Perturbative quantum corrections in flux compactifications

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0704.0737 [hep-th] (with M. Berg, E. Pajer)

Motivation

- Reduction D=10 $\longrightarrow$ D=4 leads to massless scalar fields in effective action: contradiction to 5th force exp.

- Background fluxes lead to potentials

- α', loop and non-perturbative corrections important if tree level term vanishes

- Reduction determines soft supersymmetry breaking terms (direct relevance for LHC!): role of quantum corrections?

Overview

- Review type IIB compactifications with fluxes
- Large volume scenario
- Effects of string loop corrections
- Soft susy breaking terms
- Outlook

Type IIB

- Massless (bosonic) spectrum:

$$g_{MN},\ \phi,\ B_{MN},\ C,\ C_{MN},\ C_{MNPQ}$$

- 10-dimensional action:

$$S_{IIB} \sim \int d^{10}x\sqrt{-g}\left\{e^{-2\phi}\left[R + (\partial\phi)^2\right] - F_1^2 - G_3\cdot\bar{G}_3 - \tilde{F}_5^2\right\}$$

$$+ \int e^{\phi} C_4 \wedge G_3 \wedge \bar{G}_3$$

with $G_3 = F_3 - SH_3$, $S = e^{-\phi} + iC_0$

and $\tilde{F}_5 = F_5 - \frac{1}{2}C_2 \wedge H_3 + \frac{1}{2}B_2 \wedge F_3$

- Calabi-Yau (orientifold) compactification of IIB
  $\Longrightarrow \ \mathcal{N}=1,\ d=4$ supergravity

- Moduli problem

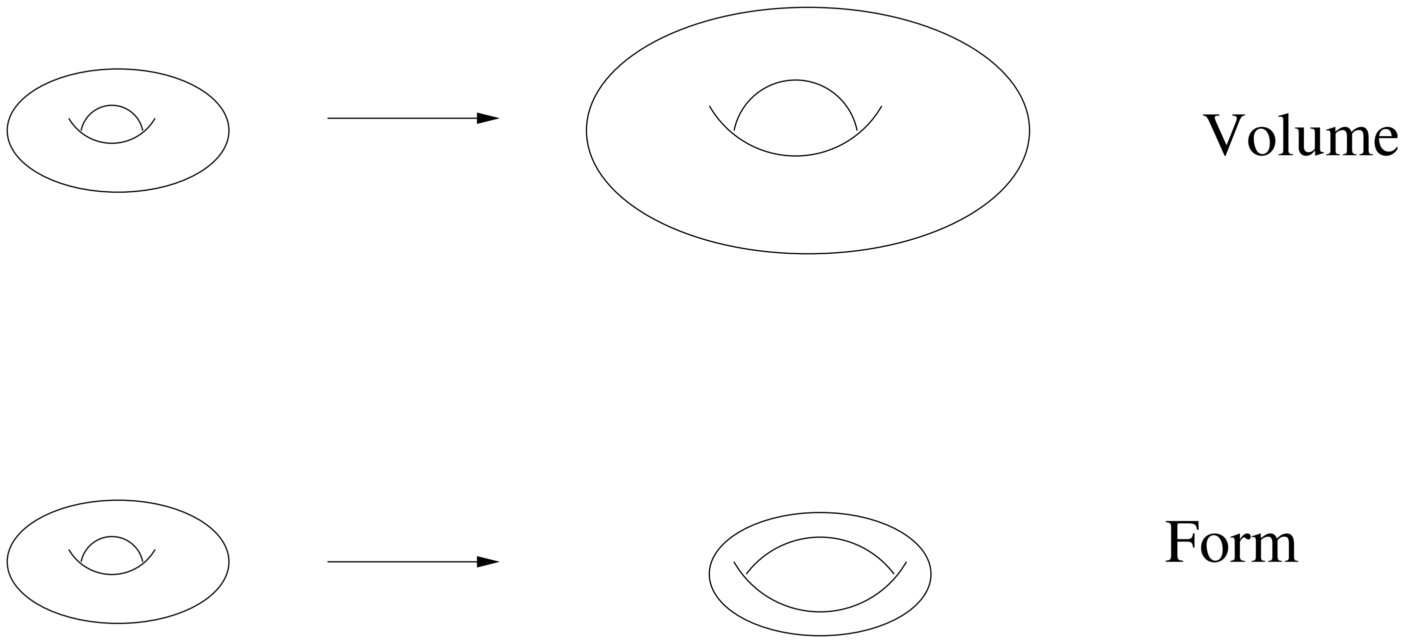
Complex structure: U^{α} (number given by $h^{2,1}$)

Kähler: $T^j = \tau^j + iC^j$ (number given by $h^{1,1}$)

τ^j: $\text{volume}(\Sigma_4^j)$

C^j: $\int_{\Sigma_4^j} C_4$

2-dim example: torus



Background Fluxes

- IIB string theory contains 2-form fields
- Kinetic term in 10 dimensions:

$$\int d^{10}x\,\sqrt{-G}F_{IJK}F_{LMN}G^{IL}G^{JM}G^{KN}$$

- Internal components G^{il} of the metric correspond to the moduli fields
- $\langle F_{ijk}\rangle \neq 0$: Potential for the moduli
  [Polchinski, Strominger]
- $\int d^{10}x\sqrt{-g}\,G_3 \cdot \bar{G}_3 \quad \Rightarrow \quad W_{\rm flux} = \int_{\rm CY} G_3 \wedge \Omega_3$

[Giddings, Kachru, Polchinski]

$\mathcal{N}=1,\ d=4$ Supergravity

$$\frac{\mathcal{L}_{\rm bos}}{(-G)^{1/2}} = \frac{1}{2\kappa^2}R - K_{,\bar{\imath}j}D_\mu\bar{\phi}^{\bar{\imath}}D^\mu\phi^j - \frac{1}{4}\mathrm{Re}(f_{ab}(\phi))F^a_{\mu\nu}F^{b\mu\nu}$$

$$-\frac{1}{8}\mathrm{Im}(f_{ab}(\phi))\epsilon^{\mu\nu\rho\sigma}F^a_{\mu\nu}F^b_{\rho\sigma} - V(\phi,\bar{\phi})$$

- $K_{,\bar{\imath}j} = \dfrac{\partial^2 K(\phi,\bar{\phi})}{\partial\bar{\phi}^{\imath}\partial\phi^j}$, K Kählerpotential
- f_{ab} gauge kinetic function (holomorphic)
- $V(\phi,\bar{\phi}) = e^K(K^{\bar{\imath}j}D_{\bar{\imath}}\overline{W}D_jW - 3|W|^2) + \mathrm{Re}(f_{ab})\mathcal{D}^a\mathcal{D}^b$
- $D_jW \equiv \partial_{\phi^j}W + \partial_{\phi^j}KW$, W Superpotential (holom.)

  ↖ $\equiv F_j$

- $W_{\rm flux}$ leads to potential for dilaton and c.s. moduli

- KKLT: [Kachru, Kallosh, Linde, Trivedi]

  Additional contribution to superpotential from D7-branes wrapped around 4-cycles Σ^j

  Gaugino condensation on D7:

$$W_{np} \sim e^{-a f^j}$$

$$f^j_{\rm tree} = T^j = \tau^j + iC^j$$

- Supersymmetric minima:

$$D_{T^j} W = D_{U^\alpha} W = D_S W = 0$$

Drawback of KKLT

- $W = W_{\rm flux} + W_{\rm np} = W(S,U) + A(S,U)e^{-aT}$

  $K = -\ln(S+\bar{S}) - 3\ln(T+\bar{T}) + K_{\rm cs}(U,\bar{U})$

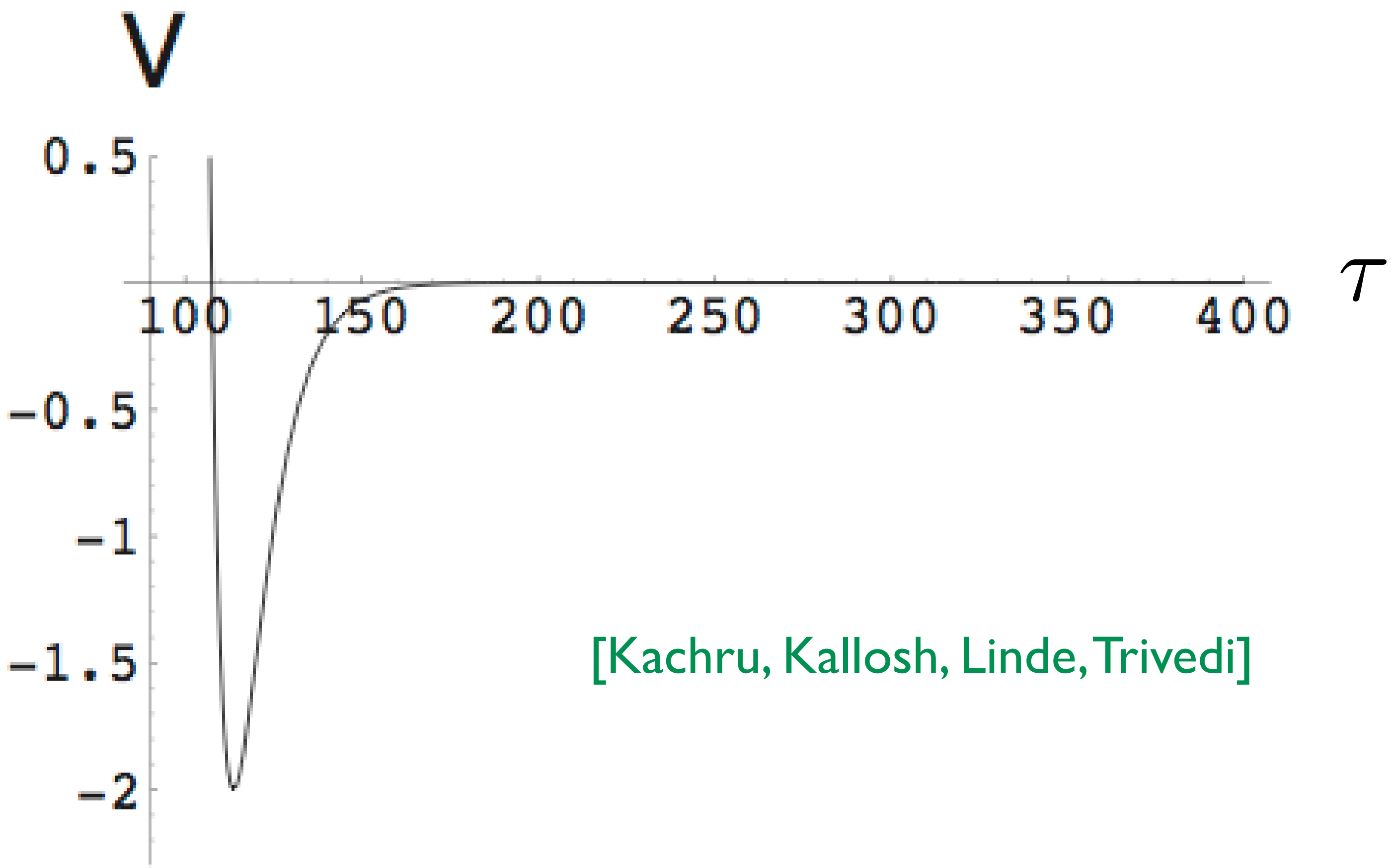
- Stabilization of S and U by $D_U W = 0 = D_S W$

  $W = W_0 + Ae^{-aT} \quad , \quad T = \tau + iC$

  $D_T W = 0 \Longrightarrow W_0 = -Ae^{-a\tau}(1 + \frac{2}{3}a\tau)$

  $\longrightarrow$ W_0 very small

- Supersymmetric minimum is AdS:



- One needs uplift mechanism

$$V = e^K \left(G^{I\bar{J}} D_I W D_{\bar{J}} \bar{W} - 3|W|^2 \right) + \frac{\epsilon}{\mathcal{V}^\alpha}$$

Large Volume Scenario (LVS)

[Balasubramanian, Berglund, Conlon, Quevedo]

- W_0 of order $\mathcal{O}(1)$ ⟵ more generic
- $D_T W \neq 0$ [Balasubramanian, Berglund]
- α'-corrections not negligible [Becker, Becker, Haack, Louis]

$$K = -2\ln(\mathcal{V}) + \ldots \rightarrow -2\ln(\mathcal{V} + \tfrac{1}{2}\xi S_1^{3/2}) + \ldots$$

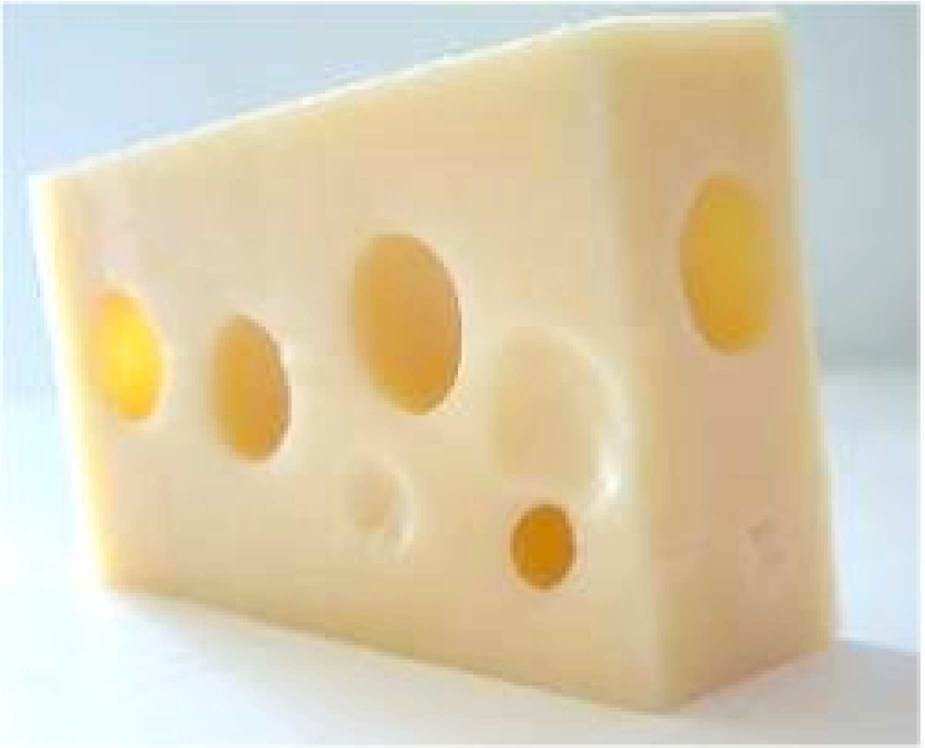
$$\xi = -\zeta(3)\chi/(2(2\pi)^3)$$

- Look at particular direction in Kähler cone

$$\mathcal{V}^{2/3} \sim \tau_b \gg 1 \quad (\mathcal{V} \sim 10^{15} l_s)$$

$$\tau_i \sim \ln \mathcal{V}$$

- “Swiss cheese” form [Conlon, Quevedo, Suruliz]



- $\mathcal{V} = \tau_b^{3/2} - \sum_i a_i \tau_i^{3/2}$

- Examples in [Denef, Douglas, Florea]
- Here for simplicity always hypersurface in $\mathbb{P}^4_{[1,1,1,6,9]}$:
  - ★ $h_{1,1} = 2$

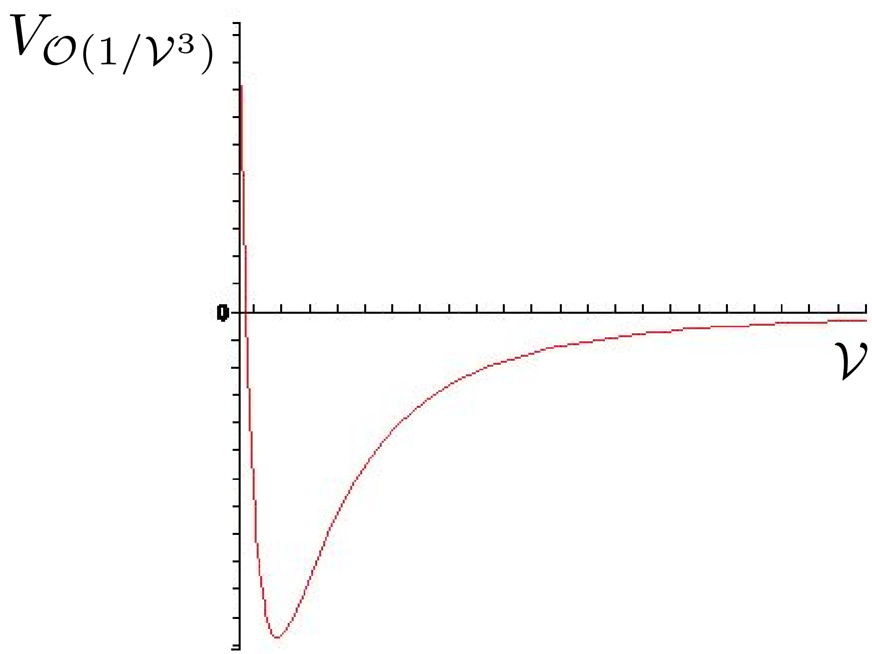
    $h_{2,1} = 272$
  - ★ $\mathcal{V} \sim \tau_b^{3/2} - \tau_s^{3/2}$

- Look for minimum with

$$\tau_b^{3/2} \sim \mathcal{V} \,, \quad a\tau_s \sim \ln \mathcal{V} \quad (\Longrightarrow \; e^{-a\tau_s} \sim \mathcal{V}^{-1})$$

- Large volume expansion of the potential (already minimized w.r.t. imaginary part in T_s)

$$V_{\mathcal{O}(1/\mathcal{V}^3)} = \frac{12\sqrt{2}|A|^2 a^2 \sqrt{\tau_s} e^{-2a\tau_s}}{\mathcal{V} S_1} - \frac{2a|AW_0|\tau_s e^{-a\tau_s}}{\mathcal{V}^2 S_1} + \xi \frac{3|W_0|^2\sqrt{S_1}}{8\mathcal{V}^3}$$



$$\Longrightarrow \xi > 0 \,, \quad \text{i.e. } \chi < 0$$

- Minimize w.r.t. $\tau_s, \mathcal{V}$:

$$\tau_s \sim S_1 \xi^{2/3} \quad , \qquad \mathcal{V} \sim \frac{\xi^{1/3}\sqrt{S_1}|W_0|}{a|A|} e^{a\tau_s}$$

- Minimum (non-supersymmetric) AdS; needs uplift

  Can be done without changing $\tau_s, \mathcal{V}$ much

[Conlon, Quevedo, Suruliz; Choi, Falkowski, Nilles, Olechowski]

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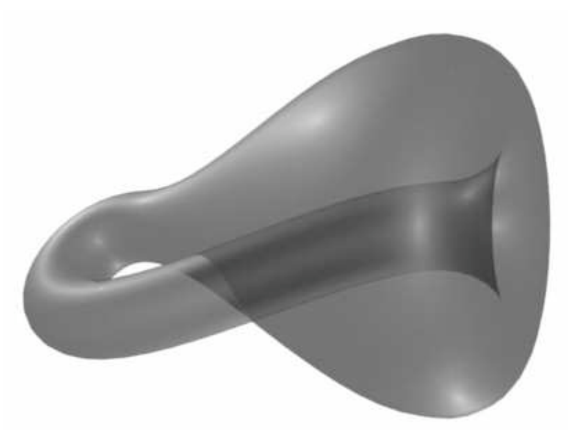
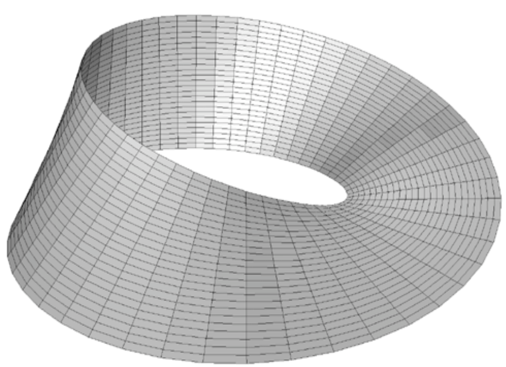
- No discussion of additional loop-corrections in presence of D-branes/O-planes:

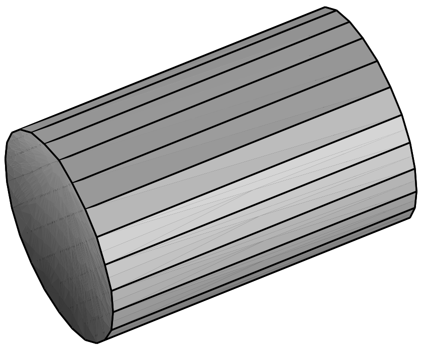
Kleinbottle:



Möbius strip:



Cylinder:



1-loop Kähler potential

- Not known for the $\mathbb{P}^4_{[1,1,1,6,9]}$ model
- Result for $\mathbb{T}^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$:

$$K^{(1)} = c \sum_{I=1}^{3} \left[\frac{E_2(U^I)}{(S+\bar{S})(T^I+\bar{T}^I)} + \frac{E_2(U^I)}{(T^J+\bar{T}^J)(T^K+\bar{T}^K)}\Big|_{K\neq I\neq J} \right]$$

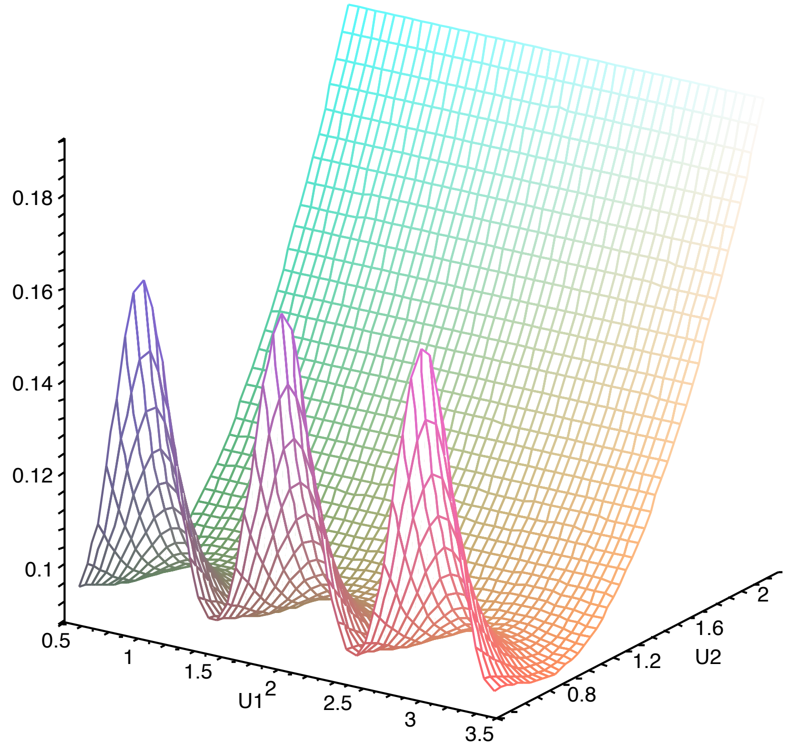
[Berg,Haack,Körs]

where $c = 15/(2\pi^6)$ and

$$E_2(U) = \sum_{(n,m)\neq(0,0)} \frac{U_2^2}{|n+mU|^4}$$

(Eisenstein series)

- The function $cE_2(U)$:



- Gets large for large U_2 : proportional to U_2^2

  $\longrightarrow$ Corrections can get large for degenerate tori

- Compared to α'-correction, 1-loop correction is suppressed in S - but leading in T-expansion

- Generalization to $\mathbb{P}^4_{[1,1,1,6,9]}$ model?

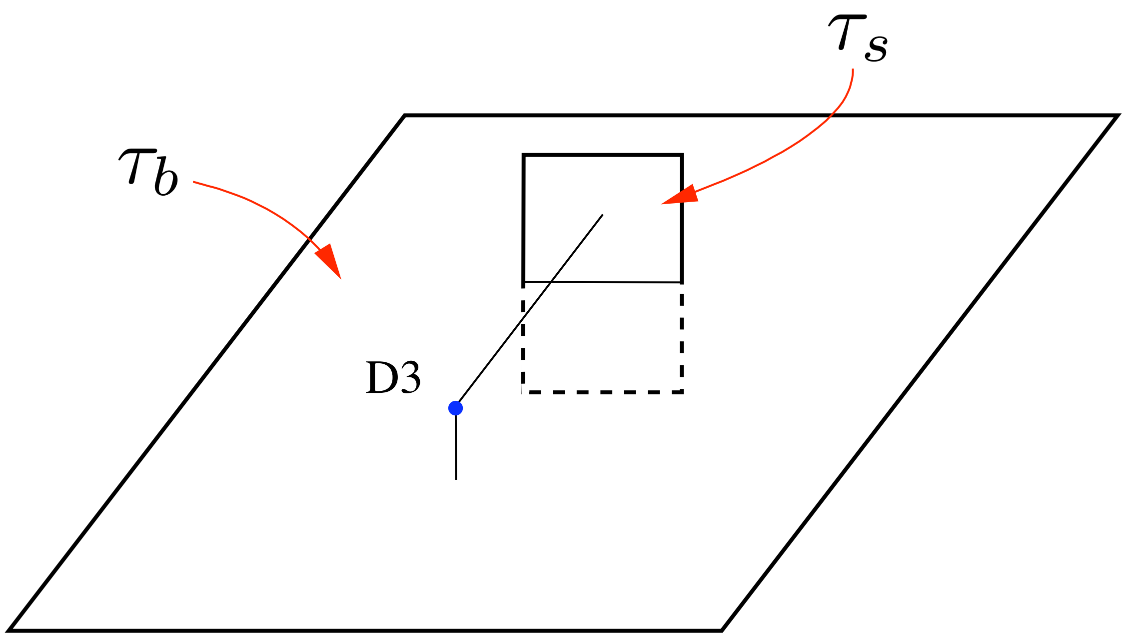
  Does one expect corrections $\delta K \sim \frac{E(U)}{S_1 \tau_s}$?

- Could lead to very strong constraints for LVS

Origin of loop corrections

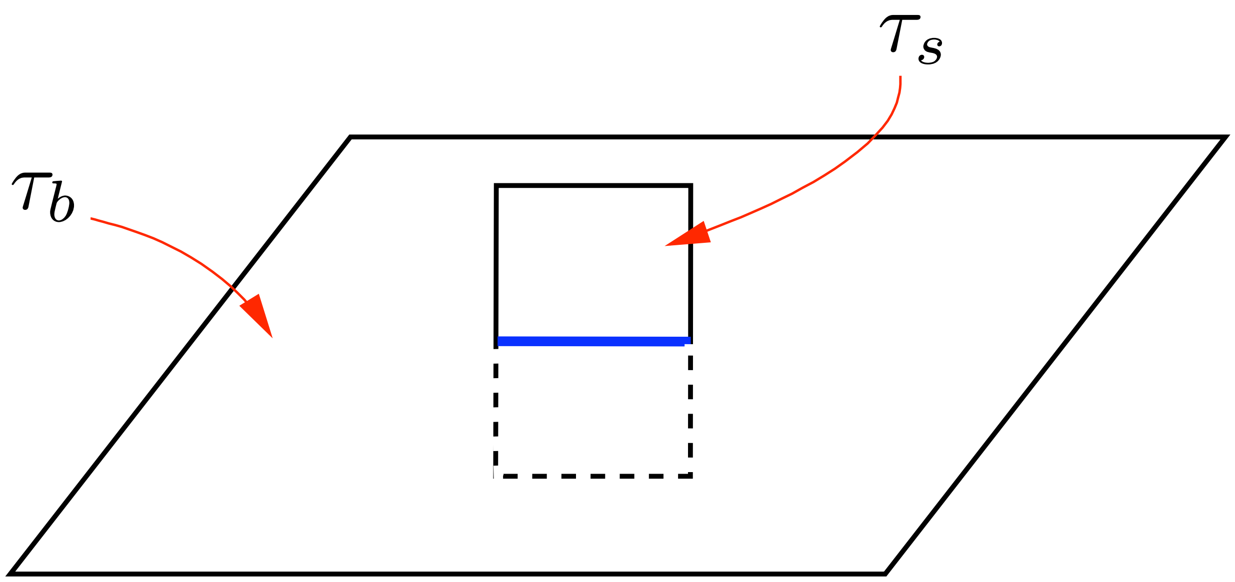
- Exchange of Kaluza-Klein modes $m_{KK}^2 \sim t^{-1}$

2-cycle volume



This effect should generalize to $\mathbb{P}^4_{[1,1,1,6,9]}$ model

- Exchange of strings, winding around 1-cycles within the intersection of two D7-brane stacks



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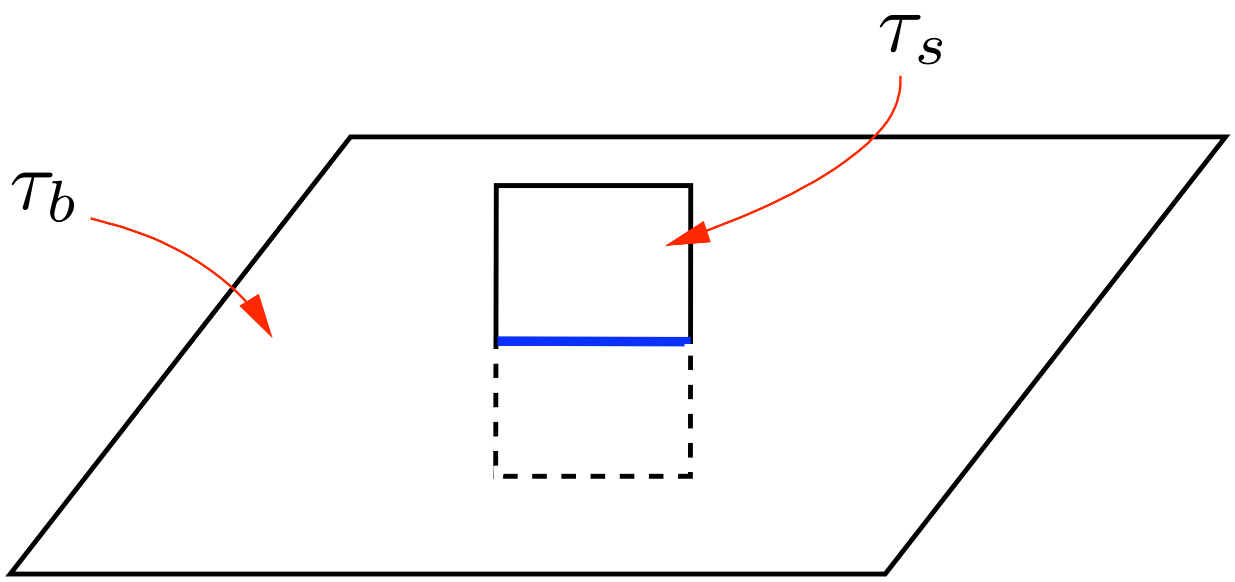


- However: In $\mathbb{P}^4_{[1,1,1,6,9]}$ model the D7-branes do not intersect $\Longrightarrow$ does not generalize (other models?)

- Plausible form of 1-loop corrections:

$$K^{(1)} = \frac{\sqrt{\tau_s} E_s^{(K)}(U)}{S_1 \mathcal{V}} + \frac{\sqrt{\tau_b} E_b^{(K)}(U)}{S_1 \mathcal{V}} \left(+ \frac{E_s^{(W)}(U)}{\sqrt{\tau_s} \mathcal{V}} + \frac{E_b^{(W)}(U)}{\sqrt{\tau_b} \mathcal{V}} \right)$$

with unknown functions $E_s^{(K)}(U), E_b^{(K)}(U)$

- Note:

$$\mathcal{V} \sim \tau_b^{3/2} \quad \Longrightarrow \frac{\sqrt{\tau_b} E_b^{(K)}}{S_1 \mathcal{V}} \sim \frac{E_b^{(K)}}{S_1 \mathcal{V}^{2/3}}$$

more leading than α'-correction

- $V = V_{\rm np1} + V_{\rm np2} + V_3$

- $V_{\rm np1} = e^{K_{cs}} \frac{24\sqrt{2} a^2 |A|^2 \tau_s^{3/2} e^{-2a\tau_s}}{\Delta \mathcal{V}}$

$$V_{\rm np2} = -e^{K_{\rm cs}} \frac{2a|AW_0|\tau_s e^{-a\tau_s}}{S_1 \mathcal{V}^2} \left[1 + \frac{6E_s^{(K)}}{\Delta}\right]$$

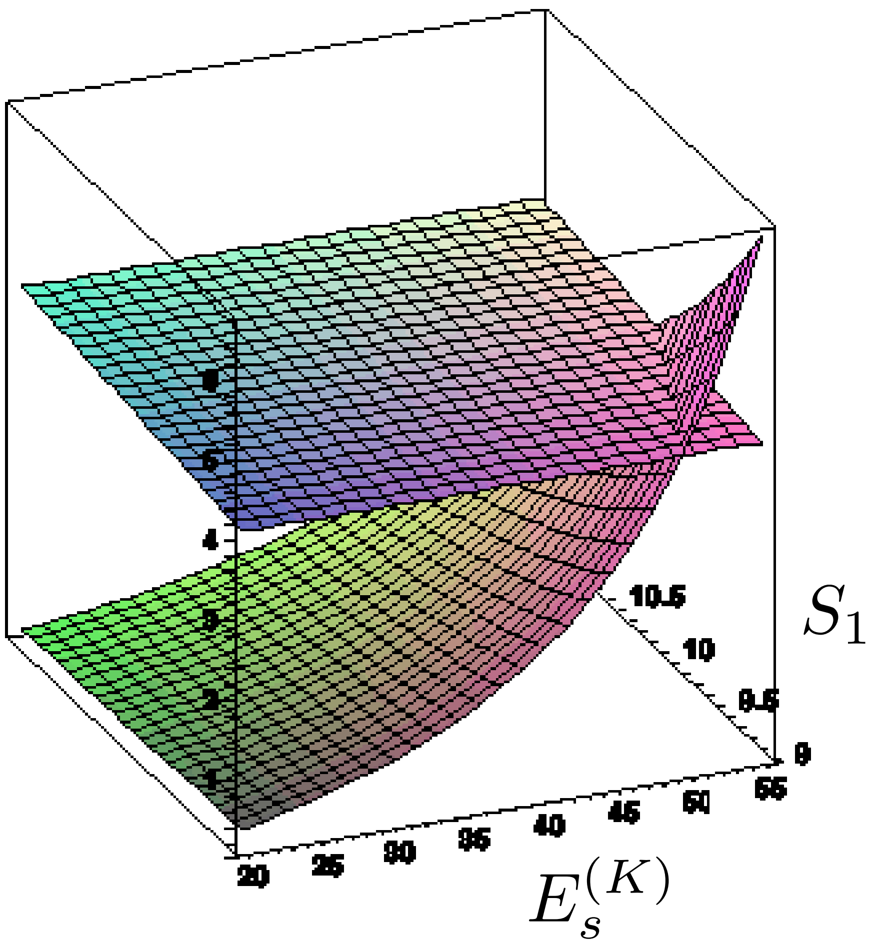
$$V_3 = e^{K_{cs}} \frac{3|W_0|^2}{8\mathcal{V}^3} \left[\sqrt{S_1}\xi\left(1 + \frac{\pi^2}{3\zeta(3)S_1^2}\right) + \frac{4\sqrt{\tau_s}(E_s^{(K)})^2}{S_1^2\Delta}\right]$$

- $\Delta \equiv \sqrt{2} S_1 \tau_s - 3E_s^{(K)}$

$E_b^{(K)}$ appears at $\mathcal{O}(\mathcal{V}^{-10/3})$

- The two terms in V_3 :

  $(A = 1, W_0 = 1,$
  $a = 2\pi/8, \xi = 1.31)$



- Mainly quantitative changes

- $\log_{10} \mathcal{V} \sim -0.129 E_s^{(K)} + 13.99$, $\tau_s \sim -0.379 E_s^{(K)} + 41.98$

- $\chi = 0$ possible?

Soft susy breaking terms

- $\mathcal{L}_{\text{soft}} = -\left(\frac{1}{2}M_a\lambda^a\lambda^a + \frac{1}{6}a^{IJK}\phi_I\phi_J\phi_K + \frac{1}{2}b^{IJ}\phi_I\phi_J\right) + \text{c.c.}$
  $-(m^2)^{IJ}\phi_I^*\phi_J$

- These are determined by moduli F-terms

- E.g. gaugino masses:

$$M_a = \frac{1}{2}\frac{1}{\text{Re}f_a}\sum_j F^j\partial_j f_a$$

$$F^j = e^{K/2}G^{ji}D_iW$$

- In LVS: SM gauge group arises from D7-branes wrapped around small cycles

$$\mathrm{Re} f_a \sim g_a^{-2} \sim \tau_s + h_a(S, U)$$

- However

$F^U = 0$ (without loop corrections)

$F^S \sim \mathcal{V}^{-2}$

$F^s \sim \mathcal{V}^{-1}$

$$\Longrightarrow \quad M_a = \frac{1}{2} \frac{1}{\mathrm{Re} f_a} F^s \overbrace{\partial_s f_a}^{=1} + (\text{suppressed in } \mathcal{V}^{-1})$$

- In $\mathbb{P}^4_{[1,1,1,6,9]}$ model:

$$F^s = 2\tau_s e^{K/2}\bar{W}_0\Big((1-\frac{3}{4a\tau_s}) - 1 + \mathcal{O}((a\tau_s)^{-2})\Big) + \mathcal{O}(\mathcal{V}^{-2})$$

$\longrightarrow$ Cancellation at leading order

- $$M_a \sim \frac{m_{3/2}}{\ln(M_p/m_{3/2})} \quad , \quad m_{3/2} \sim \frac{M_p}{\mathcal{V}}$$

[Conlon, Quevedo;
Choi, Falkowski, Nilles, Olechowski]

- Similar for other soft susy breaking terms

[Abdussalam, Conlon, Quevedo, Suruliz]

- What about loop corrections?

$$F^U \sim \mathcal{V}^{-2}$$

$$F^S \sim \mathcal{V}^{-2}$$

$$F^s = 2\tau_s e^{K/2}\bar{W}_0\Big(-\frac{3}{4a\tau_s} - \frac{9\bar{W}_0}{16a^2\tau_s} + \frac{9\bar{W}_0(12aE_s^{(K)} - S_1)}{64S_1a^3\tau_s^3} + \dots \Big)$$

- Loop corrections only appear sub-sub-leading!

Conclusion

- LVS seems surprisingly stable against 1-loop corrections
- Is our conjecture right?
- Field theory derivation? [Cicoli,Conlon,Quevedo]
- Further corrections?