# Is dark energy an effect of averaging?

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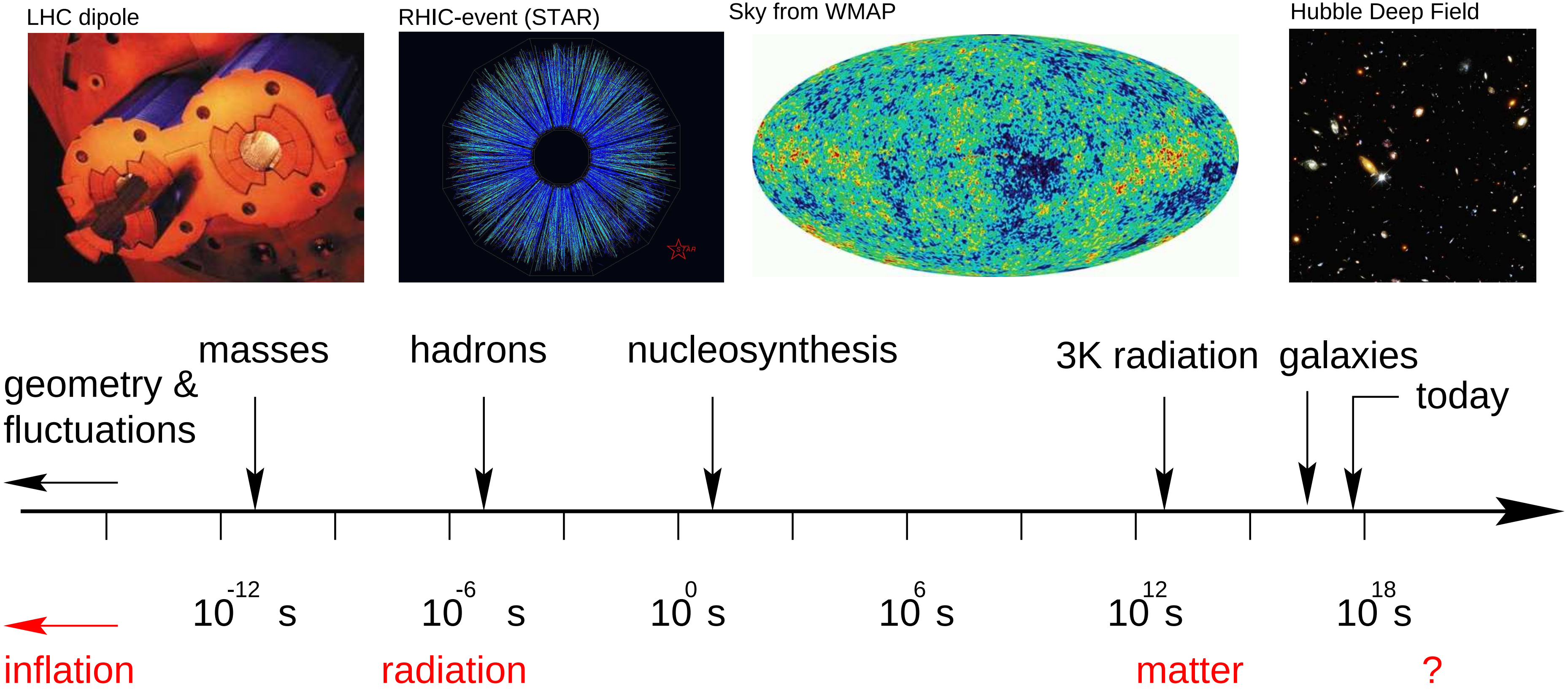
based on Li & Schwarz, arXiv:gr-qc/0702043

Balkan Workshop 2007

# History of the Universe



# Friedmann model I

isotropic & homogenous line element $(c = 1)$:

$$ds^2 = -dt^2 + a^2 \left( \frac{dr^2}{1 - Kr^2} + r^2 d\Omega^2 \right)$$

$a = a(t)$ scale factor, $K/a^2$ spatial curvature $(K = -1, 0, +1)$

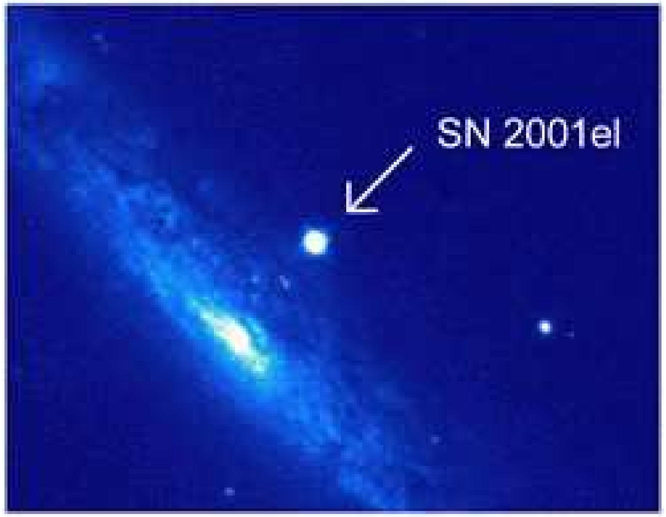
Consequences:

cosmological red-shift: $z = \frac{a_0}{a} - 1$

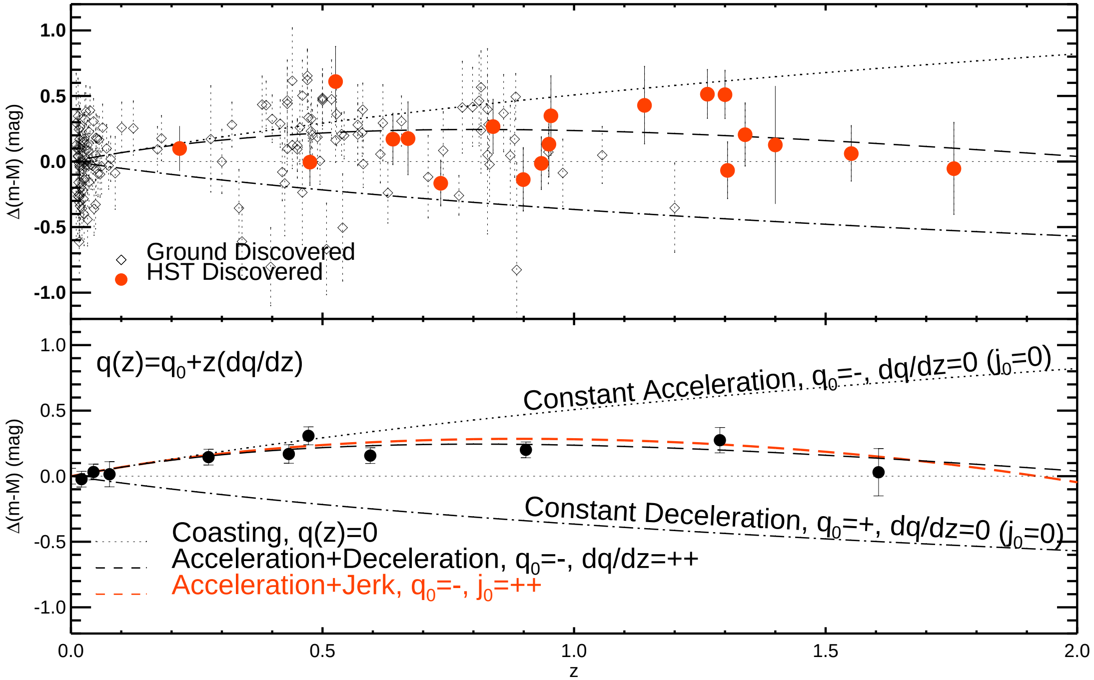
Hubble law: $H_0 d_L = z + \mathcal{O}(z^2)$; $H \equiv \dot{a}/a$ expansion rate

$H_0 \equiv 100h$ km/s/Mpc, $h = 0.72 \pm 0.03 \pm 0.07$ Freedman et al. 2001

## 1998 cosmology revolution: accelerated expansion




supernova type Ia




SN Ia data suggest $q_0 < 0$

Riess et al. 2004

deceleration $q \equiv -(\ddot{a}/a)/H^2$, jerk $j \equiv (\dddot{a}/a)/H^3$

# Friedmann model II

continuity equation and Friedmann equation

$$\dot{\epsilon} + 3H\,(\epsilon + p) = 0 \quad \text{and} \quad 3H^2 + \frac{3K}{a^2} - \Lambda = 8\pi G\epsilon$$

$\epsilon$ energy density, $p$ pressure

$\Lambda$ cosmological constant, $G$ Newton's gravitational constant

equation of state $p = p(\epsilon)$

dimensionless energy density: $\Omega \equiv 8\pi G\epsilon/3H^2$

## Einstein-de Sitter model

$\Lambda = K = p = 0$: flat dust Universe

$$a(t) = a_0 \left(\frac{t}{t_0}\right)^{2/3}, \quad t_0 = \frac{2}{3} t_{\mathrm{H}}, \quad q_0 = \frac{1}{2}$$

in conflict with age of Universe $t_0 \geq 12$ Gyr (oldest stars) and
in conflict with SN Ia Hubble diagram $q_0 < 0$

drop at least one of the assumptions of Einstein-de Sitter model

## Cosmological constant or dark energy

acceleration possible for

$$-3\frac{\ddot{a}}{a} = 4\pi G(\epsilon + 3p) - \Lambda < 0$$

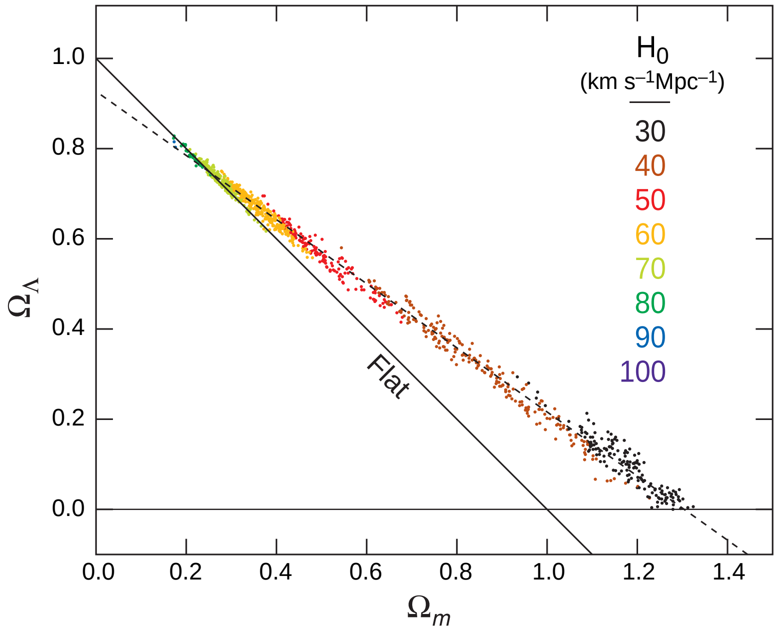
cosmological constant or “dark energy” ($p < -\epsilon/3$) required

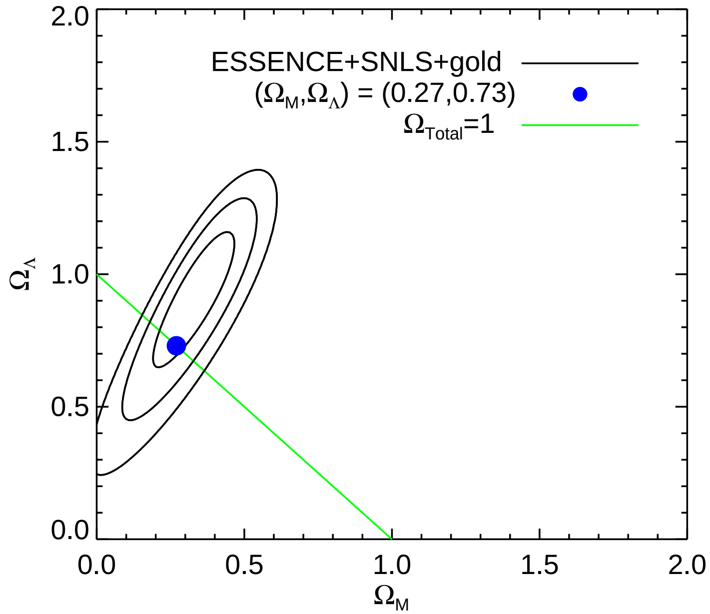
simplest model: $\Lambda > 0, p = 0, K = 0$ flat ΛCDM

age of the Universe is now ok, Hubble diagram is now ok

What about curvature? What about pressure?

# $\Lambda$CDM vs. flat $\Lambda$CDM




Spergel et al. 2006

CMB (WMAP) and $H_0$ (HST key project)

$\Omega - 1 = -0.014 \pm 0.017 \Rightarrow r_c > 21\text{Gpc}$

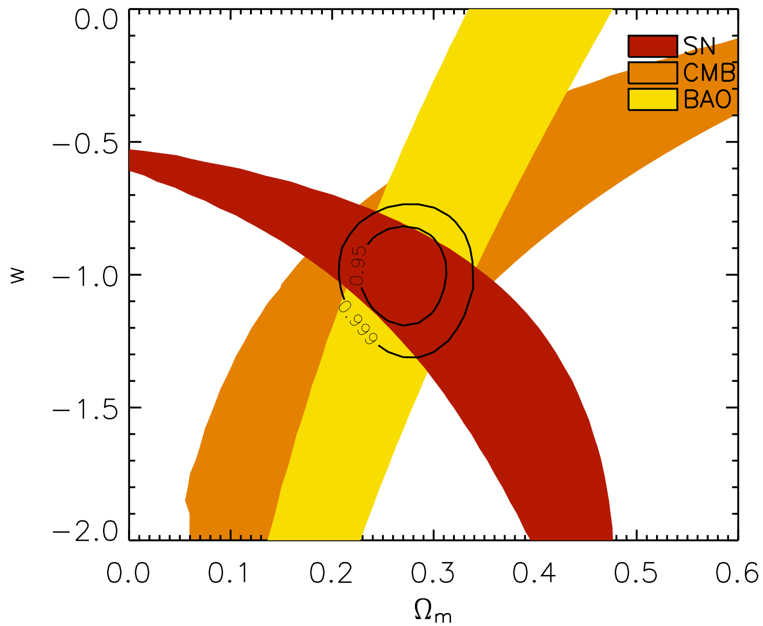
Wood-Vasey et al. 2007

supernovae Ia

$\Omega_\Lambda > 0$

## Cosmological constant vs. more general dark energy

flat cosmology, constant $w_{\mathrm{de}} = p_{\mathrm{de}}/\epsilon_{\mathrm{de}}$: $w = -1.01 \pm 0.15$




SN 1a, CMB, BAO

Davis et al. 2007

## Conceptional problems of the $\Lambda$CDM model

• no theory for vacuum energy density, i.e. cosmological constant; naive guess from quantum field theory wrong by $10^{122}$
(cosmological constant problem)

• why is $\Omega_\Lambda(t_0) \sim \Omega_m(t_0)$?
(coincidence problem)

## Ideas to solve the coincidence problem

**dynamic de:** quintessence/k-essence – another scalar field
make the dynamics trace dominant component (tracker solutions)
leads to accelerated, weaker coincidence problem, lacks fundametal justification

**unified de/dm:** e.g. generalised Chaplygin gas
no compelling physics, leads to acceleration, may solve the coincidence problem

**modify gravity:** change the large scale properties of gr
some extra dimension models provide interesting ideas
leads to acceleration, but does not solve the coincidence problem and may be in conflict with Solar system tests

**cosmological backreaction:** no new physics, non-linear effect of gr
evolution of averaged metric $\neq$ averaged evolution of real metric
nonlinear effect, hard to quantify
real effect, unclear if it leads to acceleration, could solve coincidence problem

**anthropic principle:** give up

# Cosmological backreaction: motivations

**coincidence problem(s):**

Why is $\Omega_\Lambda(t_0) \sim \Omega_\mathrm{m}(t_0)$?

Why is $z_\mathrm{nl}(R_\mathrm{eq} \sim 100\ \mathrm{Mpc}) \sim z_\mathrm{acc}$?

e.g. Shapely supercluster, Sloan great wall, biggest voids

**averaging problem:**

Einstein tensor (averaged metric) $\neq$ averaged Einstein tensor (metric)

How big is the difference?

**most observations are averages,** e.g. $H_0$, $q_0$, $P(k)$

**standard cosmology:**

linear regime: averaged metric plus small perturbations

non-linear regime: averaged metric plus Newtonian gravity

## Origin of structure: cosmological inflation

epoch of accelerated expansion in the very early Universe

Starobinsky 1979; Guth 1980

$$\ddot{a} > 0 \qquad \Leftrightarrow \qquad \epsilon + 3p < 0$$

since $-3\frac{\ddot{a}}{a} = 4\pi G\,(\epsilon + 3p)$

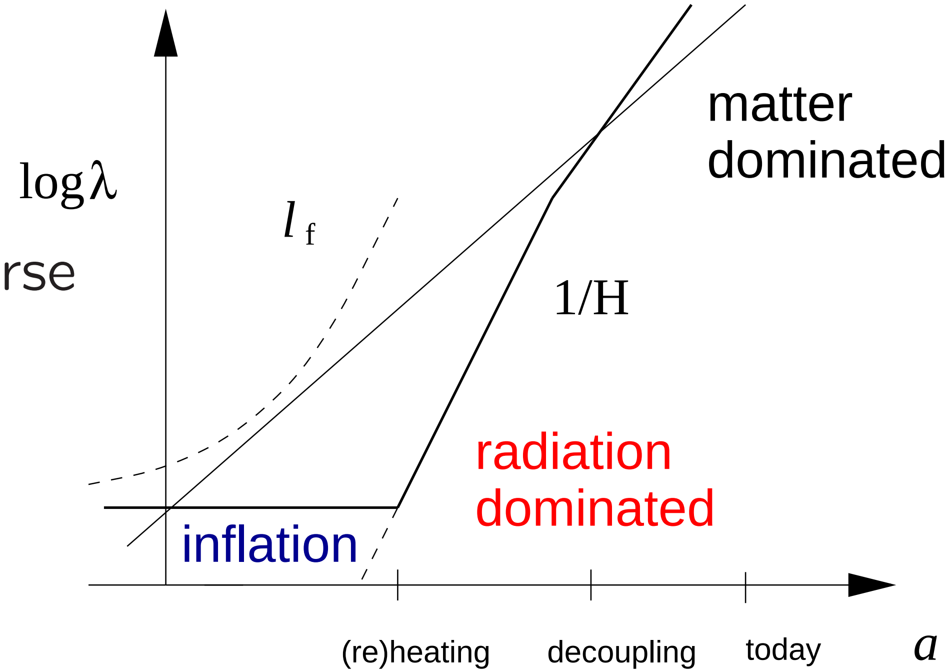
e.g. vacuum: $H =$ const and $a = a_i \exp[H(t - t_i)]$

generic prediction: $\Omega \approx 1$

# Density inhomogeneities from quantum fluctuations

quantum fluctuations of
energy density and metric
during inflation
become classical fluctuations
in the matter dominated Universe

Chibisov & Mukhanov 1981

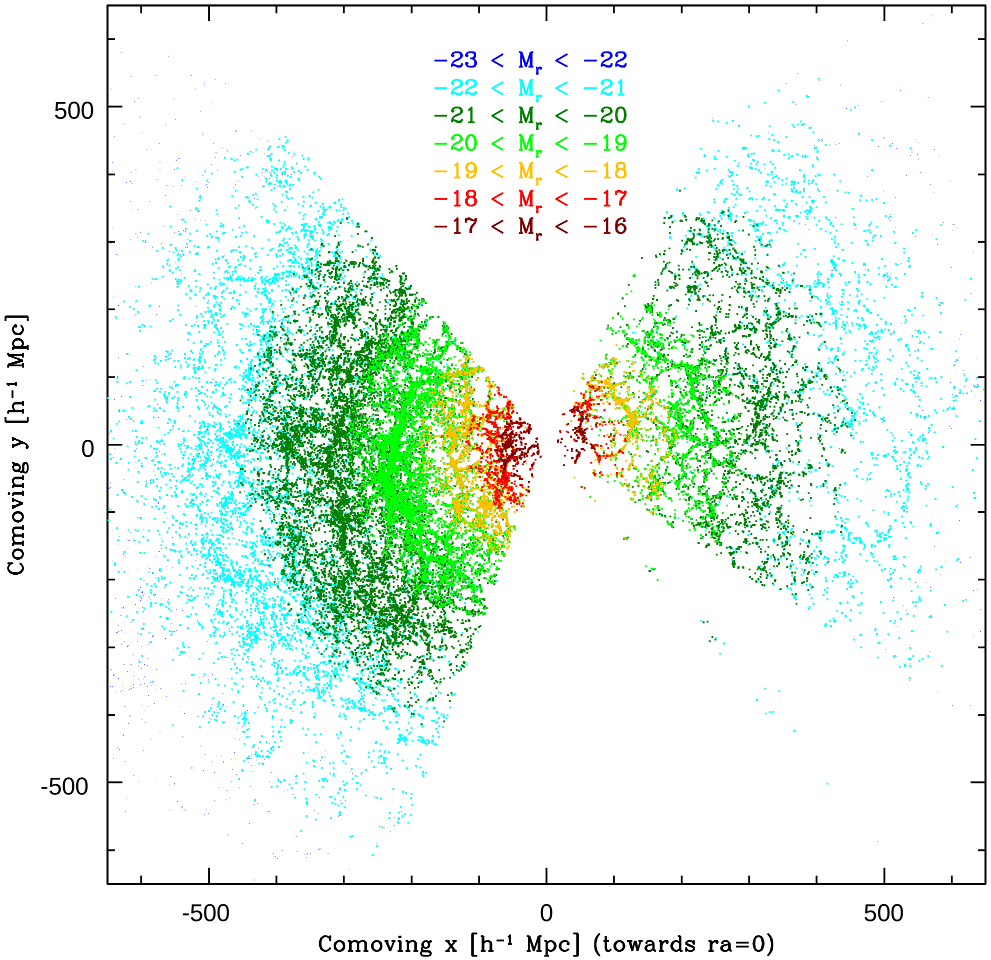



$\lambda_{\text{ph}} \equiv a\lambda$

$\lambda_{\text{ph}} \ll 1/H$ locally Minkowski

$\lambda_{\text{ph}} \gg 1/H$ no causal physics

# Large scale structure




galaxies, visible light

Sloan Digital Sky Survey

SDSS

Tegmark et al 2004

## How to measure the expansion rate $H_0$?

take a set of standard candles (if not available SN 1a)
distributed homogeneously in some volume physical $V$

measure distances $d_i$ (magnitudes) and redshifts $cz_i$

$$H_0 \equiv \frac{1}{N} \sum_{i=1}^{N} \frac{cz_i}{d_i}$$

for the idealised case $N \to \infty$ this turns into a volume average

$$H_0 = \frac{1}{V} \int \frac{cz}{d} \, \mathrm{d}V$$

NB: for large $z$ one averages over the past light cone instead of a spatial volume

# Buchert's equations

spatial average over comoving domain $D$: $\langle O \rangle \equiv \frac{1}{V_D}\int_D O\mathrm{d}V$

$a_D \propto V_D^{1/3}$, $H_D \equiv \dot{a}_D/a_D = \langle\theta\rangle/3$

for any irrotational dust Universe: Buchert 2000

$$\left(\frac{\dot{a}_D}{a_D}\right)^2 = \frac{8\pi G}{3}\rho_{\mathrm{eff}}, \qquad -3\frac{\ddot{a}_D}{a_D} = 4\pi G(\rho_{\mathrm{eff}} + 3p_{\mathrm{eff}})$$

$$\rho_{\mathrm{eff}} \equiv \langle\rho\rangle - \frac{1}{16\pi G}\left[\langle Q\rangle + \langle R\rangle\right], \qquad p_{\mathrm{eff}} \equiv -\frac{1}{16\pi G}\left[\langle Q\rangle - \frac{1}{3}\langle R\rangle\right]$$

kinematic backreaction $\langle Q\rangle \equiv \frac{2}{3}(\langle\theta^2\rangle - \langle\theta\rangle^2) - 2\langle\sigma^2\rangle$ and averaged 3-curvature $\langle R\rangle$ are related by an integrability condition

effective acceleration for $\langle Q\rangle > 4\pi G\langle\rho\rangle$

# Estimate backreaction by second order perturbation theory

Wetterich 2003, Räsänen 2004, Kolb et al. 2005

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)[(1-2\psi)\delta_{ij} + D_{ij}\chi]\mathrm{d}x^i\mathrm{d}x^j$$

growing mode of first order perturbations

$\psi = -\frac{5}{9}Ct_0^{-4/3} - \frac{1}{6}\Delta C t^{2/3}$ and $\chi = C(\mathbf{x})t^{2/3}$, with $C = C(\mathbf{x})$

$C \approx 9t_0^{4/3}\zeta/5$ at superhorizon scales

$\zeta$ hypersurface-invariant density contrast — Bardeen 1989

use integrability condition for $\langle Q\rangle$ and $\langle R\rangle$:

$$\langle Q\rangle = \frac{1}{27t^{2/3}}\left[3\left(\langle\partial^i(\partial_i C\Delta C)\rangle_1 - \langle\partial^i(\partial_j C\partial^j\partial_i C)\rangle_1\right) - 2\langle\Delta C\rangle_1^2\right]$$

$$\langle R\rangle = -\frac{20}{9t^{4/3}}\langle\Delta C\rangle_1 + \frac{5}{9t^{2/3}}\left[\left(\langle\partial^i(\partial_j C\partial^j\partial_i C)\rangle_1 - \langle\partial^i(\partial_i C\Delta C)\rangle_1\right) + 2\langle\Delta C\rangle_1^2\right]$$

$$\langle\rho\rangle = \frac{1}{6\pi G}\left[\frac{1}{t^2} - \frac{1}{2t^{4/3}}\langle\Delta C\rangle_1 + \frac{1}{28t^{2/3}}\left(2\left(\langle\partial^i(\partial_j C\partial^j\partial_i C)\rangle_1 - \langle\partial^i(\partial_j C\partial^j\partial_i C)\rangle_1\right) + 7\langle\Delta C\rangle_1^2\right)\right]$$

with $\langle O\rangle_1 \equiv \int_D O\mathrm{d}\mathbf{x}/\int_D \mathrm{d}\mathbf{x}$ — Li & Schwarz 2007

## Effective equation of state

express $w_{\text{eff}}$ as a function of $a_D$:

$$w_{\text{eff}} = -\frac{5}{18}\langle \Delta C\rangle_1 a_D - \frac{1}{9}\left[\left(\langle \partial^i(\partial_i C \Delta C)\rangle_1 - \langle \partial^i(\partial_j C \partial^j \partial_i C)\rangle_1\right) - \frac{11}{4}\langle \Delta C\rangle_1^2\right] a_D^2$$

irrotational dust $w = 0$, but $w_{\text{eff}} \neq 0$

– cosmological backreaction is real
– surface terms only
– second order grows faster than first order
– $w_{\text{eff}} = w_{\text{eff}}(t, D)$ (sign is not fixed)

Li & Schwarz 2007

## Beyond second order

ansatz: $\langle R\rangle = \sum_{n=1} R_n a_D^{n-3}$, $\langle Q\rangle = \sum_{n=2} Q_n a_D^{n-3}$

integrability constraint gives

$$Q_n = -\frac{n-1}{n+3}R_n, \quad \rho_{\text{eff}} = \rho_0 a_D^{-3} - \frac{1}{16\pi G}[R_1 a_D^{-2} - \sum_{n=2}\frac{4Q_n}{n-1}a_D^{n-3}]$$

$n$th order: $\propto (\partial^2 C)^n$;
third order terms give rise to a cosmological constant: $\Lambda = Q_3$

mapping on dark energy model: $\rho_{\text{eff}} = \rho_{\text{m}} + \rho_{\text{de}}$ with

$$\rho_{\text{m}} = \langle\rho\rangle, \quad \rho_{\text{de}} = -\frac{1}{16\pi G}\left[\langle Q\rangle + \langle R\rangle\right]$$

iff $\exists n_{\text{max}}$, $w_{\text{de}} \to -n_{\text{max}}/3$ as $a_D \to \infty$
$n_{\text{max}} > 3$: phantom de (but perturbation theory suggests there is no $n_{\text{max}}$)

# Observational consequences

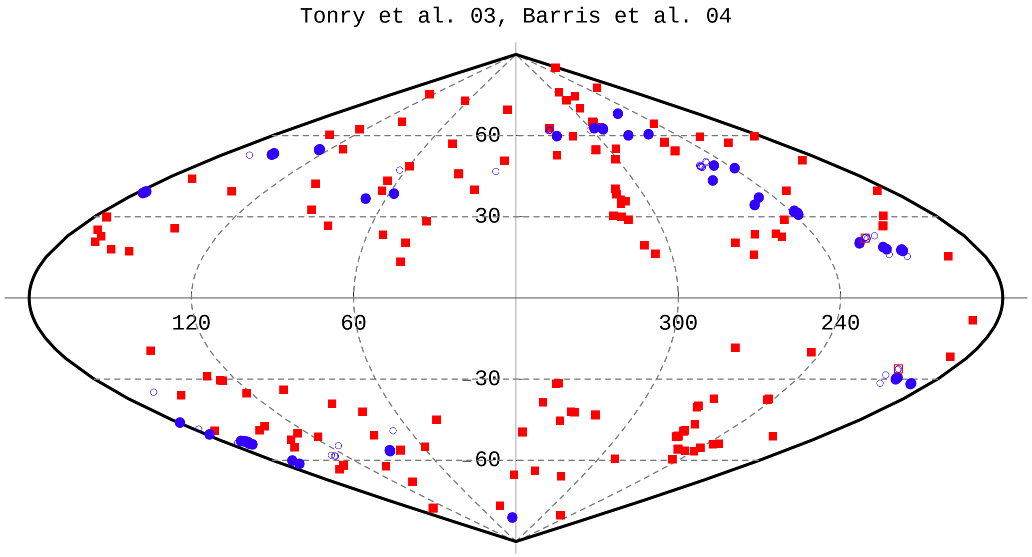
order of magnitude estimate:
typical density fluctuations from WMAP normalisation $P_\zeta = 2.4\times 10^{-9}$
$\partial \to 1/R$, $R$ typical size of domain, $h = 0.7$

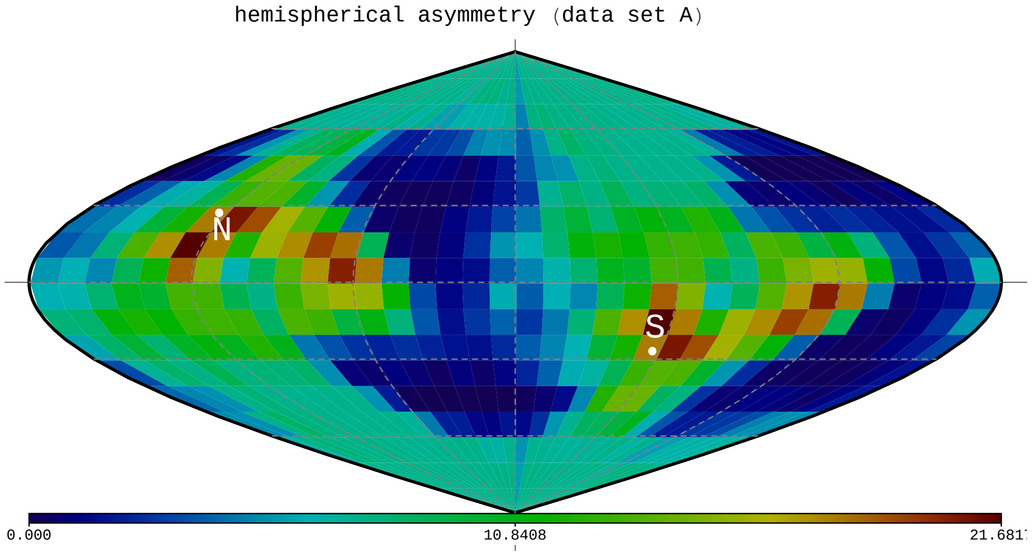
$$\frac{\langle R\rangle}{16\pi G\langle\rho\rangle} \sim \frac{0.1}{1+z}\left(\frac{70\ \mathrm{Mpc}}{R}\right)^2, \qquad \frac{\langle Q\rangle}{\langle R\rangle} \sim \frac{0.01}{1+z}\left(\frac{70\ \mathrm{Mpc}}{R}\right)^2$$

i.e. determination of Hubble constant (local measurement) could be affected and normalisation of high-z SN Ia depends on the understanding of local SN Ia $\Rightarrow$ observable consequences

effective acceleration seems possible in small domains:
$\langle Q\rangle/(4\pi G\langle\rho\rangle) \sim (20\ \mathrm{Mpc}/R)^4/(1+z)^2$ Li & Schwarz, in preparation

# (An)isotropy of the observed SN Ia Hubble diagram




$(\Delta\chi^2)_{max} \approx 22$: systematic effect or bulk flow? Schwarz & Weinhorst 2007

# Conclusions

• flat ΛCDM model has conceptual problems,
but provides a simple and good fit to all cosmological data

• cosmological backreaction is relevant for cosmology

• it seems important for $H_0$ (10% effect),
but does not seem to explain the apparent acceleration of the Universe
(as our perturbative study is limited to small effects, go beyond!)

• crucial observations: improve distance measurements (GAIA),
improve Hubble diagram by adding angular information
to infer bulk motion
largest possible sky coverage of SN Ia surveys
(e.g. SDSS, LSST, Pan-STARRS)