2\, i'}_{i''} Z^{1/2\, j'}_{j''} = \delta^i_{(i''} \delta^j_{j'')}$$

(In the background field method)

$$Z_g Z_V^{1/2} = 1$$

$\rightarrow$ Only surviving infinities are $Z^i_j \, (Z_V)$ i.e. one infinity for each field.

The 1-loop β-function of the gauge coupling is

$$\beta_g^{(1)} = \frac{dg}{dt} = \frac{g^3}{16\pi^2}\left[\sum_i \ell(R_i) - 3C_2(G)\right]$$

$\ell(R_i)$ – Dynkin index of R_i

$C_2(G)$ – quadratic Casimir of the adjoint rep.

β-functions of C_{ijk}, by virtue of the non-renormalization thm, are related with the anomalous dim. matrix γ_i^j of ϕ^i

$$\beta_{ijk}^{(1)} = \frac{dC_{ijk}}{dt} = C_{ijl}\gamma_k^l + C_{ikl}\gamma_j^l + C_{jkl}\gamma_i^l$$

$$\gamma_i^{(1)j} = Z^{-\frac{1}{2}}{}_i^k \frac{d}{dt} Z^{\frac{1}{2}}{}_k^j$$

$$= \frac{1}{32\pi^2}\left[C^{jkl}C_{ikl} - 2g^2 C_2(R_i)\delta_i^j\right]$$

$C_2(R_i)$ – quadratic Casimir of R_i

$C^{ijk} = C^*_{ijk}$

$$b_g^{(2)} = \frac{1}{(16\pi^2)^2} 2 g^5 \left[\sum_i \ell(R_i) - 3 C_2(G) \right]$$

$$- \frac{1}{(16\pi^2)^2} \frac{g^3}{r} C_2(R_i) \left[C^{jk\ell} C_{ik\ell} - 2 g^2 C_2(R_i) \delta^j_i \right]$$

Parkes, West, Jones
Mezincescu, Yau
Machacek, Vaughn

$$r : \operatorname{tr} \delta^{ab}$$

$$\gamma^{(2)i}{}_j = \frac{1}{(16\pi^2)^2} 2 g^4 C_2(R_i) \left[\sum_i \ell(R_i) - 3 C_2(G) \right]$$

$$- \frac{1}{(16\pi^2)^2} \frac{1}{2} \left[C^{ik\ell} C_{jkm} + 2 g^2 (R^a)^i_m (R^a)^\ell_j \right]$$

$$\cdot \left[C^{mpq} C_{\ell pq} - 2 \delta^m_\ell g^2 C_2(R_i) \right]$$

$$b_g^{NSVZ} = \frac{g^3}{16\pi^2} \left[\frac{\sum_i \ell(R_i)(1 - 2\gamma_i) - 3 C_2(G)}{1 - g^2 C_2(G)/8\pi^2} \right]$$

Novikov – Shifman – Vainstein – Zakharov

Finite Unification

Old days...

... divergences are "hidden under the carpet" (Dirac, Lects on Q.F.T., '64)

Recent years ...

... divergences reflect existence of a higher scale where new degrees of freedom are excited.

Not just artifacts of pert. th.

However the presence of quadratic divergences means that physics at one scale are very sensitive to unkown physics at higher scales.

⇝ SUSY ths which are free of quadratic divergences in spite of any experimental evidence...

⇝ Natural to expect that beyond unification scale the theory should be completely finite.

- $N=4$ ⇝ finite to all orders in pert.
- $N=2$ ⇝ only 1-loop contributions to β-function. Possible to arrange the spectrum so that theory is finite.

Multiplicities for massless irreducible reps with maximal helicity 1

Spin \ N	1	1	2	2	4
1	—	1	—	1	1
1/2	1	1	2	2	4
0	2	—	4	2	6

$$N=2 \; : \quad \beta(g) = \frac{2g^3}{(4\pi)^2}\left(\sum_i T(R_i) - C_2(G)\right)$$

e.g. $SU(N)$ with $2N$ fundamental reps $\rightsquigarrow \beta(g) = 0$

$SU(5)$: $p(5+\bar{5})$; $q(10+\overline{10})$; $r(15+\overline{15})$
with $p + 3q + 7r = 10$

$SO(10)$: $p(10+\overline{10})$; $q(16+\overline{16})$
with $p + 2q = 8$

E_6 : $4(27+\overline{27})$

Finite Unified Theories

$N=1$

- 1-loop finiteness conditions

$$b_g^{(1)} = 0$$

$$\gamma^{(1)i}_{\ \ j} = 0 \quad \text{- anomalous dimensions of all chiral superfields}$$

- Exists complete classification of all chiral $N=1$ models with $b_g^{(1)}=0$ Hamidi - Patera - Schwarz, Jiang - Zhou

- 1-loop finiteness $\longrightarrow$ 2-loop finiteness Parkes-West, Jones, Mezincescu

- Exist simple criteria that guarantee all loop finiteness (vanishing of all-loop beta functions)

Lucchesi-Piguet-Sibold
Ermushev Kazakov Tarasov
Leigh-Strassler

- All-loop finite SU(5) $\Longrightarrow$ top quark mass ✓

Kapetanakis Mondragon Z '92

~~Susy~~ sector

Jones Mezincescu Yao

- 1-loop finiteness conds (require in particular universal soft ~~susy~~ scalar masses $(m^2)^i_j = \frac{1}{3} M M^* \delta^i_j$)

·· 1-loop finiteness

→ 2-loop finiteness

Jack
Jones

Reduction of couplings

· Extension of method in SSB sector
+ application in min susy SU(5)

Kubo
Mondragon
Z

·· 1-loop sum rule for soft scalar masses in non-finite susy ths.

Kawamura
Kobayashi
Kubo

··· 2-loop sum rule for soft scalar masses in finite ths.

Kobayashi
Kubo
Mondragon
Z

* All-loop RGI relations in finite and non-finite ths

Yamada
Hisano, Shifman
Kazakov
Jack, Jones, Pickering

** All-loop sum rule for soft scalar masses in finite and non-finite ths — Kobayashi, Kubo, Z

•• SU(5) FUTs — Kobayashi, Kubo, Mondragon, Z

- Prediction of s-spectrum in terms of few parameters starting from several hundreds GeV.
- •• The LSP is neutralino ✓ (see e.g. Kazakov et. al. Yoshioka)
- ••• Radiative E-W breaking ✓ (see e.g. Brignole, Ibanez, Munoz)
- •••• No funny colour, charge ✓ (see e.g. Casas et. al)

* Prediction of Higgs masses

Lightest ~ 120 - 130 GeV

Similar results also for min susy SU(5)

Consider a chiral, anomaly free, $N=1$ gauge theory with group G.

The superpotential is

$$W = \frac{1}{6} Y^{ijk} \Phi_i \Phi_j \Phi_k + \frac{1}{2} \mu^{ij} \Phi_i \Phi_j$$

$$\left.\begin{array}{l} Y^{ijk} \\ \mu^{ij} \end{array}\right\} \text{gauge invariant}$$

Yukawa couplings

Φ_i - matter superfields in irreducible reps of G

Necessary and sufficient conditions for $N=1$ 1-loop finiteness

- Vanishing of $b_g^{(1)}$ implies

$$\sum_i \ell(R_i) = 3 C_2(G) \quad ||$$

$\ell(R_i)$ - Dynkin index of R_i
$C_2(G)$ - Quadratic Casimir of G (adjoint)

$\Rightarrow$ Selection of the field content (representations) of the theory

• • Vanishing of $\gamma^{(1)i}_{\ \ j}$ implies

$$Y^{ikl}\, \underset{\uparrow}{Y_{jkl}} = 2\,\delta^i_j\, \underset{\uparrow}{g^2}\, C_2(R_i)$$

Yukawa gauge

$C_2(R_i)$ – quadratic Casimir of R_i

$Y_{ijk} = (Y_{ijk})^*$

$\Rightarrow$ Yukawa and gauge couplings are related.

Note • μ^{ij} are not restricted

•• Appearence of $U(1)$ is incopatible with 1^{st} condt.

••• 2^{nd} condt forbids the presence of singlets with nonvanishing couplings.

∴ $\Rightarrow$ $\cancel{\text{Susy}}$ by G-invariant soft terms

* 1-loop finiteness condts necessary and sufficient to guarantee 2-loop finiteness

* 1-loop finiteness condts ensure that $b_g^{(3)}$ in 3-loops vanishes but in general $\gamma^{(3)}$ does not.

Grisaru - Milewski - Zanon

Parkes - West

What happens in higher loops?

So far 1-loop finiteness condts (or γs) are telling us

$$Y^{ijk} = Y^{ijk}(g)$$

$$b_Y^{(1)ijk} = 0$$

** Necessary and sufficient condts for vanishing β_g and β_{ijk} to all orders

Lucchesi
Piguet
Sibold

1. $\beta_g^{(1)} = 0$
2. $\gamma^{(1)i}_{\ j} = 0$
3. $\beta_Y^{ijk} = \beta_g \dfrac{dY^{ijk}}{dg}$

admit power series solution which in lowest order is a solution of condt 2.

3. → 3'. There exist solutions to $\gamma^{(1)i}_{\ j} = 0$ of the form
$$Y^{ijk} = \rho^{ijk} g, \quad \rho^{ijk}\text{-complex}$$

→ 4. These solutions are isolated and non-degenerate considered as solutions of $\beta_Y^{(1)ijk} = 0$

Recall

R-invariance, axial anomaly

In massless $N=1$ ths

$U(1)$ chiral transformation R:

$$A_\mu \to A_\mu \,, \quad \lambda \to e^{-i\alpha} \lambda \,,$$

$$\phi \to e^{-i\frac{2}{3}\alpha} \phi \,, \quad \psi \to e^{i\frac{1}{3}\alpha} \psi \,, \quad \ldots$$

$$\psi_D = \begin{pmatrix} \psi \\ \bar{\chi} \end{pmatrix} \longrightarrow e^{i\alpha\gamma_5} \psi_D$$

Noether current $J^\mu_R = \bar{\lambda}_D \gamma^\mu \gamma^5 \lambda_D + \cdots$

$$\leadsto \partial_\mu J^\mu_R = r \left(\epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} + \cdots \right)$$

$$r = b_g^{(1)} \,!$$

Only 1-loop contributions due to non-renormalization thm.

Adler, Bardeen, Jackiw, Pi, Shei, Zee

Recall

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$U(1)$ chiral transformation R:

$$A_\mu \to A_\mu \,, \quad \lambda \to e^{-i\alpha}\lambda \,,$$

$$\phi \to e^{-i\frac{2}{3}\alpha}\phi \,, \quad \psi \to e^{i\frac{1}{3}\alpha}\psi \,, \quad \dots$$

$$\psi_D = \begin{pmatrix} \psi \\ \bar{\chi} \end{pmatrix} \longrightarrow e^{i\alpha\gamma_5}\psi_D$$

Noether current $J^\mu_R = \bar{\lambda}_D \gamma^\mu \gamma^5 \lambda_D + \cdots$

$$\rightsquigarrow \partial_\mu J^\mu_R = r\left(\epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} + \cdots\right)$$

$$r = b_g^{(1)} \, !$$

Only 1-loop contributions due to non-renormalization thm.

Adler, Bardeen, Jackiw, Pi, Shei, Zee

Supercurrent

$$\mathcal{J} \equiv \left\{ J'^{\mu}_{R} \; , \; Q^{\mu}_{\alpha} \; , \; T^{\mu}_{\nu} \right\}, \cdots$$

vector super multiplet

associated to R-invariance

associated to susy

associated to translation inv.

Ferrara + Zumino

supercurrent is represented as vector superfield

$$V_\mu(x,\theta,\bar\theta) = R_\mu(x) - i\,\theta^\alpha Q_{\mu\alpha}(x) + i\,\bar\theta_{\dot\alpha}\bar Q^{\dot\alpha}_{\mu}(x)$$

$$-2\left(\theta\sigma^\nu\bar\theta\right)T_{\mu\nu}(x) + \cdots$$

- $J'^{\mu}_{R} \neq J^{\mu}_{R}$

$\therefore \quad J'^{\mu}_{R} = J^{\mu}_{R} + O(\hbar)$

In addition

Clark Piguet Sibold

$$S = \left\{ b_g F^{\mu\nu}F_{\mu\nu} + \cdots, \; b_g\, \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu}F_{\rho\sigma} + \cdots, \right.$$

Super trace anomaly

trace anomaly of T^{μ}_{ν}

anomaly of R-current

$$\left. b_g\, \lambda^\beta \sigma^{\mu\nu}_{\alpha\beta} F_{\mu\nu} + \cdots, \; \cdots \right\}$$

chiral super multiplet

trace anomaly of susy current

There is a relation, whose structure is independent from the renormalization scheme, although individual coefficients (except the 1-loop values of b-functions) may be scheme dependent

$$r = b_g(1 + x_g) + b_{ijk} x^{ijk} - \gamma_A r^A$$

radiative corrections

unrenormalized coefficients of anomalies associated to chiral inv. of superpotential

linear combinations of anomalous dims

Thm: If (i) no gauge anomaly

(ii) $b^{(1)}(g) = 0$ i.e. no R-current anomaly

(iii) $\gamma^{(1)i}_{j} = 0$ implies also $r^A = 0$

(iv) exist solutions to $\gamma^{(1)} = 0$ of the form $C_{ijk} = \rho_{ijk}\, g$, ρ_{ijk} - complex

(v) these solutions are isolated + non-degenerate when considered as solutions of $b^{(1)}_{ijk} = 0$.

- Then each of the solutions can be uniquely extended to a formal power series in g, and the $N=1$ Y-M models depend on the single coupling constant g with a b-function vanishing to all orders.

Proof: Inserting $b_{ijk} = b_g \dfrac{dY_{ijk}}{dg}$ in the identity and taking into account the vanishing of r, r^A

$$\rightsquigarrow \quad 0 = b_g\left(1 + O(\hbar)\right)$$

Its solution (as formal power series in $\hbar$) is: $b_g = 0$ and $b_{ijk} = 0$ too. //

2-loop RGEs for SSB parameters

Martin-Vaughn-Yamada-Jack-Jones

'94

Consider $N=1$ gauge thy with

$$W = \frac{1}{6} Y^{ijk} \Phi_i \Phi_j \Phi_k + \frac{1}{2} \mu^{ij} \Phi_i \Phi_j$$

and SSB terms

$$-\mathcal{L}_{SB} = \frac{1}{6} h^{ijk} \phi_i \phi_j \phi_k + \frac{1}{2} b^{ij} \phi_i \phi_j$$

$$+ \frac{1}{2} (m^2)^i_j \phi^{*i} \phi_j + \frac{1}{2} M \lambda\lambda + h.c.$$

- 1-loop finiteness conditions

$$h^{ijk} = -M Y^{ijk}$$

$$(m^2)^i_j = \frac{1}{3} M M^* \delta^i_j$$

universality

in addition to $\beta_g^{(1)} = \gamma^{(1)i}_j = 0$

- • 1-loop finiteness

$\to$ 2-loop finiteness

Assuming

- $b_g^{(1)} = \gamma^{(1)j}_{\ \ i} = 0$
- the reduction eq

$$\beta_Y^{ijk} = \beta_g \, dY^{ijk}/dg$$

admits power series solution

$$Y^{ijk} = g \sum_{n=0} \rho_{(n)}^{ijk} g^{2n}$$

- $(m^2)^i_j = m_j^2 \delta^i_j$

$$\rightsquigarrow (m_i^2 + m_j^2 + m_k^2)/MM^* = 1 + \frac{g^2}{16\pi^2} \Delta^{(2)}$$

for i, j, k with $\rho_{(0)}^{ijk} \neq 0$

where $\Delta^{(2)} = -2 \sum_l \left[(m_l^2/MM^*) - \frac{1}{3} \right] \ell(R_l)$

- $\Delta^{(2)} = 0$ for $N=4$ with STr cond
- $\Delta^{(2)} = 0$ for the $N=1$, $SU(5)$ FUTs !

The SU(5) finite model

Kapetanakis, Mondragon, Z
Kobayashi, Kubo, Mondragon, Z

Content

$$3(\bar{5}+10)+4(\overset{H_\alpha}{5}+\overset{\bar{H}_\alpha}{\bar{5}})+24$$

fermion supermultiplets

scalar supermultiplets

Hamidi-Schwarz
Jones-Raby
Quiros et. al.
Kazakov
Babu-Enkhbat-Gogoladze

Imposing a discrete symmetry

$$\rightarrow W=\sum_{i=1}^{3}\sum_{\alpha=1}^{4}\left[\frac{1}{2}g^u_{i\alpha}\,10_i\,10_i\,H_\alpha + g^d_{i\alpha}\,10_i\,\bar{5}_i\,\bar{H}_\alpha\right]+\sum_{\alpha=1}^{4}g^f_\alpha H_\alpha\,24\,\bar{H}_\alpha + \frac{g^\lambda}{3}(24)^3$$

with $g^{u,d}_{i\alpha}=0$ for $i\neq\alpha$

We find

$$b_g^{(1)} = 0$$

$$b_{i\alpha}^{u\,(1)} = \frac{1}{16\pi^2}\Big[-\frac{96}{5}g^2 + \sum_{b=1}^{4}\left(g_{ib}^u\right)^2 + 3\sum_{j=1}^{3}\left(g_{ja}^u\right)^2$$

$$+\frac{24}{5}\left(g_\alpha^f\right)^2 + 4\sum_{b=1}^{4}\left(g_{ib}^d\right)^2\Big]g_{i\alpha}^u$$

$$b_{i\alpha}^{d\,(1)} = \frac{1}{16\pi^2}\Big[-\frac{84}{5}g^2 + 3\sum_{b=1}^{4}\left(g_{ib}^u\right)^2 + \frac{24}{5}\left(g_\alpha^f\right)^2$$

$$+4\sum_{j=1}^{3}\left(g_{j\alpha}^d\right)^2 + 6\sum_{b=1}^{4}\left(g_{ib}^d\right)^2\Big]g_{i\alpha}^d$$

$$b^{\lambda\,(1)} = \frac{1}{16\pi^2}\Big[-30g^2 + \frac{63}{5}\left(g^\lambda\right)^2 + 3\sum_{\alpha=1}^{4}\left(g_\alpha^f\right)^2\Big]g^\lambda$$

$$b_\alpha^{f\,(1)} = \frac{1}{16\pi^2}\Big[-\frac{98}{5}g^2 + 3\sum_{i=1}^{3}\left(g_{i\alpha}^u\right)^2$$

$$+4\sum_{i=1}^{3}\left(g_{i\alpha}^d\right)^2 + \frac{48}{5}\left(g_\alpha^f\right)^2$$

$$+\sum_{b=1}^{4}\left(g_b^f\right)^2 + \frac{21}{5}\left(g^\lambda\right)^2\Big]g_\alpha^f$$

Considering g as the primary coupling, we solve the Red. Eqs.

$$\beta_g = \beta_a \frac{dg}{dg_a}$$

requiring power series ansatz.

$$\Rightarrow \left(g^u_{ii}\right)^2 = \frac{8}{5} g^2 + \cdots, \left(g^d_{ii}\right)^2 = \frac{6}{5} g^2 + \cdots$$

$$\left(g^\lambda\right)^2 = \frac{15}{7} g^2 + \cdots, \left(g^f_4\right)^2 = g^2, \left(g^f_\alpha\right)^2 = 0 + \cdots \; (\alpha = 1,2,3)$$

Higher order terms can be uniquely determined.

$\Rightarrow$ All 1-loop β-functions vanish

Moreover

All 1-loop anomalous dimensions of chiral fields vanish.

$$\gamma^{(1)}_{10i} = \frac{1}{16\pi^2}\left[-\frac{36}{5}g^2 + 3\sum_{b=1}^{4}\left(g^u_{ib}\right)^2 + 2\sum_{b=1}^{4}\left(g^d_{ib}\right)^2\right]$$

$$\gamma^{(1)}_{\bar{5}i} = \frac{1}{16\pi^2}\left[-\frac{24}{5}g^2 + 4\sum_{b=1}^{4}\left(g^d_{ib}\right)^2\right]$$

$$\gamma^{(1)}_{H\alpha} = \frac{1}{16\pi^2}\left[-\frac{24}{5}g^2 + 3\sum_{i=1}^{3}\left(g^u_{i\alpha}\right)^2 + \frac{24}{5}\left(g^f_\alpha\right)^2\right]$$

$$\gamma^{(1)}_{\bar{H}\alpha} = \frac{1}{16\pi^2}\left[-\frac{24}{5}g^2 + 4\sum_{i=1}^{3}\left(g^d_{i\alpha}\right)^2 + \frac{24}{5}\left(g^f_\alpha\right)^2\right]$$

$$\gamma^{(1)}_{24} = \frac{1}{16\pi^2}\left[-\frac{10}{5}g^2 + \sum_{\alpha=1}^{4}\left(g^f_\alpha\right)^2 + \frac{21}{5}\left(g^\lambda\right)^2\right]$$

$\Rightarrow$ Necessary and sufficient conds for finiteness to all orders are satisfied

- SU(5) Breaks down to the standard model due to $\langle 24 \rangle$

- Use the freedom in mass parameters to obtain only a pair of Higgs fields light, acquiring v.e.v.

- Get rid of unwanted triplets rotating the Higgs sector (after a fine tuning)

  see Quiros et. al., Kazakov et. al
  Yioshioka

- Adding soft terms we can achieve SUSY breaking.

1) Gauge Couplings Unification

$\sin^2\theta_W,\ \alpha_{em} \rightsquigarrow \alpha_3(M_Z)$

Marciano+Senjanović
⋮
Amaldi et. al.

2) Bottom-Tau Yukawa Unif.

SU(5)-type

$\rightsquigarrow m_t \sim 100-200$ GeV

Barger et. al.
Carena et. al.

*3) Top-Bottom-Tau Yuk Unif.

$h_t^2 = \frac{4}{3} h_{b,\tau}^2$ in SU(5)-FUT

Similar to SO(10)

Ananthanarayan et. al.
Barger et. al.
Carena et. al.

$\rightsquigarrow m_t \sim 160-200$ GeV

*4) Gauge-Top-Bottom-Tau Unif.

e.g. FUT-SU(5): $h_t^2 = \frac{8}{5} g_U^2\ ;\ h_{b,\tau}^2 = \frac{6}{5} g_U^2$

M_s [GeV]	$\alpha_{3(5f)}(M_Z)$	$\tan\beta$	M_{GUT} [GeV]	M_b [GeV]	M_t [GeV]
300	0.123	54.1	2.2×10^{16}	5.3	183
500	0.122	54.2	1.9×10^{16}	5.3	183
10^3	0.120	54.3	1.5×10^{16}	5.2	184

FUTA

M_s [GeV]	$\alpha_{3(5f)}(M_Z)$	$\tan\beta$	M_{GUT} [GeV]	M_b [GeV]	M_t [GeV]
800	0.120	48.2	1.5×10^{16}	5.4	174
10^3	0.119	48.2	1.4×10^{16}	5.4	174
1.2×10^3	0.118	48.2	1.3×10^{16}	5.4	174

FUTB

M_s [GeV]	$\alpha_{3(5f)}(M_Z)$	$\tan\beta$	M_{GUT} [GeV]	M_b [GeV]	M_t [GeV]
300	0.123	47.9	2.2×10^{16}	5.5	178
500	0.122	47.8	1.8×10^{16}	5.4	178
1000	0.119	47.7	1.5×10^{16}	5.4	178

MIN SU(5)

The predictions for the three models for different M_s

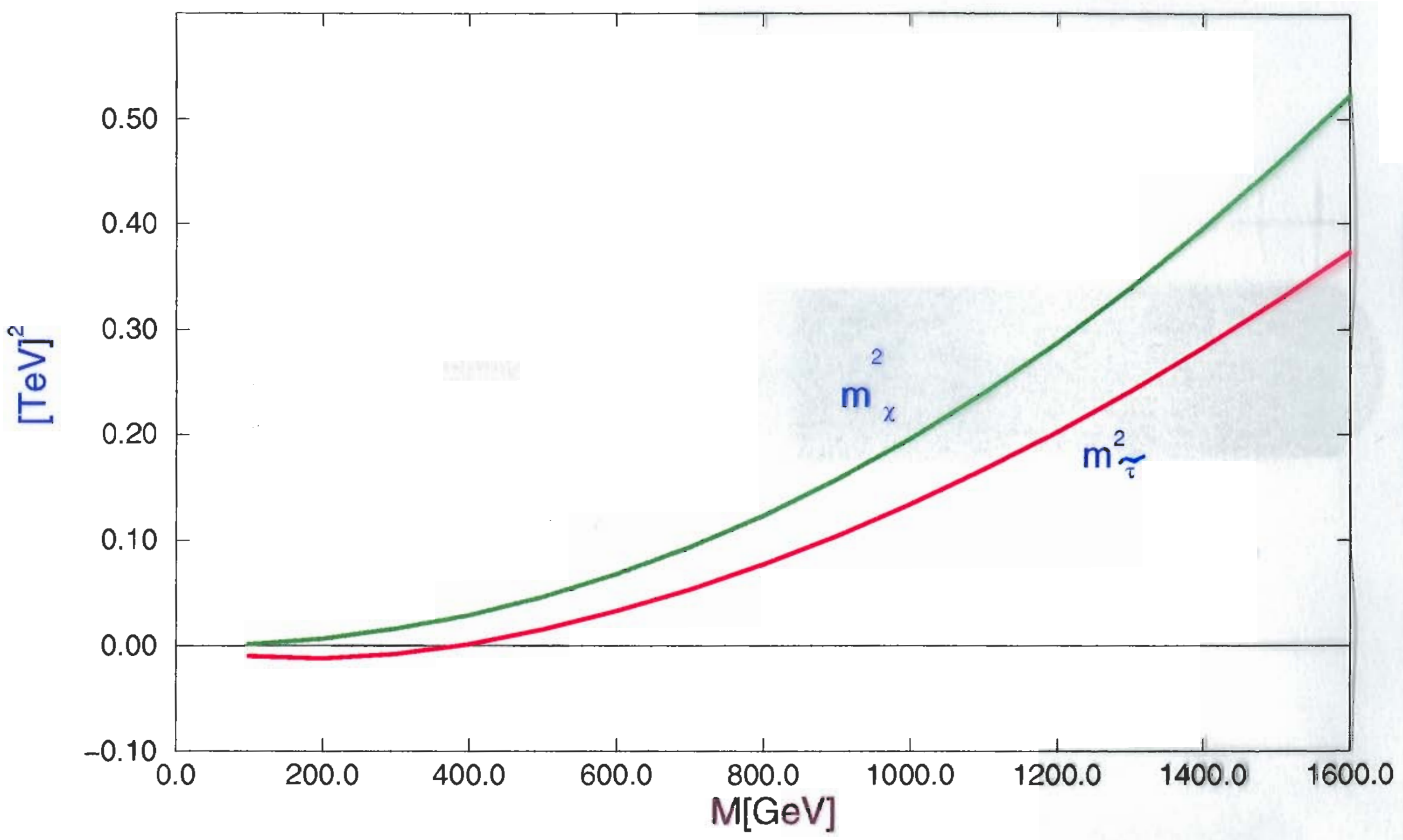
With theoretical corrections and uncertainties

$\sim 4\%$

$M_t = 173.8 \pm 5$ GeV; 178.0 ± 4.3 GeV
CDF + D0

Model A

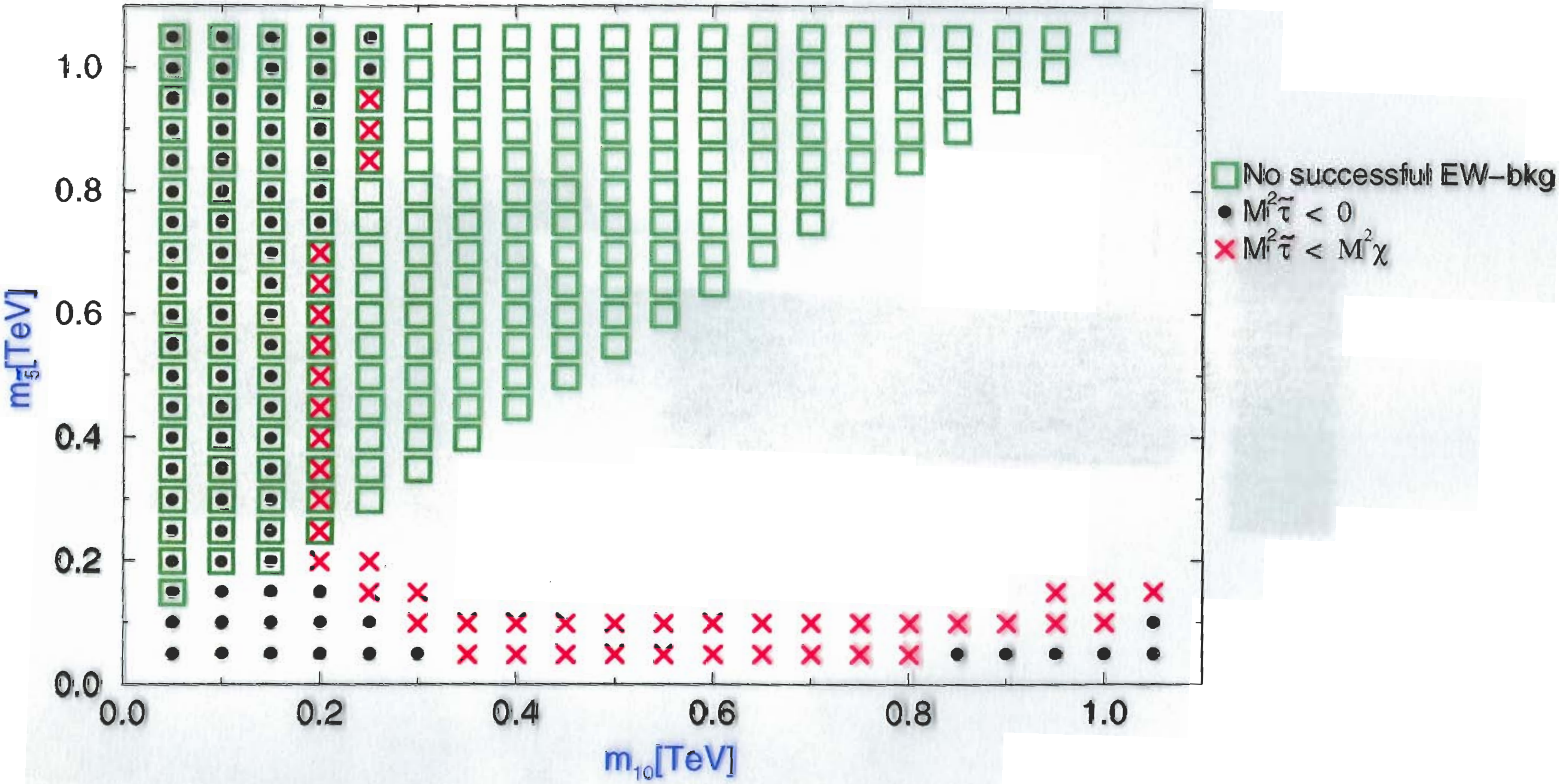
Similar behaviour holds for Model B too



$m^2_{\tilde\tau}$ and m^2_χ for the universal choice of soft scalar masses

Model A

$M_{susy} = 0.3$ TeV



The empty region yields a neutralino as LSP

Djouadi
Heinemeyer
Mondragon
Z

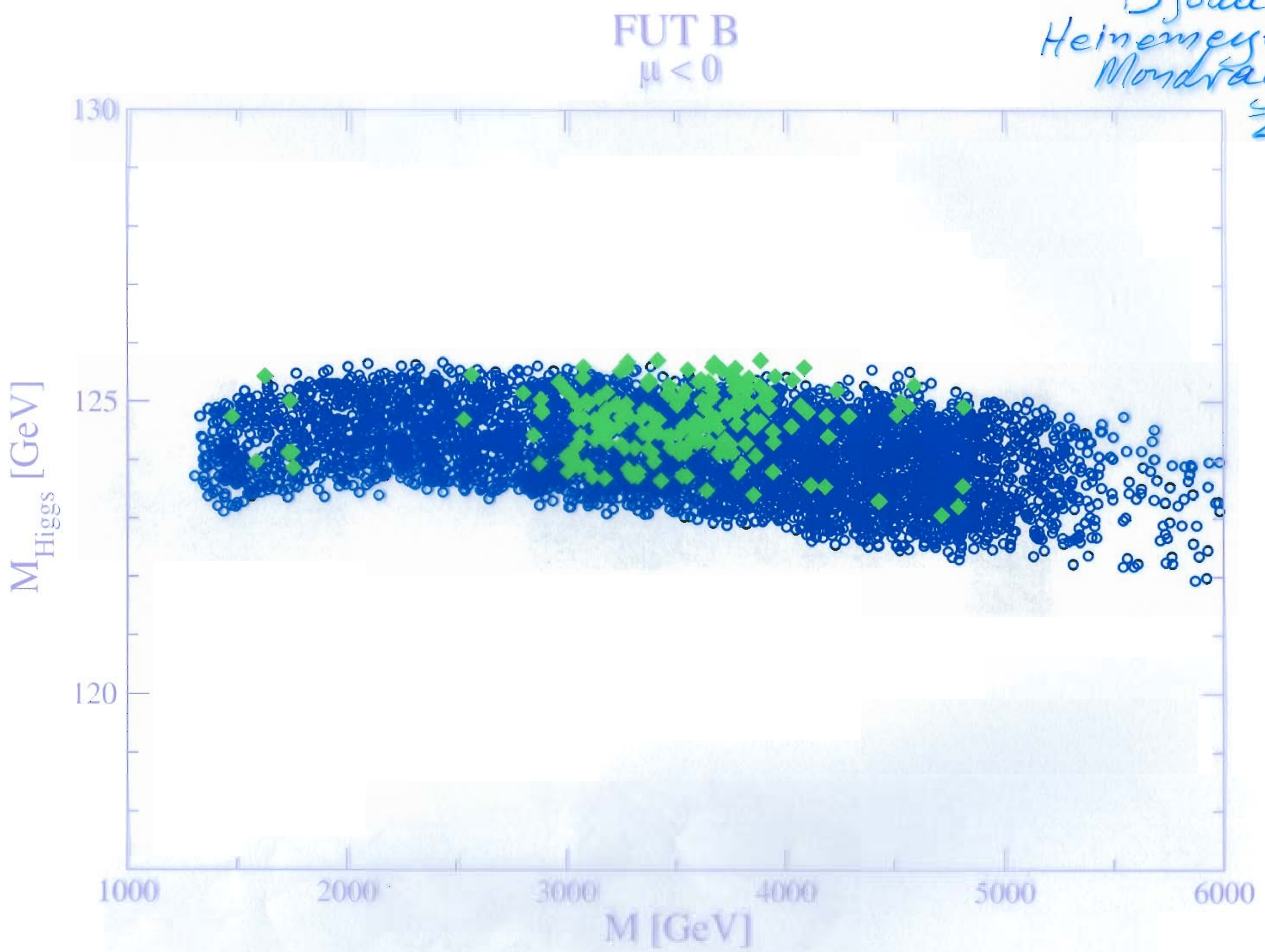


Table 3: A representative example of the predictions for the s-spectrum for the model **A**.

$m_\chi \equiv m_{\chi_1}$ (TeV)	0.22	$m_{\tilde{b}_2}$ (TeV)	1.06
m_{χ_2} (TeV)	0.41	$m_{\tilde{\tau}} = m_{\tilde{\tau}_1}$ (TeV)	0.33
m_{χ_3} (TeV)	0.93	$m_{\tilde{\tau}_2}$ (TeV)	0.54
m_{χ_4} (TeV)	0.94	$m_{\tilde{\nu}_1}$ (TeV)	0.41
$m_{\chi_1^\pm}$ (TeV)	0.41	m_A (TeV)	0.44
$m_{\chi_2^\pm}$ (TeV)	0.94	$m_{H^\pm}$ (TeV)	0.45
$m_{\tilde{t}_1}$ (TeV)	0.94	m_H (TeV)	0.44
$m_{\tilde{t}_2}$ (TeV)	1.09	m_h (TeV)	0.12
$m_{\tilde{b}_1}$ (TeV)	0.86		

Table 4: A representative example of the predictions of the s-spectrum for the model **B**.

$m_\chi \equiv m_{\chi_1}$ (TeV)	0.44	$m_{\tilde{b}_2}$ (TeV)	1.79
m_{χ_2} (TeV)	0.84	$m_{\tilde{\tau}} = m_{\tilde{\tau}_1}$ (TeV)	0.47
m_{χ_3} (TeV)	1.38	$m_{\tilde{\tau}_2}$ (TeV)	0.69
m_{χ_4} (TeV)	1.39	$m_{\tilde{\nu}_1}$ (TeV)	0.62
$m_{\chi_1^\pm}$ (TeV)	0.84	m_A (TeV)	0.74
$m_{\chi_2^\pm}$ (TeV)	1.39	$m_{H^\pm}$ (TeV	0.75
$m_{\tilde{t}_1}$ (TeV)	1.60	m_H (TeV	0.74
$m_{\tilde{t}_2}$ (TeV)	1.82	m_h (TeV)	0.12
$m_{\tilde{b}_1}$ (TeV)	1.56		

Coset Space Dimensional Reduction
(CSDR)

Original motivation
Use higher dimensions
- to unify the gauge and Higgs sectors
- to unify the fermion interactions with gauge and Higgs fields

Supersymmetry provides further unification (fermions in adj reps)

Forgacs + Manton ; Manton
Chapline + Slansky
Kubyshin + Mourao + Rudolph + Volobujev-book
Kapetanakis + G. Z. - Phys. Rept
Manousselis + G. Z., Phys. Lett. B504, 122 (01); PLB518, 171 (01); JHEP03, 002 (02); JHEP11, 025 (04)

Further successes

(a) chiral fermions in 4 dims from vector-like reps in the higher dim thy.

(b) the metric can be deformed (in certain non-symmetric coset sp) and more than one scales can be introduced

(c) Wilson flux Breaking can be used

ADD

- Softly broken susy chiral ths in 4 dims can result from a higher dimensional susy theory

Theory in D dims $\longrightarrow$ Thy in 4 dims

1. Compactification $\underset{x^M}{M^D} \longrightarrow \underset{x^\mu}{M^4} \times \underset{y^a}{B}$

B - a compact space

$\dim B = D - 4 = d$

2. Dimensional Reduction

Demand that $\mathcal{L}$ is independent of the extra y^a coordinates

- One way: Discard the field dependence on y^a coordinates

- - An elegant way: Allow field dependence on y^a and employ a symmetry of the Lagrangian to compensate.

Obvious choice: Gauge Symmetry

Allow a non-trivial dependence on y^a, but impose the condition that a symmetry transformation by an element of the isometry group S of B is compensated by a gauge transformation.

$\Longrightarrow$ $\mathcal{L}$ independent of y^a just because is gauge invariant.

Integrate out extra coordinates

CSDR: $B = S/R$

$$S: \; Q_A = \{ \underset{R}{Q_i}, \underset{S/R}{Q_a} \}$$

$$[Q_i, Q_j] = f_{ij}^{\;\,k} Q_k \quad , \quad [Q_i, Q_a] = f_{ia}^{\;\,b} Q_b$$

$$[Q_a, Q_b] = f_{ab}^{\;\,i} Q_i + f_{ab}^{\;\,c} Q_c$$

vanishes in symmetric S/R

Fuzzy CSDR

Aschieri
Madore
Manousselis
Z

JHEP0404('04)34
hep-th/0401200
hep-th/0503039

$$M^D = M^4 \times (S/R)_F$$

finite matrix manifold
e.g. fuzzy sphere S^2_F

Instead of considering the algebra of functions

$$\mathrm{Fun}(M^D) \sim \mathrm{Fun}(M^4) \times \mathrm{Fun}(S/R)$$

we consider the algebra

$$A = \mathrm{Fun}(M^4) \times M_N$$

M_N - finite dim NC (non-com) algebra of matrices that approximates the functions on $(S/R)_F$

On A we still have the action of symmetry group S $\rightsquigarrow$ we can apply CSDR

Fuzzy Sphere

Madore

Nice example of $(S/R)_F$ is the fuzzy sphere S^2_F, a matrix approximation of S^2. The algebra of functions on S^2 (spanned by spherical harmonics) is truncated at a given angular momentum and becomes finite dimensional. The algebra becomes that of $N \times N$ matrices.

The associativity of the algebra is nicely achieved by relaxing commutativity

The algebra of functions on S^2 can be generated by the coordinates of R^3 modulo the relation $\sum_{a=1}^{3} x_a^2 = r^2$

Scalar functions on S^2 can be expanded

$$f(\theta,\phi) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} f_{\ell m}\, Y_{\ell m}(\theta,\phi)$$

spherical harmonics

$Y_{\ell m}(\theta,\phi)$ can be expressed in terms of the cartesian coordinates X_a, $a=1,2,3$ in $\mathbb{R}^3$

$$Y_{\ell m}(\theta,\phi) = \sum_a f^{(\ell m)}_{a_1\ldots a_\ell}\, X^{a_1}\ldots X^{a_\ell}$$

traceless symmetric tensor of $SO(3)$ with rank ℓ

Similarly we can expand $N\times N$ matrices of a matrix theory on a fuzzy sphere

$$\hat{f} = \sum_{\ell=0}^{N-1} \sum_{m=-\ell}^{\ell} f_{\ell m}\, \hat{Y}_{\ell m}$$

$$\hat{Y}_{\ell m} = r^{-\ell} \sum_a f^{(\ell m)}_{a_1\ldots a_\ell}\, \hat{X}^{a_1}\ldots \hat{X}^{a_\ell}$$

where $f^{(\ell m)}_{a_1\ldots a_\ell}$ the same as in S^2, while

$$\hat{X}_a = r\frac{i}{\sqrt{N^2-1}}\, X_a\ , \qquad \hat{X}_a^{+} = \hat{X}_a$$

are $N\times N$ hermitian matrices proportional to the N-dimensional rep of $SU(2)$

They satisfy

$$\sum_{a=1}^{3} \hat{X}_a \hat{X}_a = r^2 \quad , \quad [X_a, X_b] = \epsilon_{abc} X_c$$

$\hat{Y}_{lm}$ - fuzzy spherical harmonics

they obey $\quad Tr_N(\hat{Y}^+_{lm} \hat{Y}_{l'm'}) = \delta_{ll'} \delta_{mm'}$

Obvious relation

$$\hat{f} = \sum_{l=0}^{N-1} \sum_{m=-l}^{l} f_{lm} \hat{Y}_{lm} \longrightarrow \sum_{l=0}^{N-1} \sum_{m=-l}^{l} f_{lm} Y_{lm}(\theta, \phi)$$

Similarly

$$\frac{1}{N} Tr_N \longrightarrow \frac{1}{4\pi} \int d\Omega \quad , \quad d\Omega = \sin\theta d\theta d\phi$$

In addition on S^2_F there is a natural $SU(2)$ covariant differential calculus. The derivations of a function f along X_a are given by

$$e_a(f) = [X_a, f] \quad , \quad a = 1, 2, 3$$

i.e. this calculus is 3-dimensional.

These are essentially the angular momentum operators

$$J_a f = i e_a f = [i X_a, f]$$

which satisfy the $SU(2)$ Lie algebra

$$[J_a, J_b] = i \epsilon_{abc} J_c$$

In the limit $N \to \infty$ the e_a become

$$e_a = \epsilon_{abc} x_b \partial_c$$

i.e. 2-dimensional

The exterior derivative is given by

$$df = [X_a, f] \theta^a$$

θ^a - 1-forms dual to e_a, $\langle e_a, \theta^b \rangle = \delta^b_a$

1-forms are generated by θ^a

$$\omega = \sum_{a=1}^{3} \omega_a \theta^a, \quad \omega \text{ any 1-form}$$

1-form on $M^4 \times S^2_F$: $A = A_\mu dx^\mu + A_a \theta^a$

with $A_\mu = A_\mu(x^\mu, x_a)$, $A_a = A_a(x^\mu, x_a)$

Non Commutative gauge fields and transformations

Consider a field $\phi(X_a)$ on a fuzzy space described by non-com coordinates X_a. An infinitesimal gauge transformation

$$\delta\phi(X_a) = \lambda(X_a)\,\phi(X_a)$$

$\lambda(X_a)$ - gauge transformation parameter

$U(1)$ if $\lambda(X_a)$ antihermitian function of X_a

$U(P)$ if $\lambda(X_a)$ is valued in Lie algebra of $P \times P$ matrices

Coordinates X_a invariant under gauge transformation $\quad \delta X_a = 0$

- $\delta(X_a\phi) = X_a\lambda(X_a)\phi \neq \lambda(X_a)X_a\phi$

- $\delta(\phi_a\phi) = \lambda(X_a)\phi_a\phi$

covariant coordinates

which holds if $\delta(\phi_a) = [\lambda(X_a), \phi_a]$

- $\phi_a = X_a + A_a$

NC analogue of covariant derivative

interpreted as gauge fields

note that $\delta A_a = -[X_a, \lambda] + [\lambda, A_a]$

supporting the interpretation of A_a

Correspondingly define

- $$F_{ab} = [X_a, A_b] - [X_b, A_a] + [A_a, A_b] - C^c{}_{ab}A_c$$
$$= [\phi_a, \phi_b] - C^c{}_{ab}\phi_c$$ analogue of field strength

$\rightsquigarrow$ $\delta F_{ab} = [\lambda, F_{ab}]$

- $\delta\psi = [\lambda, \psi]$, spinor ψ in the adjoint

Actions in higher dimensions seen as 4-dim actions (expansion in Kaluza-Klein modes)

$G = U(P)$ on $M^4 \times (S/R)_F$

$$A_{YM} = \frac{1}{4} \int d^4x \, Tr \, tr_G \, F_{MN} F^{MN}$$

integration over $(S/R)_F$

$$F_{MN} \longrightarrow (F_{\mu\nu}, F_{\mu b}, F_{ab})$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu]$$

$$F_{\mu a} = \partial_\mu A_a - [X_a, A_\mu] + [A_\mu, A_a]$$

$$= \partial_\mu \phi_a + [A_\mu, \phi_a] = D_\mu \phi_a$$

$$F_{ab} = [\phi_a, \phi_b] - C^c{}_{ab} \phi_c$$

$$\rightsquigarrow A_{YM} = \int d^4x \, Tr \, tr_G \left(\frac{1}{4} F_{\mu\nu}^2 + \frac{1}{2} (D_\mu \phi_a)^2 - V(\phi) \right)$$

$$V(\phi) = -\frac{1}{4} Tr \, tr_G \sum_{ab} F_{ab} F_{ab}$$

$$= -\frac{1}{4} Tr \, tr_G \sum_{ab} \left([\phi_a, \phi_b] - C^c{}_{ab} \phi_c \right) \left([\phi_a, \phi_b] - C^c{}_{ab} \phi_c \right)$$

The infinitesimal G gauge transf with parameter $\lambda(x^\mu, X^a)$ can be interpreted as M^4 gauge transformation

$$\lambda(x^\mu, X^a) = \lambda^\alpha(x^\mu, X^a)\,\Upsilon^\alpha$$
$$= \lambda^{h,\alpha}(x^\mu)\,T^h\,\Upsilon^\alpha$$

Υ^α - generators of $U(P)$

$\lambda^\alpha(x^\mu, X^a)$ - $N \times N$ matrices, therefore expressible as

$$\left.\begin{array}{l}\text{Kaluza-Klein} \\ \text{modes of} \\ \lambda(x^\mu, X^a)^\alpha\end{array}\right\} - \lambda(x^\mu)^{\alpha,h}\,\underset{\substack{| \\ \text{generators of } U(N)}}{T^h}$$

Considering on equal footing the indices h and α we interpret $\lambda^{h,\alpha}(x^\mu)$ as a field valued in the tensor product $Lie(U(N)) \otimes Lie(U(P)) = Lie(U(NP))$

Similarly we write the gauge field A_ν as

$$A_\nu(x^\mu, X^a) = A_\nu^\alpha(x^\mu, X^a)\, \Upsilon^\alpha$$
$$= A_\nu^{h,\alpha}(x^\mu)\, T^h\, \Upsilon^\alpha$$

and interpret it as $Lie(U(NP))$ valued gauge field on M^4.

Similarly for ϕ_a

Then we reduce the number of gauge fields and scalars by applying the CSDR principle.

e.g. $G = U(1)$, $(S/R)_F = S^2_F$

CSDR constraints are satisfied by embedding $SU(2)$ in $U(N)$.

We find in four dimensions

- No H group (due to the fact that the differential calculus is based on $\dim S$ derivations instead of $\dim S - \dim R$ in ordinary case)
- $K = C_{U(N)}(SU(2)) = U(N-2) \times U(1)$ as the final gauge group
- a harmless (singlet) surviving Higgs

Similar results are obtained for $G = U(P)$

CSDR for more general $(S/R)_F$

(e.g. CP^M described by $N \times N$ matrices)

CSDR constraints are satisfied by embedding S in $U(N \cdot P)$ and the 4-dim gauge group is

$$K = C_{U(N \cdot P)}(S)$$

Concerning fermions, to solve the corresponding constraints we embed

$$S \subset SO(\dim S)$$

$$U(N \cdot P) \supset S_{U(N \cdot P)} \times K$$

$$\mathrm{adj}\, U(N \cdot P) = (\mathrm{adj}\, S, 1) + (1, \mathrm{adj}\, K) + \sum_i (s_i, k_i)$$

$$\begin{array}{lcl} SO(\dim S) & \supset & S \\ \text{spinor } \sigma & = & \sum_i \sigma_i \end{array}$$

for $s_i = \sigma_i \rightsquigarrow k_i$ survive in 4 dims

Major difference among ordinary and fuzzy-CSDR

- 4-dim gauge theory appears already spontaneously broken

⤳ in 4 dims appears only the physical Higgs that survives SSB

⤳ Yukawa sector

(i) massive fermions

(ii) interactions among fermions and physical Higgs fields.

⟹ if we obtain in fuzzy-CSDR the SM ⤳ large extra dims

Fundamental differences among ordinary and fuzzy-CSDR:

- A non-abelian gauge group is not necessary in high dims.

The presence of a $U(1)$ in the higher-dim theory is enough to obtain non-abelian gauge theories in 4 dims.

- The theory is renormalizable in the sense that divergencies can be removed by a finite number of counterterms.

Aschieri
Grammatikopoulos
Steinacker
Z
hep-th/0606021
JHEP
hep-th/07060398

We have constructed a renormalizable 4-dim $SU(N)$ gauge theory with suitable multiplet of scalar fields.

The symmetry breaking pattern and low-energy gauge group are determined dynamically in terms of a few free parameters of the potential. Depending on these parameters the final gauge group can be

$$SU(n) \quad \text{or} \quad SU(n_1) \times SU(n_2) \times U(1)$$

We explicitly found the tower of massive K-K modes, consistent with an interpretation as dimensionally reduced higher-dim gauge theory over an S^2_F.