Nonsupersymmetric Attractors in String Theory

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Outline

- Introduction and Motivation
- Attractor Mechanism and Supersymmetry
- Attractor Mechanism and Extrimality
- Nonsupersymmetric Attractors
- Examples
- Conclusion

- Refs. arXiv: hep-th/0511117 by Tripathy,Trivedi
  arXiv:0705.4554 [hep-th] by Nampuri,Tripathy,Trivedi

References

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- Ferrara, Kallosh, Strominger, hep-th/9508072.
- Gibbons, Kallosh, Kol, hep-th/9607018.
- Ferrara, Gibbons, Kallosh, hep-th/9702103.
- Denef, hep-th/0005049.
- Goldstein, Iizuka, Jena, Trivedi, hep-th/0507096.
- Kallosh et. al., Cardoso et.al. ...

Introduction and Motivation

- Black holes are solutions to Einstein's equations with a horizon. The thermodynamic interpretation of black holes suggests that they carry entropy.
- Bekenstein-Hawking formula

$$S = \frac{A}{4G} \, .$$

- String theory gives a microscopic description of the entropy of a black hole.
- In string theory, they correspond to branes wrapping various cycles of a Calabi-Yau manifold. Degeneracy of these microstates corresponding to these brane configurations give entropy of the black holes.

- Compactifications on Calabi-Yau spaces give rise to moduli. So the corresponding black hole solutions in supergravity should also depend on the moduli.
- In particular the entropy should depend on these moduli, which are continuous parameters!
- Attractor Mechanism gives the explanation to this puzzle.
- Most of the discussion in literature is on susy preserving black holes and the corresponding brane configurations. So we have consistent microscopic description of entropy for susy preserving black holes.
- What happens to the black holes which don't preserve any supersymmetry?

Attractor Mechanism

- Consider Einstein's theory coupled to a $U(1)$ field. There are two possible vacuum configurations.
  - Flat Minkowski Vacuum.
  - $AdS_2 \times S^2$ (Bertotti-Robinson space).
- A black hole interpolate these two vacua.
- For, $N = 2$ supergravity theory in $4D$ coupled to 'n' vector multiplets, in addition to metric and gauge fields, we also have scalars.
- The scalar fields take arbitrary VEVs at the Minkowskin vacuum. At the $AdS_2 \times S^2$ vacuum they flow to a fixed point (which is completely determined by the charges of the black hole). $\implies$ Attractor!

- In order to understand the attractor mechanism better analyse solutions to the spinor conditions (for a static, spherically symmetric black hole solution). The metric and gauge fields are

$$\begin{aligned} ds^2 &= -e^{2U(r)}dt^2 + e^{-2U(r)}d\mathbf{r}^2 \\ \hat{F}_r^\Lambda &= \frac{p^\Lambda}{r^2}e^{U(r)} \, . \end{aligned}$$

  - The gravitino transformation law gives

$$\partial_\rho U = -\sqrt{f(p,x)}e^U \ , \rho = \frac{1}{r} \, .$$

  - The gaugino transformation law gives

$$\partial_\rho x^a = -\sqrt{g(p,x)}(x^a p^0 - p^a) \, .$$

- These equations were derived using special geometry.
  - It can be shown that they imply

$$\partial_\rho^2 x^a + h(p, x, \partial_\rho x)\partial_\rho x^a = 0 \ .$$

  - This equation is independent of U.
  - Can be viewed as a generalized geodesic equation which describe how x^a evolve as one moves into the core of the black hole.
  - x^a will evolve until it runs into the fixed point

$$x^a_{\text{fixed}} = \frac{p^a}{p^0} \ .$$

(Ferrara, Kallosh, Strominger)

Attractor Mechanism and Extrimality

- Consider the Lagrangian

$$R + (\partial x^a)^2 + F^2 + F \wedge F \ .$$

- In static, spherically symmetric ansatz this reduces to

$$\left(\frac{dU}{dr}\right)^2 + g_{ab}\frac{dx^a}{dr}\frac{dx^b}{dr} + e^{2U}V(x,p,q)$$

- In addition, we have the constraint

$$\left(\frac{dU}{dr}\right)^2 + g_{ab}\frac{dx^a}{dr}\frac{dx^b}{dr} - e^{2U}V(\phi,p,q) = 2ST$$

where S is the entropy and T is the temperature.

- Geometry as well as moduli are regular near horizon implies

$$\frac{A}{4\pi} = V(x_h^a, p, q) \ ,$$

and

$$\left(\frac{\partial V}{\partial x^a}\right)_h = 0 \ .$$

(Ferrara, Gibbons, Kallosh).

- Goldstein, Iizuka, Jena, Trivedi obtained nonsusy attractors by doing a numerical analysis.

Nonsusy Attractors

- We will consider the type IIA compactification on a Calabi-Yau manifold at large volume. The low energy theory is $N = 2$ sugra coupled to n vector multiplets.
- It is described by the prepotential

$$F = D_{abc} \frac{X^a X^b X^c}{X^0} \ .$$

- For convenience, define

$$D_{ab} = D_{abc} p^c \ , \ D_{ab} D^{bc} = \delta_a^c \ , \ D_a = D_{ab} p^b \ , \ D = D_a p^a \ .$$

- We can have $D0, D2, D4, D6$ branes with charges q_0, q_a, p^a, p^0 respectively.

- The Kähler potential and superpotential are derived from F using:

$$
\begin{aligned}
K &= -\log\left[i\sum\left(X^a\partial_a F - X^a(\partial_a F)^*\right)\right] \\
W &= \sum\left(q_a X^a - p^a\partial_a F\right) .
\end{aligned}
$$

- The effective potential is given in terms of these quantities as

$$
V = e^K\left[g^{a\bar{b}}\nabla_a W\overline{\nabla_b W} + |W|^2\right] ,
$$

where $\nabla_a W = \partial_a W + \partial_a K W$.

- A regular horizon exists if

$$
\begin{aligned}
&g^{b\bar{c}}\nabla_a\nabla_b W\overline{\nabla_c W} + 2\nabla_a W\overline{W} \\
&\quad + \partial_a g^{b\bar{c}}\nabla_b W\overline{\nabla_c W} = 0 .
\end{aligned}
$$

- We have susy solution if $\nabla_a W = 0$, else nonsusy.

Examples

- We will first consider the case when there are no $D6$ branes. This system can be reduced to the $D0-D4$ system by a redefinition of the scalars X^a and the $D0$ charge q_0:

$$\begin{aligned} q_0 &\rightarrow q_0 - \frac{1}{12} D^{ab} q_a q_b \\ x^a &\rightarrow x^a + \frac{1}{6} D^{ab} q_b \ . \end{aligned}$$

- So we set $p^0 = q_a = 0$ and also consider the ansatz $x^a = p^a t$. Substituting it in the equation of motion we find

$$\frac{6i}{t}\left(q_0 - Dt^2\right)\left(q_0 + Dt^2\right) = 0 \ .$$

- The susy solution corresponds to the value $t = i\sqrt{\frac{q_0}{D}}$ with entropy $S = 2\pi\sqrt{Dq_0}$.
- The nonsusy solution corresponds to $t = i\sqrt{-\frac{q_0}{D}}$ with entropy $S = 2\pi\sqrt{-Dq_0}$.
- Susy solution is guaranteed to be stable. How can we make sure there are no tachyonic directions for the nonsusy case?
- Compute the eigen values of the mass matrix.
  - Set $x^a = ip^a t + \delta\xi^a + i\delta y^a$, and expand the potential.

$$S_{\text{mass}} = A_{ab}\left(\delta\xi^a\delta\xi^b + \delta y^a\delta y^b\right) + B_{ab}\left(\delta\xi^a\delta\xi^b - \delta y^a\delta y^b\right)$$

  - For susy solution $A > 0$ and $B = 0$, hence all the eigen values of the mass matrix are positive.

- For nonsusy solution

$$
\begin{aligned}
A_{ab} &= 24q_0 e^{K_0}\left(D_{ab} - 3\frac{D_a D_b}{D}\right) \; ; \\
B_{ab} &= -24 q_0 e^{K_0} D_{ab} \; .
\end{aligned}
$$

Hence

$$
\begin{aligned}
S_{\text{mass}} &= 48 q_0 e^{K_0}\left(D_{ab} - \frac{3D_a D_b}{2D}\right)\delta y^a \delta y^b \\
&+72 e^{K_0}\left(-\frac{q_0}{D}\right) D_a D_b \delta\xi^a \delta\xi^b \\
&= 32 q_0^2 e^{K_0} g_{a\bar{b}} \delta y^a \delta y^b + 72 e^{K_0}\left(-\frac{q_0}{D}\right) D_a D_b \delta\xi^a \delta\xi^b \; .
\end{aligned}
$$

- Hence $D_a \delta\xi^a$ and all δy^a are massive. There are $(n-1)$ mass less modes.

- Thus, in order to know whether we have a stable solution, we need to look at the quartic terms (which in general is quite complicated to evaluate).
- We may exploit the following invariances of the effective potential:
  - The $GL(N,R)$ invariance

$$\begin{aligned} x^a &\rightarrow A^a_b x^b \ , \\ p^a &\rightarrow A^a_b p^b \ , \\ q_a &\rightarrow q_b \left(A^{-1}\right)^b_a \ , \\ D_{abc} &\rightarrow D_{def} \left(A^{-1}\right)^d_a \left(A^{-1}\right)^e_b \left(A^{-1}\right)^f_c \ . \end{aligned}$$

  - Invariance under the transformation $x^a \leftrightarrow -\bar{x}^a$. This tells only even powers of $\delta\xi^a$ appear in the expansion.

- The most general quadratic term allowed by this symmetry is

$$V_{\text{quadr}} = \sqrt{-\frac{D}{q_0}}\left(C_1 D_{ab} + C_2 \frac{D_a D_b}{D}\right)\delta\xi^a \delta\xi^b + \sqrt{-\frac{D}{q_0}}\left(C_3 D_{ab} + C_4 \frac{D_a D_b}{D}\right)\delta y^a \delta y^b \; .$$

- We can find the coefficients C_i by comparing it with STU model, which has a prepotential

$$F = -\frac{X^1 X^2 X^3}{X^0} \; .$$

- This gives

$$C_1 = 0 \, , C_2 = -9 \, , C_3 = 6 \, , C_4 = -9 \; .$$

- We can similarly obtain the cubic terms.

$$V_{\text{cubic}} = \frac{1}{q_0}\Big(C_1 D D_{abc} + C_2 D_{ab} D_c + C_3 D_a D_{bc} + C_4 \frac{D_a D_b D_c}{D} \Big) \delta\xi^a \delta\xi^b \delta y^c \ .$$

- Comparing with STU we find

$$C_1 = 3 \, , C_2 = -9 \, , C_3 = 18 \, , C_4 = 27 \ .$$

- Similarly the quartic term can be found to be

$$V_4 = -\frac{9}{2D}\left(-\frac{D}{q_0}\right)^{3/2} \left(D_{ab}\delta\xi^a \delta\xi^b\right)^2 \ .$$

- The potential is of the form

$$V = V_0 + \frac{1}{2}M^2\Phi^2 + \lambda_1\phi^2\Phi + \lambda_2\phi^4 \ .$$

- Integrating out the massive field, we obtain

$$V_{\text{quartic}} = \left(\lambda_2 - \frac{\lambda_1^2}{2M^2}\right)\phi^4 \ .$$

- Using similar steps we can find the quartic terms

$$V_{\text{quartic}} = \frac{9}{4D}\left(\frac{-D}{q_0}\right)^{3/2}\Big[-\left(D_{lm}\delta\xi^l\delta\xi^m\right)^2 + \frac{1}{4}\left(\frac{-D}{q_0}\right)\left(g^{a\bar{b}}D_{alm}\delta\xi^l\delta\xi^m D_{bpq}\delta\xi^p\delta\xi^q\right)\Big] .$$

- Two competing terms with opposite sign.

- In the $D0 - D4 - D6$ case the non-susy extremum is located at, $x^a = x_0^a = p^a(t_1 + it_2)$. The values of t_1, t_2 are determined by the charges. It is useful to define a variable $s > 0$ given by,

$$s = \sqrt{(p^0)^2 - \frac{4D}{q_0}}.$$

The two branches correspond to $|s/p^0| < 1$ and $|s/p^0| > 1$ respectively. t_1 is given by

$$t_1 = \begin{cases} \frac{2}{s} \frac{\left(1+\frac{p^0}{s}\right)^{1/3} - \left(1-\frac{p^0}{s}\right)^{1/3}}{\left(1+\frac{p^0}{s}\right)^{4/3} + \left(1-\frac{p^0}{s}\right)^{4/3}} & \left|\frac{s}{p^0}\right| > 1 \\ \frac{2}{p^0} \frac{\left(1-\frac{s}{p^0}\right)^{1/3} + \left(1+\frac{s}{p^0}\right)^{1/3}}{\left(1-\frac{s}{p^0}\right)^{4/3} + \left(1+\frac{s}{p^0}\right)^{4/3}} & \left|\frac{s}{p^0}\right| < 1 \end{cases}$$

- The expression for t_2 is given by:

$$t_2 = \begin{cases} \frac{4s}{(s^2-(p^0)^2)^{1/3}\left((s+p^0)^{4/3}+(s-p^0)^{4/3}\right)} & \left|\frac{s}{p^0}\right| > 1 \\ \frac{4s}{((p^0)^2-s^2)^{1/3}\left((|p^0|+s)^{4/3}+(|p^0|-s)^{4/3}\right)} & \left|\frac{s}{p^0}\right| < 1 \end{cases}$$

- For $D0 - D6$ system we need to take $p^a \to 0$. In this limit (in the $|s/p^0| < 1$ branch)

$$t_1 = \frac{2}{p^0}\ , t_2 = \left(-\frac{q_0}{D|p^0|}\right)^{1/3} \ .$$

- Hence we have a $(n-1)$ dimensional moduli space:

$$D_{abc}y^a y^b y^c = -q_0/|p^0|\ .$$

Conclusion

- For $N = 2$ sugra coupled to n vector multiplets, scalars run into a fixed point at the horizon.
- This is a consequence of extrimality. Thus nonsusy attractors also exist.
- For IIA on Calabi-Yau, we have exact solution for $D0 - D4 - D6$ system. The $D0 - D4$ system has $(n - 1)$ mass less fields. They have quartic terms and depending on the charges we may or may not have stable solutions.
- For $(D0 - D6)$ system we have a $(n - 1)$ dimensional moduli space.