A LOWER BOUND FOR *S*-PARAMETER FOR WALKING TECHNICOLOR

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Plan:

1) Introduction
2) ACD Method
3) *S*-parameter
4) Conclusions

1) Introduction

- Electroweak *S*-parameter – Peskin and Takeuchi (1990)
- Operator Product Expansion (OPE) – Shifman et al. (1979)
- QCD Sum Rules
- Analytic Continuation by Duality (ACD) – Nasrallah et al. (1982-)

Technicolor:

QCD-like
Walking

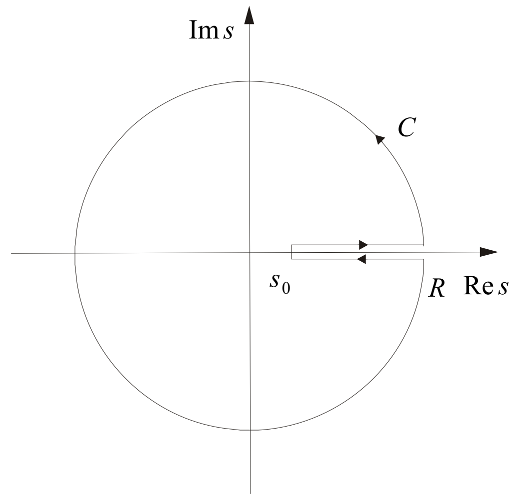
S, *T*, *U* parameters

$$S = -0.13 \pm 0.10 \qquad (m_H = 117\,\text{GeV})$$

2) ACD Method

Cauchy's integral formula:

$$F(t)=\frac{1}{2\pi i}\oint_C \frac{F(s)}{s-t}ds.$$



Discontinuity relation:

$$F(s+i\varepsilon)-F(s-i\varepsilon)=2i\,\mathrm{Im}\,F(s+i\varepsilon)$$

$$F(t)=\frac{1}{2\pi i}\oint_{|s|=R}\frac{F(s)}{s-t}ds+\frac{1}{\pi}\int_{s_0}^{R}\frac{\mathrm{Im}\,F(s)}{s-t-i\varepsilon}ds.$$

limit $t\to 0$

$$F(0)=\frac{1}{2\pi i}\oint_{|s|=R}\frac{F(s)}{s}ds+\frac{1}{\pi}\int_{s_0}^{R}\frac{\mathrm{Im}\,F(s)}{s}ds$$

polynomial

$$p_N(s)=\sum_{n=0}^{N}a_n(N)s^n$$

Cauchy's theorem

$$0=\frac{1}{2\pi i}\oint_C p_N(s)F(s)ds=\frac{1}{2\pi i}\oint_{|s|=R}p_N(s)F(s)ds+\frac{1}{\pi}\int_{s_0}^{R}p_N(s)\,\mathrm{Im}\,F(s)ds$$

Subtraction

$$F(0)=\frac{1}{2\pi i}\oint_{|s|=R}\left[\frac{1}{s}-p_N(s)\right]F(s)ds+\frac{1}{\pi}\int_{s_0}^{R}\left[\frac{1}{s}-p_N(s)\right]\mathrm{Im}\,F(s)ds$$

First approximation

$$G \equiv F(0) = \frac{1}{2\pi i}\oint_{|s|=R}\left[\frac{1}{s} - p_N(s)\right]F(s)ds + \delta G_{\rm fit}(N) = G_N + \delta G_{\rm fit}(N)$$

$\delta G_{\rm fit}(\mathrm{N})$ – *fit error*

L_p norm

$$\| d \| = \left[\int_a^b | d(x) |^p \; w(x)dx\right]^{\frac{1}{p}},$$

where

$$d(x) = \frac{1}{s} - p_N(s)$$

large-*s* expansion

$$F(s) = \sum_{m=1}^{M}\frac{h_m(s)}{s^m} + O\left(\frac{1}{s^{M+1}}\right)$$

around the circle $|s| = R$

Second Approximation

$$G_N = G_{N,M} + \delta G_{\rm trunc}(N,M)$$

$$G_{N,M} = \frac{1}{2\pi i}\oint_{|s|=R}\left[\frac{1}{s} - p_N(s)\right]\sum_{m=1}^{M}\frac{h_m(s)}{s^m}ds$$

$$\delta G_{\rm trunc}(N,M) = \frac{1}{2\pi i}\oint_{|s|=R}\left[\frac{1}{s} - p_N(s)\right]\left[F(s) - \sum_{m=1}^{M}\frac{h_m(s)}{s^m}\right]ds$$

Third approximation

$$h_m(s) \approx h_m(-R) \equiv \hat{h}_m$$

$$G_{N,M} = G_{\mathrm{ACD}} + \delta G_{\mathrm{AC}}(N,M)$$

$$G_{\mathrm{ACD}} = \frac{1}{2\pi i} \oint_{|s|=R} \left[\frac{1}{s} - p_N(s) \right] \sum_{m=1}^{M} \frac{\hat{h}_m}{s^m} ds$$

$$\delta G_{\mathrm{AC}}(N,M) = \frac{1}{2\pi i} \oint_{|s|=R} \left[\frac{1}{s} - p_N(s) \right] \sum_{m=1}^{M} \frac{h_m(s) - \hat{h}_m}{s^m} ds$$

ACD estimate

$$G_{\mathrm{ACD}} = - \sum_{n=0}^{\min(N,M-1)} \hat{h}_{n+1} a_n(N)$$

ACD Estimates for Model Spectra

1) introduce several toy model functions for $F(s)$ such that the imaginary part along the real axis, $\mathrm{Im}\, F(s)$, the large-s expansion coefficients $\hat{h}_m$ and the corresponding G are known exactly;
2) apply both the dispersion relation and ACD technique to the model functions;
3) check if the dispersion relation approach reproduces the exact result;
4) calculate the three types of error to see if they are under control.

a) Vector-meson dominance model with δ-functions:

$$F(s) = \frac{f_V^2}{s - m_V^2 + i\varepsilon} - \frac{f_A^2}{s - m_A^2 + i\varepsilon},$$

$$\operatorname{Im} F(s) = -\pi[f_V^2 \delta(s - m_V^2) - f_A^2 \delta(s - m_A^2)], \qquad G = \frac{f_A^2}{m_A^2} - \frac{f_V^2}{m_V^2};$$

b) Vector-meson dominance model with Breit-Wigner resonances:

$$F(s) = \frac{f_V^2}{s - m_V^2 + i\sqrt{s}\Gamma_V} - \frac{f_A^2}{s - m_A^2 + i\sqrt{s}\Gamma_A},$$

$$\operatorname{Im} F(s) = -\sqrt{s}\ \Theta(s - s_0)\left[\frac{f_V^2 \Gamma_V}{(s - m_V^2)^2 + s\Gamma_V^2} - \frac{f_A^2 \Gamma_A}{(s - m_A^2)^2 + s\Gamma_A^2}\right], \quad G = \frac{f_A^2}{m_A^2} - \frac{f_V^2}{m_V^2};$$

Applications of the ACD Method

- low-energy QCD
- QCD-like Technicolor
- Walking Technicolor

Electroweak parameter

$$S = -4\pi F(0),$$

Recent Modification of the Method

$$\frac{1}{s} = \frac{1}{R}\sum_{n=0}^{N} a_n(N)\left(\frac{s}{R}\right)^n + O\left(\frac{1}{R^{N+2}}\right)$$

$$a_n(N) = \frac{(-1)^n}{R^{n+1}}\binom{N+1}{n+1}$$