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Black holes in heterotic string theory

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M. Cvitan, P.D.P., S. Pallua & I. Smolić, hep-th/0706.1167
M. Cvitan, P.D.P. and A. Ficnar, soon.

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BH's in string theory

Heavy strings $\longrightarrow$ BH's (argued $r_s < r_{Sch}$)

- microstate counting at $g_s \sim 0 \longrightarrow S_{stat}$

$$g_s \downarrow ??$$

Low energy effective action (some SUGRA)

- BH solutions $\longrightarrow S_{bh}$

Problem: generally S_{bh} depends on g_s!

BPS states $\Rightarrow$ short multiplets $\Rightarrow$ No. of states protected $\Rightarrow$ $S_{stat} = S_{bh}$

Extremality essential $\Rightarrow$ non-BPS BH's

Nontrivial test of string theory!

Heterotic string on $\mathcal{M}_D \times S^1 \times T^{9-D}$

1/2 BPS states in perturbative spectrum

$$M = |k_R| \qquad k_{R,L} = \frac{n}{R} \pm \frac{wR}{\alpha'}$$

Using mass-shell conditions

$$M^2 = \frac{4}{\alpha'}(N-1) + k_L^2 = \frac{4}{\alpha'}(\tilde{N} - \delta_{NS}) + k_R^2$$

From the second equation follows

$$\tilde{N} = \frac{1}{2}\delta_{NS} \;\Rightarrow\; \text{SUSY} - \text{sector not excited}$$

From the first equation follows

$$N = nw + 1$$

Using asymptotic formula for number of states

$$\mathcal{N} \sim \exp(4\pi\sqrt{N}), \qquad N \gg 1$$

we obtain the statistical entropy of 2-charge BPS states as

$$S_{stat} = \ln \mathcal{N} \sim 4\pi\sqrt{nw}\ , \qquad nw \gg 1$$

Het. string on $\mathcal{M}_4 \times S^1 \times \tilde{S}^1 \times T^4$

8-charge BPS states:

n , $\tilde{n}$ – momenta Nos.
w , $\tilde{w}$ – winding Nos.
N , $\tilde{N}$ – Nos. of Kaluza-Klein monopoles
W , $\tilde{W}$ – Nos. of H-monopoles

Putting $\tilde{n} = \tilde{w} = N = W = 0 \rightarrow$ 4-charge BPS states with statistical entropy

$$S_{stat} = 2\pi\sqrt{nw\,(\tilde{N}\tilde{W} + 4)}\ , \qquad nw \gg 1$$

Entropy formulae exact in α'!

Heterotic LEEA for $S^1 \times \tilde{S}^1 \times T^4$

Lowest order in α' and g_s:

$$\begin{aligned}\mathcal{L}_0 &= R + S^{-2}(\partial S)^2 - T^{-2}(\partial T)^2 - \tilde{T}^{-2}(\partial \tilde{T})^2 \\ &\quad -T^2\left(F^{(1)}_{\mu\nu}\right)^2 - T^{-2}\left(F^{(3)}_{\mu\nu}\right)^2 \\ &\quad -\tilde{T}^2\left(F^{(2)}_{\mu\nu}\right)^2 - \tilde{T}^{-2}\left(F^{(4)}_{\mu\nu}\right)^2\end{aligned}$$

S – dilaton $(1/g_s^2)$, $\quad T$, $\tilde{T}$ – radia of S^1, $\tilde{S}^1$
$A^{(1)}_\mu$, $A^{(2)}_\mu$ – from $g_{\mu 4}$, $g_{\mu 5} \rightarrow (n,\, N)$, $(w,\, W)$
$A^{(3)}_\mu$, $A^{(4)}_\mu$ – from $B_{\mu 4}$, $B_{\mu 5} \rightarrow (\tilde{n},\, \tilde{N})$, $(\tilde{w},\, \tilde{W})$

For 4-charge case $\rightarrow N = W = \tilde{n} = \tilde{w} = 0$.

Asymptotically flat BH solutions exist. For extremal and sphericaly symmetric one gets:

$$S(r_H) \sim \sqrt{\left|\frac{nw}{\tilde{N}\tilde{W}}\right|}\,, \qquad (F^{(a)})^2 \sim \frac{1}{\tilde{N}\tilde{W}}$$

$$S_{BH} = 2\pi\sqrt{|nw\tilde{N}\tilde{W}|}\,, \qquad T(r_H) = \sqrt{\frac{n}{w}}$$

$S_{BH} \neq S_{stat}$

Observations

- For $\tilde{N}, \tilde{W} \gg 1$, $S_{stat} \to S_{BH}$. It is obvious that expansion in $1/\tilde{N}\tilde{W}$ is α' expansion. To explain discrepancy we need higher terms in α' in the effective action. In the string frame

$$r_H^2 \propto \alpha' \tilde{N}\tilde{W} \gg \alpha' \sim l_{string}$$

i.e., BH is large and α' expansion well defined.

- For $nw \gg \tilde{N}\tilde{W}$, $g_{eff}^2 \sim 1/S \ll 1$. Tree level in string coupling OK.

- Solutions of LEEA for all signs of charges $\Rightarrow$ non-BPS extremal BH's
  - (attractor mechanism) + ($g_{eff}^2 \ll 1$) $\Rightarrow$ Entropy also "protected"
  - From string side – statistical entropy

$$S_{stat} = 2\pi\sqrt{|nw|\,(\tilde{N}\tilde{W}+2)}\;, \qquad |nw| \gg 1$$

- For $\tilde{N}\tilde{W} = 0 \longrightarrow S_{BH} = 0$, horizon becomes null-singular!!? BH is small and α' expansion breaks down.

  $\mathcal{L}_{eff} =$ "R" $+ \alpha'$ "R^2" $+ \ldots + \alpha'^n$ "R^n" $+ \ldots$

  As now $R \sim 1/\alpha'$, all terms a priory equally important (infinite number). Full LEEA needed to account for 2-charge BH's?

Entropy in generalized gravity

For manifestly diffeomorphism invariant Lagrangians

$$L = L(g_{ab}, R_{abcd}, \nabla R_{abcd}, \psi, \nabla \psi, ...)$$

BH entropy is given by Wald formula:

$$S = -2\pi \int_{\mathcal{H}} \epsilon_{D-2} \frac{\delta L}{\delta R_{abcd}} \eta_{ab} \eta_{cd} \qquad (1)$$

If

$$L = L(g_{ab}, R_{abcd}, \nabla R_{abcd}, \psi, \nabla \psi, ...)$$

Recently generalised to theories with Chern-Simons terms.

Entropy function formalism for extremal BH's

Extremality $\rightarrow AdS_2 \times S^{D-2}$ near-horizon geometry $\rightarrow SO(2,1) \times O(D-1)$ symmetry

$$ds^2 = v_1 \left(-x^2 dt^2 + \frac{dx^2}{x^2} \right) + v_2\, d\Omega^2_{D-2}$$

$$\phi_s = u_s$$

$$F^{(i)}_2 = -e_i\, \epsilon_2 \ , \qquad H^{(a)}_{D-2} = p_a\, \epsilon_{D-2}$$

all other fields & cov. derivatives vanishing.

If one defines

$$f(\vec{u}, \vec{v}, \vec{e}, \vec{p}) = \int_{S^{D-2}} \sqrt{-g}\, \mathcal{L}$$

then EOM's near the horizon become

$$\frac{\partial f}{\partial u_s} = 0 \ , \qquad \frac{\partial f}{\partial v_i} = 0$$

Entropy and electric charges are

$$S_{BH} = 2\pi \left(\sum_i e_i\, q_i - f \right) , \qquad q_i = \frac{\partial f}{\partial e_i}$$

Alternatively, one defines the entropy function

$$\mathcal{E}(\vec{u}, \vec{v}, \vec{e}, \vec{q}, \vec{p}) = 2\pi \left(\sum_i e_i \, q_i - f \right)$$

Extremisation of $\mathcal{E}$ gives EOM's and connects electric fields with charges

$$0 = \frac{\partial \mathcal{E}}{\partial u_s} = \frac{\partial \mathcal{E}}{\partial v_i} = \frac{\partial \mathcal{E}}{\partial e_i}$$

and a value of $\mathcal{E}$ at the extremum is the entropy

$$S_{BH} = \mathcal{E}$$

Comments on $\mathcal{E}$-function method

- Very practical way for near-horizon analyses of extremal BH's.
- Attractor mechanism - direct consequence.
- Directly applicable only for actions with manifest gauge and diffeomorfism invariance (with Chern-Simons terms more effort needed).
- Has been extended to BTZ-type BH's.

α' corrections in LEEA

- Full effective string action has ∞ No. of terms (even on tree level)

$$\mathcal{A}_{eff} = \sum_{n=1}^{\infty} (\alpha' \text{``}R\text{''})^n$$

up to $n = 3$ known completely.
$\Rightarrow$ low order perturbative analyses possible

- Taking some truncated actions by adding:
  1. Gauss-Bonnet term

$$(R_{abcd})^2 - 4(R_{ab})^2 + R^2$$

  2. SUSY-zation of gravitational Chern-Simons term

$$A \wedge R \wedge R + \ldots \qquad (\text{in } D = 5)$$

Results in $D = 4$

- $S_{stat} = S_{bh}$ perturbatively up to α'^2-order (large; BPS and non-BPS)
- $S_{stat} = S_{bh}$ for both GB and gCS actions (large and small; BPS)
  - also same near-horizon solutions

  Why!?
- OSV conjecture in $N = 2$ SUGRA

  $$Z_{bh} = |Z_{top.\,string}|^2$$

  (large and small; BPS)
- AdS$_3$-view $\to$ $S_{stat} = S_{bh}$ (large and small; BPS and non-BPS)
  - only anomalies important (i.e., CS terms)
  - partial explanation (non-BPS BH's, in some cases no AdS$_3$)

Topological origin?

$D = 5$ 3-charge heterotic BH's

Heterotic string on $T^4 \times S^1$ - LEEA is $N = 2$ SUGRA with prepotential $\mathcal{N} = M_1 M_2 M_3$
Connection with dilaton and modulus:

$$M_1 = S^{-1/3} T^{-1}, \quad M_2 = S^{-1/3} T, \quad M_3 = S^{2/3}$$

3-charge BH solutions with entropy

$$S_{bh}^{(0)} = 2\pi \sqrt{nwm}$$

n and w electric (Maxwell), m magnetic ($B_{\mu\nu}$) integer charge.

For $n, w, m \geq 0$ BPS.

Natural candidate!

Results in $D = 5$

- String side $\rightarrow$ $S_{stat} = ??$
- ADS$_3$-argument $\rightarrow$ extension to $D = 5$??
- gCS (SUSY)-action gives:

$$\begin{aligned} \mathcal{S}_{bh} &= 2\pi\sqrt{nw(m+3)}, && \text{(BPS)} \\ \mathcal{S}_{bh} &= 2\pi\sqrt{|n|w(m+1/3)}, && \text{(non-BPS)} \end{aligned}$$

- Gauss-Bonnet action gives entropy which starts to differ at α'^2 for BPS, and α' for non-BPS BH's
- Perturbative results up to α'^2 agree with gCS (SUSY) for BPS, and for non-BPS suggest

$$\mathcal{S}_{bh} = 2\pi\sqrt{|n|w(m+1)}$$

- OSV conjecture $4D/5D$ properly uplifted $\rightarrow$ confirms gCS (SUSY) for BPS

$\Longrightarrow$ SUSY wins, GB loses? Not that simple.

Small BH's in $D = 5$

For $m = 0 \implies$ small black holes:

- String theory – DH states:

$$\mathcal{S}_{stat} = 4\pi\sqrt{nw}, \qquad \text{(BPS)}$$
$$\mathcal{S}_{stat} = 2\sqrt{2}\,\pi\sqrt{|n|w}, \qquad \text{(non-BPS)}$$

- Gauss-Bonnet action:

$$S_{bh} = 4\pi\sqrt{|nw|}$$

Agrees for BPS

- gCS (SUSY) action:

$$\mathcal{S}_{bh} = 2\sqrt{3}\,\pi\sqrt{nw} \qquad \text{(BPS)}$$
$$\mathcal{S}_{bh} = ? \qquad \text{(non-BPS)}$$

$S_{bh} \neq S_{stat}$

In $D = 5$ large/small BH's limit non-trivial !

Small BH's → Lovelock type gravity

Attempt: in general D try with generalized Gauss-Bonnet densities:

$$\begin{aligned}\mathcal{L}_m &= \lambda_m \mathcal{L}_m^{GB} \\ &= \frac{\lambda_m}{2^m}\,\delta^{\rho_1\sigma_1\ldots\rho_m\sigma_m}_{\mu_1\nu_1\ldots\mu_m\nu_m}\, R^{\mu_1\nu_1}{}_{\rho_1\sigma_1}\cdots R^{\mu_m\nu_m}{}_{\rho_m\sigma_m}\ ,\end{aligned}$$

to construct the action

$$\mathcal{A} = \frac{1}{16\pi G_N}\int d^Dx\sqrt{-g}\,S\sum_{m=1}\alpha'^{m-1}\mathcal{L}_m$$

Incredibly, there is a unique fixed choice of λ_m, given with

$$\lambda_m = \frac{4}{4^m m!}$$

for which

$$S_{BH} = 2\pi\sqrt{nw} = S_{stat}\ , \qquad \forall D$$

Conclusion and Outlook

- Many examples (in $D = 4$ and $D = 5$)
- Extension of AdS_3-argument to $D = 5$?
- Something more general?
  - near horizon new type of effective action?
- BPS $\rightarrow$ extremal $\stackrel{?}{\rightarrow}$ realistic BH's
  - anomalies
  - near-horizon constraints (reguarity, ...)