Emergent Gravity from Noncommutative Gauge Theory

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Introduction

- Classical space-time meaningless at Planck scale
  due to gravity $\leftrightarrow$ Quantum Mechanics
- $\Rightarrow$ "Quantized" (noncommutative?) spaces:,
  e.g. $[x_i, x_j] = i\theta_{ij}$
  space-time uncertainty relations $\Delta x_i \Delta x_j \geq \theta_{ij}$
  realized in string theory (D-branes with B-field)
- Physics on quantized space:
  Noncommutative Quantum Field Theory
  well developed; some problems

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Main Message:

- NC gauge theory (as Matrix Model) does contain gravity
  surprising, intrinsically NC mechanism
  gravity tied with NC
  cf. stringy Matrix Models (IKKT)
- Not precisely general relativity
  appears to agree with GR at low energies (?)
  - gravitational waves
  - Newtonian limit
  - linearized metric: $R_{ab} \sim 0$

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Main result:

The model:

$$S_{YM} = -Tr[X^a, X^b][X^{a'}, X^{b'}]\,\eta_{aa'}\eta_{bb'}$$

where $X^a \in L(\mathcal{H})$ ... matrices (operators), $a = 0, 1, 2, 3$

low-energy effective action:

$$S_{YM} = \int d^4y\,\rho(y) tr\left(4\eta(y) - G^{cc'}G^{dd'}F_{cd}F_{c'd'}\right) + 2\int \eta(y)\, trF \wedge F$$

where

$G^{ab}(y) = -\theta^{ac}(y)\theta^{bd}(y)\, g_{cd}$ effective dynamical metric

F_{ab} ... $\mathfrak{su}(n)$ field strength

contains dynamical gravity, close to general relativity

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Outline

- NC gauge theory as Matrix Model
  $\rightarrow$ dynamical quantum spaces
- Effective metric, geometry
- Low-energy effective action and emergent gravity
- Some checks:
  - Gravitational waves, linearized metric
  - Newtonian Limit
- Remarks on quantization, UV/IR
- Conclusion

NC $U(1)$ gauge theory from Matrix Model

Consider Matrix Model:

$$S_{YM} = -Tr\left(([X^a, X^b] - i\bar\theta^{ab})([X^{a'}, X^{b'}] - i\bar\theta^{a'b'})\right)\eta_{aa'}\eta_{bb'}$$

$\bar\theta^{ab}$ ... antisymmetric tensor, nondegenerate $a = 0, 1, 2, 3$

dynamical objects:

$$X^a = \overline{Y}^a + A^a \quad \in L(\mathcal{H})$$

... hermitian matrices / operators ("covariant coordinates")

$$[\overline{Y}^a, \overline{Y}^b] = i\bar\theta^{ab} \qquad \text{“quantum plane”}$$

(cf. Q.M. phase space, Heisenberg-algebra)

"conventional" point of view:

- describes $U(1)$ Yang-Mills on quantum plane $\mathbb{R}^4_\theta$
- $\rightarrow$ usual $U(1)$ Yang-Mills on $\mathbb{R}^4$ for $\theta \rightarrow 0$

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why? ("standard" analysis)

let $[\overline{Y}^a, \overline{Y}^b] = i\overline{\theta}^{ab}$ ... quantum plane $\mathbb{R}^4_\theta$,

then

$[\overline{Y}^a, f(\overline{Y})] \sim i\theta^{ab}\partial_b f(\overline{y})$ for "smooth function" $f(\overline{Y}) \approx f(\overline{y}),\ \theta \approx 0$

let $X^a = \overline{Y}^a + \overline{\theta}^{ab} A_b$ then

$$\begin{aligned}
[X^a, X^b] - i\overline{\theta}^{ab} &= \overline{\theta}^{aa'}\overline{\theta}^{bb'}\,(\partial_{a'} A_{b'} - \partial_{b'} A_{a'} + [A_{a'}, A_{b'}]) \\
&= \overline{\theta}^{aa'}\overline{\theta}^{bb'}\,F_{a'b'}
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so

$$S_{YM} \sim \int F_{ab} F_{a'b'}\, \overline{g}^{aa'}\overline{g}^{bb'}, \qquad \overline{g}^{ab} = -\overline{\theta}^{aa'}\overline{\theta}^{bb'}\,\eta_{a'b'}$$

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gauge fields ... fluctuations of covariant coordinates however:

- $\exists$ versions for compact ("fuzzy") spaces $S^2_N \times S^2_N$, $\mathbb{C}P^2$
  H. Grosse, H.S; W. Behr, F. Meyer, H.S

  space itself obtained as "vacuum" of similar matrix model
  $\Rightarrow$ space is dynamical;

  fluctuations of covariant coords $X^a \leftrightarrow$ gravity ?!

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- NC $\mathfrak{u}(1)$ gauge theory $\leftrightarrow$ gravity proposed in
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Geometry from NC $u(n)$ gauge theory:

- $u(n)$, naive:

$$X^a = \overline{Y}^a \otimes \mathbf{1}_n + \overline{\theta}^{ab}(A_b^0 \otimes \mathbf{1}_n + A_b^\alpha \otimes \lambda_\alpha) \quad \in \mathcal{A} \otimes \mathfrak{u}(n),$$

$\mathcal{A}$ ... functions on $\mathbb{R}^4_\theta$

... obtain $\mathfrak{u}(n)$ Yang-Mills

- here: separate $\mathfrak{u}(1)$ and $\mathfrak{su}(n)$ components

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$\mathfrak{u}(1)$ components Y^a $\leftrightarrow$ general Poisson structure:

$$[Y^a, Y^b] = i\theta^{ab}(y)$$

then

$$[Y^a, \Phi(y)] = i\theta^{ab}(y)\partial_b\Phi(y) \qquad + O(\theta^2)$$

consider additional scalar Φ in adjoint

$$\begin{aligned} S[\Phi] &= -Tr\, \eta_{aa'}[X^a, \Phi][X^{a'}, \Phi] \\ &= Tr\, G^{ab}(y)\,(\partial_a + [A_a, .])\Phi(\partial_b + [A_b, .])\Phi \end{aligned}$$

where

$$G^{ab}(y) = -\theta^{ac}(y)\theta^{bd}(y)\,\eta_{cd}$$

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nonabelian gauge fields (heuristic)

set $X^a = Y^a + \theta^{ab}(y)A_b(y)$ obtain

$$
\begin{aligned}
[X^a, X^b] &= i\theta^{ab}(y) + i\theta^{ac}\theta^{bd}(\partial_c A_d - \partial_d A_c + [A_c, A_d] \ + O(\theta^{-1}\partial\theta)) \\
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S_{YM} &= -Tr[X^a, X^b][X^{a'}, X^{b'}]\eta_{aa'}\eta_{bb'} \\
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\end{aligned}
$$

using $Tr(\theta^{ab}(y)F^{ab}) \approx 0$

similar to $\mathfrak{su}(n)$ YM coupled to metric $G^{ab}(y)$

nonabelian gauge fields (correct)

Seiberg-Witten map:

$$X^a = Y^a + \theta^{ab} A_b - \frac{1}{2}(A_c[Y^c, \theta^{ad} A_d] + A_c F^{ca}) \quad + O(\theta^3)$$

- expresses $\mathfrak{su}(n)$ d.o.f. in terms of commutative $\mathfrak{su}(n)$ gauge fields A_a
- relates NC g.t. $i[\Lambda, X^a]$ in terms of standard $\mathfrak{su}(n)$ g.t. of A_a

Volume element:

$$\begin{aligned}(2\pi)^2 Tr\, f(y) &= \int d^4 y\, \rho(y)\, f(y), \\ \rho(y) &= \sqrt{\det(\theta^{-1}_{ab})} = (\det(\eta_{ab})\det(G_{ab}))^{1/4}\end{aligned}$$

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effective action to leading order:

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linearized NC gravity:

flat space: Moyal-Weyl $\overline{\theta}^{ab} = \text{const}$

$\Rightarrow G^{ab} = -\overline{\theta}^{ac}\,\overline{\theta}^{bd}\,\eta_{cd} =: \overline{\eta}^{ab}$ ... flat Minkowski metric

small fluctuations: $Y^a = \overline{Y}^a + \overline{\theta}^{ab}\,A^0_b$ ($\mathfrak{u}(1)$ component)

$$\theta^{ab}(y) = -i[Y^a, Y^b] = \overline{\theta}^{ab} + \overline{\theta}^{ac}\overline{\theta}^{bd}\,F^0_{cd}(y)$$

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where

$$\boxed{h_{ab} = \bar{\eta}_{bb'}\bar{\theta}^{b'c} F^0_{ca} + \bar{\eta}_{aa'}\bar{\theta}^{a'c} F^0_{cb}}$$

... linearized metric fluctuation (cf. Rivelles [hep-th/0212262])

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$$[Y^a, \theta^{ab}(y)] = 0 \;\Leftrightarrow\; G^{ac}\partial_c\, \theta^{-1}_{ab}(y) = 0$$

implies vacuum equations of motion (linearized)

$$R_{ab} = 0 \; + O(\theta^2)$$

moreover $R_{abcd} = O(\bar\theta) \neq 0$ ... nonvanishing curvature

$\Rightarrow$ on-shell d.o.f. of gravitational waves on Minkowski space

note

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Newtonian limit

Question: sufficient d. o. f. in G^{ab} for gemetries with matter?

Answer: o.k. at least for Newtonian limit

$$ds^2 = -c^2 dt^2 \left(1 + \frac{2U}{c^2}\right) + d\vec{x}^2 \left(1 + O(\frac{1}{c^2})\right)$$

where $\Delta_{(3)} U(y) = 4\pi G \rho(y)$ and ρ ...static mass density

can show: $\exists$ sufficient d.o.f. in G^{ab} for arbitrary $\rho(y)$

moreover, vacuum e.o.m. imply

$$ds^2 = -c^2 dt^2 \left(1 + \frac{2U}{c^2}\right) + d\vec{x}^2 \left(1 - \frac{2U}{c^2}\right)$$

as in G.R.

Question: what about the Einstein-Hilbert action?

Answer:

- tree level: e.o.m. for gravity follow from $\mathfrak{u}(1)$ sector:

  $G^{ac}\partial_c \theta^{-1}_{ab}(y) = 0$ implies $R_{ab} \sim 0$,

  at least for linearized gravity.
- one-loop: gauge or matter (scalar) fields couple to G_{ab}
  $\Rightarrow$ (Sakharov) induced Einstein-Hilbert action:

$$S_{1-loop} \sim \int d^4y\sqrt{G}\left(c_1\Lambda^4_{UV} + c_2\Lambda^2_{UV}\, R[G] + O(\log(\Lambda_{UV}))\right)$$

however, modifications due to different role of scaling factor $\det(G)$ in density

Relation with UV/IR mixing

Recall UV/IR mixing:

- Quantization of NC field theory $\rightarrow$ new divergences in IR, similar to UV divergences; non-renormalizable ?
- for NC gauge theories: restricted to trace-$\mathfrak{u}(1)$ sector
- here: trace-$\mathfrak{u}(1)$ sector understood as geometric d. o. f., $\mathfrak{su}(n)$ YM coupled to G_{ab}

  $\Rightarrow$ expect new divergences in IR due to induced gravity (E-H action)

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- simple, intrinsically NC mechanism to generate gravity

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- solves problem how to define NC $\mathfrak{su}(n)$ gauge theory

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- *not* same as G.R., but close to G.R. for small curvature
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