Single Particle Representation of Parabose Extension of Conformal Supersymmetry

Igor Salom

Institute of physics, Belgrade

Motivation

- There is no experimental data on (spacetime) supersymmetry, so how do we so surely know that it should be of the standard Poencare (conformal) type?

  - We don't! (HLS presumptions are over constraining)

- Yet, why is 99.9% of supersymmetry work and of empirical predictions based on (N-dimensional) Poencare (conformal) susy?

  - Indeed, why???

Is this really necessary?

$\{Q_\alpha, Q_\beta\}=0$

$\{Q_\alpha, S_{\dot\beta}\}=0$

$\{S_\alpha, S_\beta\}=0$

$\{Q_{\dot\alpha}, Q_{\dot\beta}\}= 0$ (=central)

$\{Q_{\dot\alpha}, S_\beta\}= 0$ (=central)

$\{S_{\dot\alpha}, S_{\dot\beta}\}= 0$ (=central)

generalized supersymmetry

We demonstrate:

- Two simple defining relations of parabose algebra are equivalent to dozens of generalized conformal superalgebra relations (written in different basis)
- Sacrificing of manifest Lorentz covariance reveals a picture of spacetime with two rotation groups
- Mere introduction of preferred direction with respect to one of these groups [i.e. breaking it to U(1)] can recover observable Poincare symmetry
- The simplest representation of this algebra has interesting properties (chiral symmetry = e.m. duality, motion equations = mathematical identities, form of supersymmetry transformations)

Change of basis – part I

- Start with four pairs of parabose operators $\boldsymbol{a}$ and $\boldsymbol{a}^{+}$ which satisfy:

$$[\{\hat{a}_\alpha, \hat{a}_\beta\}, \hat{a}_\gamma] = 0, \qquad [\{\hat{a}_\alpha, \hat{a}^\dagger_\beta\}, \hat{a}_\gamma] = -2\delta^\gamma_\beta \hat{a}_\alpha$$

- Switch to hermitian combinations:

$$S^\alpha \equiv (\hat{a}_\alpha + \hat{a}^\dagger_\alpha), \qquad Q_\alpha \equiv -i(\hat{a}_\alpha - \hat{a}^\dagger_\alpha).$$

consequently satisfying:

$$[\{Q_\alpha, Q_\beta\}, Q_\gamma] = 0,$$
$$[\{Q_\alpha, Q_\beta\}, S^\gamma] = -4i\delta^\gamma_\beta Q_\alpha - 4i\delta^\gamma_\alpha Q_\beta,$$
$$[\{S^\alpha, Q_\beta\}, S^\gamma] = 4i\delta^\gamma_\beta S^\alpha,$$

$$[\{S^\alpha, S^\beta\}, S^\gamma] = 0,$$
$$[\{S^\alpha, S^\beta\}, Q_\gamma] = 4i\delta^\beta_\gamma S^\alpha + 4i\delta^\alpha_\gamma S^\beta,$$
$$[\{Q_\alpha, S^\beta\}, Q_\gamma] = 4i\delta^\beta_\gamma Q_\alpha.$$

Change of basis – part IIa
Basis of 4 by 4 real matrices

6 antisymmetric matrices:

$$[\sigma_i, \sigma_j] = 2\varepsilon_{ijk}\sigma_k$$

$$[\tau_{\underline{i}}, \tau_{\underline{j}}] = 2\varepsilon_{\underline{i}\underline{j}\underline{k}}\tau_{\underline{k}}$$

$$[\sigma_i, \tau_{\underline{j}}] = 0$$

$$\sigma_1 = \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix},$$

$$\tau_1 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}, \quad \tau_2 = \begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}, \quad \tau_3 = \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}.$$

10 symmetric matrices:

$$\alpha_{\underline{i}j} \equiv \tau_{\underline{i}}\sigma_j$$

$$\alpha_0 = 1$$

$$\alpha_{11} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad \alpha_{12} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}, \quad \alpha_{13} = \begin{pmatrix} 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix},$$

$$\alpha_{21} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}, \quad \alpha_{22} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad \alpha_{23} = -\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix},$$

$$\alpha_{31} = -\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \quad \alpha_{32} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}, \quad \alpha_{33} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

Change of basis – part IIb

Introduce a new basis for expressing of parabose anticommutators:

$$\hat{J}_i \equiv \tfrac{1}{8}(\sigma_i)^{\alpha}{}_{\beta}\{Q_\alpha, S^\beta\}, \qquad Y_{\underline{i}} \equiv \tfrac{1}{8}(\tau_{\underline{i}})^{\alpha}{}_{\beta}\{Q_\alpha, S^\beta\},$$

$$\hat{N}_{\underline{i}j} \equiv \tfrac{1}{8}(\alpha_{\underline{i}j})^{\alpha}{}_{\beta}\{Q_\alpha, S^\beta\}, \qquad \hat{D} \equiv (\alpha_0)^{\alpha}{}_{\beta}\{Q_\alpha, S^\beta\},$$

$$\hat{P}_{\underline{i}j} \equiv \tfrac{1}{8}(\alpha_{\underline{i}j})^{\alpha\beta}\{Q_\alpha, Q_\beta\}, \qquad \hat{P}_0 \equiv \tfrac{1}{8}(\alpha_0)^{\alpha\beta}\{Q_\alpha, Q_\beta\},$$

$$\hat{K}_{\underline{i}j} \equiv -\tfrac{1}{8}(\alpha_{\underline{i}j})_{\alpha\beta}\{S^\alpha, S^\beta\}, \qquad \hat{K}_0 \equiv \tfrac{1}{8}(\alpha_0)_{\alpha\beta}\{S^\alpha, S^\beta\},$$

Starting parabose relations obtain a new form:

$$[\{\hat{a}_\alpha, \hat{a}_\beta\}, \hat{a}_\gamma] = 0, \qquad [\{\hat{a}_\alpha, \hat{a}^\dagger_\beta\}, \hat{a}_\gamma] = -2\delta^\gamma_\beta \hat{a}_\alpha$$

$$\Downarrow$$

$$[\hat{J}_i, Q_\alpha] = -i(\tfrac{\sigma_i}{2})_\alpha^{\ \beta} Q_\beta, \quad [Y_i, Q_\alpha] = -i(\tfrac{\tau_i}{2})_\alpha^{\ \beta} Q_\beta, \quad [\hat{N}_{ij}, Q_\alpha] = i(\tfrac{\alpha_{ij}}{2})_\alpha^{\ \beta} Q_\beta,$$

$$[\hat{J}_i, S^\alpha] = -i(\tfrac{\sigma_i}{2})^\alpha_{\ \beta} S^\beta, \quad [Y_i, S^\alpha] = -i(\tfrac{\tau_i}{2})^\alpha_{\ \beta} S^\beta, \quad [\hat{N}_{ij}, S^\alpha] = -i(\tfrac{\alpha_{ij}}{2})^\alpha_{\ \beta} S^\beta,$$

$$[\hat{K}_0, Q_\alpha] = i(\alpha_0)_{\alpha\beta} S^\beta, \quad [\hat{K}_{ij}, Q_\alpha] = -i(\alpha_{ij})_{\alpha\beta} S^\beta, \quad [\hat{K}_0, S^\alpha] = [\hat{K}_{ij}, S^\alpha] = 0,$$

$$[\hat{P}_0, S^\alpha] = -i(\alpha_0)^{\alpha\beta} Q_\beta, \quad [\hat{P}_{ij}, S^\alpha] = -i(\alpha_{ij})^{\alpha\beta} Q_\beta, \quad [\hat{P}_0, Q_\alpha] = [\hat{P}_{ij}, Q_\alpha] = 0,$$

$$[\hat{D}, Q_\alpha] = i(\tfrac{1}{2}) Q_\alpha, \quad [\hat{D}, S^\alpha] = -i(\tfrac{1}{2}) S^\alpha.$$

Algebra of anticommutators

$$[\hat J_i, \hat J_j] = i\,\varepsilon_{ijk}\hat J_k, \quad [\hat Y_{\underline{i}}, \hat Y_{\underline{j}}] = i\,\varepsilon_{\underline{ijk}}\hat Y_{\underline{k}},$$
$$[\hat J_i, \hat Y_{\underline{j}}] = 0.$$

$$[\hat J_i, \hat N_{\underline{j}k}] = i\varepsilon_{ikl}\hat N_{\underline{j}l}, \qquad [\hat Y_{\underline{i}}, \hat N_{\underline{j}k}] = i\varepsilon_{\underline{ijl}}\hat N_{\underline{l}k},$$
$$[\hat N_{\underline{i}j}, \hat N_{\underline{k}l}] = -i\left(\delta_{jl}\varepsilon_{\underline{ikm}}\hat Y_{\underline{m}} + \delta_{\underline{ik}}\varepsilon_{jlm}\hat J_m\right),$$
$$[\hat J_i, \hat D] = [\hat Y_{\underline{i}}, \hat D] = [\hat N_{\underline{i}j}, \hat D] = 0.$$

$$[\hat J_i, \hat P_{\underline{j}k}] = i\varepsilon_{ikl}\hat P_{\underline{j}l}, \qquad [\hat Y_{\underline{i}}, \hat P_{\underline{j}k}] = i\varepsilon_{\underline{ijl}}\hat P_{\underline{l}k},$$
$$[\hat N_{\underline{i}j}, \hat P_{\underline{k}l}] = i\delta_{\underline{ik}}\delta_{jl}\hat P_0 + i\varepsilon_{\underline{ikm}}\varepsilon_{jln}\hat P_{\underline{m}n},$$
$$[\hat N_{\underline{i}j}, \hat P_0] = i\hat P_{\underline{i}j}, \qquad [\hat D, \hat P_{\underline{i}j}] = i\hat P_{\underline{i}j},$$
$$[\hat D, \hat P_0] = i\hat P_0, \qquad [\hat J_i, \hat P_0] = [\hat Y_{\underline{i}}, \hat P_0] = 0.$$

$$[\hat J_i, \hat K_{\underline{j}k}] = i\varepsilon_{ikl}\hat K_{\underline{j}l}, \qquad [\hat Y_{\underline{i}}, \hat K_{\underline{j}k}] = i\varepsilon_{\underline{ijl}}\hat K_{\underline{l}k}, \qquad \ldots$$
$$[\hat P_{\underline{i}j}, \hat K_{\underline{k}l}] = 2i\left(-\delta_{\underline{ik}}\delta_{jl}\hat D - \varepsilon_{\underline{ikm}}\varepsilon_{jln}\hat N_{\underline{m}n} + \delta_{\underline{ik}}\varepsilon_{jlm}\hat J_m + \delta_{jl}\varepsilon_{\underline{ikm}}\hat Y_{\underline{m}}\right),$$
$$[\hat P_{\underline{i}j}, \hat K_0] = -2i\hat N_{\underline{i}j}, \qquad [\hat P_0, \hat K_{\underline{i}j}] = -2i\hat N_{\underline{i}j},$$
$$[\hat P_0, \hat K_0] = -2i\hat D,$$

Two “rotation-like” groups in the core

Isomorphic to *sp*(8)

Symmetry breaking

D	J_1	J_2	J_3
$Y_{\underline{1}}$	$N_{\underline{1}1}$	$N_{\underline{1}2}$	$N_{\underline{1}3}$
$Y_{\underline{2}}$	$N_{\underline{2}1}$	$N_{\underline{2}2}$	$N_{\underline{2}3}$
$Y_{\underline{3}}$	$N_{\underline{3}1}$	$N_{\underline{3}2}$	$N_{\underline{3}3}$

{Q,S} operators

?

Potential $\sim(Y_{\underline{3}})^2$

What remains along $\underline{3}^{rd}$ axis is C(1,3) conformal algebra!

P_0			
	$P_{\underline{1}1}$	$P_{\underline{1}2}$	$P_{\underline{1}3}$
	$P_{\underline{2}1}$	$P_{\underline{2}2}$	$P_{\underline{2}3}$
	$P_{\underline{3}1}$	$P_{\underline{3}2}$	$P_{\underline{3}3}$

{Q,Q} operators

K_0			
	$K_{\underline{1}1}$	$K_{\underline{1}2}$	$K_{\underline{1}3}$
	$K_{\underline{2}1}$	$K_{\underline{2}2}$	$K_{\underline{2}3}$
	$K_{\underline{3}1}$	$K_{\underline{3}2}$	$K_{\underline{3}3}$

{S,S} operators

"Extended" conformal superalgebra

$$[\{\hat{a}_\alpha, \hat{a}_\beta\}, \hat{a}_\gamma] = 0, \quad [\{\hat{a}_\alpha, \hat{a}^\dagger_\beta\}, \hat{a}_\gamma] = -2\delta^\gamma_\beta \hat{a}_\alpha$$

change of notation

Choice of basis

$$\{Q_\alpha, Q_\beta\} = (\alpha_0)_{\alpha\beta}\, P_0 + (\alpha_{\underline{ij}})_{\alpha\beta}\, P_{\underline{ij}},$$

$$\{S^\alpha, S^\beta\} = (\alpha_0)^{\alpha\beta}\, K_0 - (\alpha_{\underline{ij}})^{\alpha\beta}\, K_{\underline{ij}},$$

$$\{S^\alpha, Q_\beta\} = (\alpha_0)^\alpha{}_\beta\, D + (\alpha_{\underline{ij}})^\alpha{}_\beta\, N_{\underline{ij}} + (\sigma_i)^\alpha{}_\beta\, J_i + (\tau_{\underline{i}})^\alpha{}_\beta\, Y_{\underline{i}}$$

$$[\hat{J}_i, Q_\alpha] = -i(\tfrac{\sigma_i}{2})_\alpha{}^\beta\, Q_\beta, \quad [\hat{Y}_{\underline{i}}, Q_\alpha] = -i(\tfrac{\tau_{\underline{i}}}{2})_\alpha{}^\beta\, Q_\beta, \quad [\hat{N}_{\underline{ij}}, Q_\alpha] = i(\tfrac{\alpha_{\underline{ij}}}{2})_\alpha{}^\beta\, Q_\beta,$$

$$[\hat{J}_i, S^\alpha] = -i(\tfrac{\sigma_i}{2})^\alpha{}_\beta\, S^\beta, \quad [\hat{Y}_{\underline{i}}, S^\alpha] = -i(\tfrac{\tau_{\underline{i}}}{2})^\alpha{}_\beta\, S^\beta, \quad [\hat{N}_{\underline{ij}}, S^\alpha] = -i(\tfrac{\alpha_{\underline{ij}}}{2})^\alpha{}_\beta\, S^\beta,$$

$$[\hat{K}_0, Q_\alpha] = i(\alpha_0)_{\alpha\beta}\, S^\beta, \quad [\hat{K}_{\underline{ij}}, Q_\alpha] = -i(\alpha_{\underline{ij}})_{\alpha\beta}\, S^\beta, \quad [\hat{K}_0, S^\alpha] = [\hat{K}_{\underline{ij}}, S^\alpha] = 0,$$

$$[\hat{P}_0, S^\alpha] = -i(\alpha_0)^{\alpha\beta}\, Q_\beta, \quad [\hat{P}_{\underline{ij}}, S^\alpha] = -i(\alpha_{\underline{ij}})^{\alpha\beta}\, Q_\beta, \quad [\hat{P}_0, Q_\alpha] = [\hat{P}_{\underline{ij}}, Q_\alpha] = 0,$$

$$[\hat{D}, Q_\alpha] = i(\tfrac{1}{2})Q_\alpha, \quad [\hat{D}, S^\alpha] = -i(\tfrac{1}{2})S^\alpha.$$

+ bosonic part of algebra

Green's ansatz representations

- Green's ansatz (combined with Klain's transformation):

$$Q_\alpha = \sum_{a=1}^{p} \hat{I}_{(1)} \hat{I}_{(2)} \cdots \hat{I}_{(a-1)} \hat{Q}_\alpha^a \qquad S^\alpha = \sum_{a=1}^{p} \hat{I}_{(1)} \hat{I}_{(2)} \cdots \hat{I}_{(a-1)} \hat{S}_a^\alpha$$

$$\hat{I}_{(a)}^{-1} \hat{Q}_\alpha^b \hat{I}_{(a)} = (-)^{\delta_{ab}} \hat{Q}_\alpha^b, \qquad \hat{I}_{(a)}^{-1} \hat{S}_b^\alpha \hat{I}_{(a)} = (-)^{\delta_{ab}} \hat{S}_b^\alpha$$

$$[\hat{S}_a^\alpha, \hat{Q}_\beta^b] = i\delta_\beta^\alpha \delta_a^b \, \hat{1}; \qquad [\hat{Q}_\alpha^a, \hat{Q}_\beta^b] = [\hat{S}_a^\alpha, \hat{S}_b^\beta] = 0; \;\; a, b = 1, 2, \ldots p$$

- In Green's ansatz p=1 representation parabose algebra reduces to bose algebra and operators Q and S satisfy Heisenberg algebra

$$[\hat{S}^\alpha, \hat{Q}_\beta] = i\delta_\beta^\alpha \, \hat{1} \qquad [\hat{Q}_\alpha, \hat{Q}_\beta] = [\hat{S}^\alpha, \hat{S}^\beta] = 0$$

Important identities for p=1

$$(\alpha_0)_{\alpha\beta}(\alpha_0)_{\gamma\delta} + \sum_{\underline{i}j}(\alpha_{\underline{i}j})_{\alpha\beta}(\alpha_{\underline{i}j})_{\gamma\delta} - \sum_{\underline{i}}(\tau_{\underline{i}})_{\alpha\beta}(\tau_{\underline{i}})_{\gamma\delta} - \sum_{i}(\sigma_i)_{\alpha\beta}(\sigma_i)_{\gamma\delta} = 4\delta_{\beta\gamma}\delta_{\alpha\delta}$$

- Poencare mass is zero:

$$\eta^{\mu\nu}\hat{P}_\mu\hat{P}_\nu \stackrel{\mathrm{d}}{=} (\hat{P}_0)^2 - (\hat{P}_1)^2 - (\hat{P}_2)^2 - (\hat{P}_3)^2 = 0$$

- Y_3 is helicity: $\vec{\hat{P}}\vec{\hat{J}} = \hat{P}^0\hat{Y}_{\underline{3}}$

- Also, e.g.: $\varepsilon_{\underline{i}\underline{j}\underline{k}}\varepsilon_{lmn}P_{\underline{i}l}P_{\underline{j}m} = P_0P_{\underline{k}n}$

$$\hat{P}_{\underline{i}j}\hat{P}_{\underline{i}k} = \delta_{jk}(\hat{P}_0)^2; \qquad \hat{P}_{\underline{j}i}\hat{P}_{\underline{k}i} = \delta_{\underline{j}\underline{k}}(\hat{P}_0)^2$$

p=1 Hilbert space bases

- The most straightforward basis

$$\mathcal{S} = \left\{ |S^1, S^2, S^3, S^4\rangle \mid S^1, S^2, S^3, S^4 \in \mathcal{R} \right\}; \qquad \hat{S}^\alpha |S^1, S^2, S^3, S^4\rangle = S^\alpha |S^1, S^2, S^3, S^4\rangle$$

$$\mathcal{Q} = \{ |Q_1, Q_2, Q_3, Q_4\rangle \mid Q_1, Q_2, Q_3, Q_4 \in \mathcal{R} \}; \qquad \hat{Q}_\alpha |Q_1, Q_2, Q_3, Q_4\rangle = Q_\alpha |Q_1, Q_2, Q_3, Q_4\rangle$$

- Momentum-helicity representation

$$Y_{\underline{3}} |p, \theta, \varphi, \mathrm{y}_{\underline{3}}\rangle = \mathrm{y}_{\underline{3}} |p, \theta, \varphi, \mathrm{y}_{\underline{3}}\rangle, \qquad \mathrm{y}_{\underline{3}} = 0, \pm\tfrac{1}{2}, \pm 1, \pm\tfrac{3}{2}, \ldots$$

$$P^1 |p, \theta, \varphi, \mathrm{y}_{\underline{3}}\rangle = p \sin\theta \cos\varphi \, |p, \theta, \varphi, \mathrm{y}_{\underline{3}}\rangle, \qquad p \in [0, \infty)$$

$$P^2 |p, \theta, \varphi, \mathrm{y}_{\underline{3}}\rangle = p \sin\theta \sin\varphi \, |p, \theta, \varphi, \mathrm{y}_{\underline{3}}\rangle, \qquad \theta \in [0, \pi]$$

$$P^3 |p, \theta, \varphi, \mathrm{y}_{\underline{3}}\rangle = p \cos\theta \, |p, \theta, \varphi, \mathrm{y}_{\underline{3}}\rangle, \qquad \varphi \in [0, 2\pi)$$

Scalar field representation

- Define states: $$|\phi(x)\rangle \stackrel{\mathrm{d}}{=} \int_{\mathcal{R}^3} \frac{d^3p}{(2\pi)^{3/2}} \frac{1}{\sqrt{2p^0}} e^{ip_\mu x^\mu} |\vec{p}, \mathrm{y}_{\underline{3}} = 0\rangle$$

$$\phi_f(x) \stackrel{\mathrm{d}}{=} \langle \phi(x) | f \rangle \qquad \phi_f(x) \xrightarrow{\Lambda} \phi_f{}'(x) = \phi_f(\Lambda^{-1} x)$$

- Symmetry generators act in the standard way

$$P_\mu \phi_f(x) \stackrel{\mathrm{d}}{=} \langle \phi(x) | \hat{P}_\mu | f \rangle = i \frac{\partial}{\partial x^\mu} \phi_f(x)$$

$$M_{\mu\nu} \phi_f(x) \stackrel{\mathrm{d}}{=} \langle \phi(x) | \hat{M}_{\mu\nu} | f \rangle = i(x_\mu \partial_\nu - x_\nu \partial_\mu) \phi_f(x)$$

- Klain-Gordon equation

$$0 = \langle \phi(x) | (-P^\mu P_\mu) | f \rangle = \partial^\mu \partial_\mu \phi_f(x)$$

Spinor field representation

- Define

$$|\psi_\alpha(x)\rangle \stackrel{\mathrm{d}}{=} \sqrt{2}\hat{Q}_\alpha|\phi(x)\rangle$$

$$\psi_{f\alpha}(x) \equiv \langle\psi_\alpha(x)|f\rangle$$

- Y_3 generates chiral symmetry

$$Y_{\underline{3}}\psi_{f\alpha}(x) \stackrel{\mathrm{d}}{=} \langle\psi_\alpha(x)|\hat{Y}_{\underline{3}}|f\rangle = i(\frac{\tau_{\underline{3}}}{2})_\alpha^{\ \beta}\psi_{f\beta}(x) = \frac{1}{2}(\gamma_5)_\alpha^{\ \beta}\psi_{f\beta}(x)$$

- Dirac equation:

$$0 \;=\; \langle\psi_\alpha(x)|\hat{P}_0\hat{Y}_{\underline{3}} + \sum_i \hat{P}_i\hat{J}_i|f\rangle \quad \Longrightarrow \quad i\gamma^\mu\partial_\mu\psi_f(x) = 0$$

$$\boxed{\gamma_0 = i\tau_{\underline{2}}, \qquad \gamma_i = \gamma_0\,\alpha_{\underline{3}i} = i\tau_{\underline{1}}\sigma_i, \qquad \gamma_5 = -i\gamma_0\gamma_1\gamma_2\gamma_3 = i\tau_{\underline{3}}.}$$

Helicity ±1field representation

- Define

$$|E_i(x)\rangle \overset{\mathrm{d}}{=} 2\hat{P}_{\underline{1}i}|\phi(x)\rangle \qquad |B_i(x)\rangle \overset{\mathrm{d}}{=} -2\hat{P}_{\underline{2}i}|\phi(x)\rangle$$

- Y_3 generates e.m. duality symmetry

$$|f\rangle \to \exp(i\phi\hat{Y}_{\underline{3}})|f\rangle \Longrightarrow \begin{cases} E_{f_i}(x) \longrightarrow E'_{f_i}(x) = E_{f_i}(x)\cos\phi - B_{f_i}(x)\sin\phi, \\ B_{f_i}(x) \longrightarrow B'_{f_i}(x) = E_{f_i}(x)\sin\phi + B_{f_i}(x)\cos\phi. \end{cases}$$

- Maxwell equations:

$$\vec{\nabla}\vec{E}_f(x) = \langle\phi(x)|2(\hat{P}_{\underline{3}1}\hat{P}_{\underline{1}1} + \hat{P}_{\underline{3}2}\hat{P}_{\underline{1}2} + \hat{P}_{\underline{3}3}\hat{P}_{\underline{1}3})|f\rangle = 0,$$

$$\vec{\nabla}\vec{B}_f(x) = \langle\phi(x)|2(\hat{P}_{\underline{3}1}\hat{P}_{\underline{2}1} + \hat{P}_{\underline{3}2}\hat{P}_{\underline{2}2} + \hat{P}_{\underline{3}3}\hat{P}_{\underline{2}3})|f\rangle = 0,$$

$$(\vec{\nabla}\times\vec{E}_f(x))_i = \langle\phi(x)|2\varepsilon_{ijk}\hat{P}_{\underline{3}j}\hat{P}_{\underline{1}k}|f\rangle = \langle\phi(x)|2\hat{P}_0\hat{P}_{\underline{2}i}|f\rangle = -\partial_0 B_{f_i}(x),$$

$$(\vec{\nabla}\times\vec{B}_f(x))_i = -\langle\phi(x)|2\varepsilon_{ijk}\hat{P}_{\underline{3}j}\hat{P}_{\underline{2}k}|f\rangle = \langle\phi(x)|2\hat{P}_0\hat{P}_{\underline{1}i}|f\rangle = \partial_0 E_{f_i}(x).$$

Field representation of arbitrary helicity

- Define

$$|F_{h,s}(x)\rangle = (\hat{u}_{\frac{1}{2}})^{h+s}(\hat{u}_{-\frac{1}{2}})^{h-s}|\phi(x)\rangle, \quad h \geq 0$$

$$|F_{h,s}(x)\rangle = (\hat{v}_{\frac{1}{2}})^{|h|+s}(\hat{v}_{-\frac{1}{2}})^{|h|-s}|\phi(x)\rangle, \quad h < 0$$

where

$$\hat{u}_{\frac{1}{2}} \stackrel{\mathrm{d}}{=} \hat{Q}_1 + i\hat{Q}_3, \qquad \hat{u}_{-\frac{1}{2}} \stackrel{\mathrm{d}}{=} \hat{Q}_2 + i\hat{Q}_4$$

$$\hat{v}_{\frac{1}{2}} \stackrel{\mathrm{d}}{=} \hat{Q}_2 - i\hat{Q}_4, \qquad \hat{v}_{-\frac{1}{2}} \stackrel{\mathrm{d}}{=} \hat{Q}_1 - i\hat{Q}_3$$

- Field representation of helicity h is

$$F_{f\,s}^{(h)}(x) = \langle F_{h,s}(x)|f\rangle \quad s = -|h|, -|h|+1, \ldots, |h|.$$

Supersymmetry transformations

- Supermultiplets are infinite:

$$\delta\phi_f(x) = i\bar{\xi}^\alpha\langle\phi(x)|\sqrt{2}\hat{Q}_\alpha|f\rangle = i\bar{\xi}^\alpha\psi_{f\alpha}(x)$$

$$\begin{aligned}\delta\psi_{f\alpha}(x) &= i\bar{\xi}^\beta\langle\psi_\alpha(x)|\sqrt{2}\hat{Q}_\beta|f\rangle \\ &= -\xi^\beta(\gamma^\mu)_{\beta\alpha}\partial_\mu\phi_f(x) + i\xi^\beta(\sigma_{\mu\nu})_{\beta\alpha}F_f{}^{\mu\nu}(x)\end{aligned}$$

$$\delta F_f{}^{\mu\nu}(x) = \cdots$$

Comparison

	Standard supersymmetry	Generalized supersymmetry
Complexity = min number of defining rel.	Significantly more than 2	2
Need for symmetry breaking	yes	yes
Which symmetry is higher?	<	
Spacetime metric introduced by hand?	yes	no
Existence of fully developed models	yes	no

Awaits for investigation...