CSDR and Semi-Realistic Particle Physics Models

Classification Results

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Outline

1. Introduction and Motivation
   - Theoretical Framework
   - Previous Work
   - Current Investigation
2. Reduction Scheme & Wilson Flux Breaking Mechanism
   - CSDR Scheme
   - Wilson Flux Breaking Mechanism in CSDR
3. Summary & Results

Quest of Unification

Kaluza-Klein theories

- **Unification** of observed interactions: **always** a **perspective**
- **EW Unification** (Energy Scale $\mathcal{O}(100GeV)$)
- **Strong** and **Electroweak Unification** (**GUTs**)
- **Kaluza-Klein** proposal: Unify **gravity** with **electromagnetism** ($5-\mathrm{dim}$ gravity over $M^4 \times S^1$)
- Compactifications of **higher dimensional gravity** [*Pauli,De-Witt*]
- **Internal spaces** with **non-abelian isometries** $\Rightarrow$ **YM fields** in $4-\mathrm{dim}$
    - **geometrical unification** of **gravity** with other **non-abelian gauge interactions**?

Obstacles of Kaluza-Klein program

$4-\dim$ chiral fermions

- **Inability** to obtain $4-\dim$ **chiral fermions**
- **Addition** of **YM fields** in **higher dimensions** can **resolve** this!
- . . . it *could* also *provide* **gauge-Higgs unification**

CSDR Proposal
Short description

- *Suggests* **gauge-Higgs unification**
- **YM-Dirac** gauge theory, G, over $M^4 \times S/R$
- *Non-trivial* **dimensional reduction** to M^4
- **Fields' transformation** under the S/R **symmetries** *compensated* by **gauge transformations**
- **constraints** on the **surviving** $4-\mathrm{dim}$ **gauge symmetry** and **fields**

Approach achievements

- **Chiral fermions** in $4-\mathrm{dim}$
- **Higgs** and **Yukawa sector** *emerge*
- **Two stages** of **Spontaneous Symmetry Breaking**
  - **Geometrical Breaking** due to CSDR
  - **SSB** by ordinary **Higgs Mechanism**
- For **cosets** with **more than one radius** $\exists$ **potential mass terms** proportional with

$$\left(\frac{\alpha_1}{R_1^2}-\frac{\alpha_2}{R_2^2}\right)$$

$\Rightarrow$ **EW breaking** can be *incorporated nicely*.

Recent CSDR-related publications

Further improvements of the **scheme**:

- **D. Kapetanakis and G. Zoupanos**, "*Discrete symmetries and coset space dimensional reduction*", Phys. Lett. B **232**, 104 (1989).
- **P. Manousselis and G. Zoupanos**, "*Dimensional reduction over coset spaces and supersymmetry breaking*", JHEP **0203**, 002 (2002)
- **P. Manousselis and G. Zoupanos**, "*Dimensional reduction of ten-dimensional supersymmetric gauge theories in the N = 1, D = 4 superfield formalism*", JHEP **0411**, 025 (2004)
- **P. Manousselis, N. Prezas and G. Zoupanos**, "*Supersymmetric compactifications of heterotic strings with fluxes and condensates*", Nucl. Phys. B **739**, 85 (2006)

Short Description of current investigation

Coset Space Dimensional Reduction

- Examine **YM-Dirac** Lagrangian defined
  - with **gauge group** $G = E_8$ and
  - over $10 - \mathrm{dim}$ **compactified space** $M^4 \times S/R$
- which is suggested by $E_8 \times E_8$ **heterotic superstring theory.**
- Perform **CSDR** and require
  - $4 - \mathrm{dim}$ GUT
    (H=E_6, $SO(10)$, $SU(5)$)
  - with *chiral fermions*
    (**constrain** possible $6 - \mathrm{dim}$ coset spaces).
- **Consider** simultaneous **Wilson Flux Symmetry breaking** as a proposal of **superstrong breaking.**
  (non-trivial topological properties of the extra dimensions)

Compact Coset Spaces

- Let $B = S/R$, $6-\dim$ coset space.
- Split the $\mathcal{S}$ Lie algebra as:

$$\begin{aligned} &\mathcal{S} = \mathcal{R} \oplus \mathcal{K} \\ &\mathcal{S}: \ \{Q_A, A = 1, \cdots, \dim(\mathcal{S})\} \\ &\mathcal{R}: \ \{Q_i, i = 1, \cdots, \dim(\mathcal{R})\} \\ &\mathcal{K}: \ \{Q_a, a = \dim(\mathcal{R}) + 1, \cdots, \dim(\mathcal{S})\} \end{aligned} \tag{1}$$

- Commutation relations of **reductive** S/R:

$$\begin{aligned} [Q_i, Q_j] &= f_{ij}{}^k Q_k \\ [Q_i, Q_a] &= f_{ia}{}^b Q_b \\ [Q_a, Q_b] &= f_{ab}{}^i Q_i + f_{ab}{}^c Q_c \end{aligned} \tag{2}$$

($f_{ab}{}^c = 0 \Rightarrow$ symmetric coset)

Reduction of $D - \dim$ YM-Dirac Lagrangian

YM-Dirac Lagrangian

Let **YM-Dirac** theory with **gauge group** G defined over $M^D = M^4 \times S/R$, ($D = 10 = 4 + d$, $d = 6 = \dim(S/R)$)

$$A = \int d^4x\, d^dy \sqrt{-g} \left[-\frac{1}{4} F_{MN} F^{MN} + \frac{i}{2} \overline{\psi} \Gamma^M D_M \psi \right], \qquad (3)$$

$$D_M = \partial_M - \theta_M - A_M, \qquad \theta_M = \frac{1}{2} \theta_{MN\Lambda} \Sigma^{N\Lambda} \text{ (spin connection)}$$

$\Sigma^{N\Lambda}$: $SO(d)$ generators and

$$F_{MN} = \partial_M A_N - \partial_N A_M - [A_M, A_N]$$

Reduction of D-dim YM-Dirac Lagrangian

A_M and ψ Symmetric Properties

$A_M(x^\mu, y^\alpha)$ and $\psi(x^\mu, y^\alpha)$:
any transformation under S/R symmetries compensated by gauge transformation
ξ_A^α, $A = 1, \ldots, dimS$: Killing vectors generating S/R symmetries
W_A: compensating gauge transformation associated with ξ_A^α
$\delta_A \equiv \mathcal{L}_{\xi_A}$ (infinitesimal coordinate transformation),
$[\delta_A, \delta_B] = f_{AB}{}^C \delta_C$

$$\begin{aligned} \delta_A \phi &= \xi_A^\alpha \partial_\alpha \phi = D(W_A)\phi \\ \delta_A A_\alpha &= \xi_A^\beta \partial_\beta A_\alpha + \partial_\alpha \xi_A^\beta A_\beta = \partial_\alpha W_A - [W_A, A_\alpha] \\ \delta_A \psi &= \xi_A^\alpha \psi - \frac{1}{2} G_{Abc} \Sigma^{bc} \psi = D(W_A)\psi \end{aligned} \tag{4}$$

Reduction of D-dim YM-Dirac Lagrangian

Solutions of CSDR constraints ($4-\mathrm{dim}$ gauge group)

- $4-\mathrm{dim}$ gauge group
  $[A_\mu(x,y)]$: $H = C_G(R_G)$ & $\partial_y A_\mu(x,y) = 0$

Note

GUT's gauge symmetries of interest: E_6, $SO(10)$, $SU(5)$
Their **commutant part** in E_8

$$\mathcal{R} = C_{E_8}(\mathcal{H}) = \begin{cases} SU(3), & \mathcal{H} = E_6 \\ SO(6) \sim SU(4), & \mathcal{H} = SO(10) \\ SU(5), & \mathcal{H} = SU(5)\,. \end{cases}$$

We **examined** all the **possible**

$$R \hookrightarrow \mathcal{R}$$

embeddings and the **resulting CSDR models**.

$R \hookrightarrow E_8$

Reduction of *D*-dim YM-Dirac Lagrangian

Solutions of CSDR constraints ($4-\mathrm{dim}$ scalar fields)

- $A_\alpha(x,y)$: $4-\mathrm{dim}$ surviving scalars denoted $\phi_\alpha(x,y)$
- ϕ_α **transformation properties**
  Given the embeddings

$$\begin{aligned} S &\supset R \\ \mathrm{adj}S &= \mathrm{adj}R + \mathrm{v}, \quad \mathrm{v} = \sum s_i \end{aligned} \tag{5}$$

$$\begin{aligned} G &\supset R_G \times H \\ \mathrm{adj}G &= (\mathrm{adj}R, 1) + (1, \mathrm{adj}H) + \sum (r_i, h_i) \end{aligned} \tag{6}$$

$\forall\ (r_i, s_i)$ pair: $r_i \equiv s_i \Rightarrow \phi_\alpha \in h_i$ of H gauge group.

Reduction of D-dim YM-Dirac Lagrangian

Solutions of CSDR constraints ($4-\mathrm{dim}$ fermions)

ψ transformation properties

Given the decomposition of spinor $SO(6)$ under R

$$SO(6) \supset R \qquad \sigma_d = \sum \sigma_j \tag{7}$$

and the one of some rep F of G

$$G \supset R_G \times H \qquad F = \sum (t_i, h_i) \tag{8}$$

$\forall\ (t_i, \sigma_i)$ pair: $t_i \equiv \sigma_i \Rightarrow \psi \in h_i$ of H gauge group.

For **chiral fermions** we need **coset spaces** of $\mathrm{dim}: D = 4n + 2$ and $\mathrm{rank}(R) = \mathrm{rank}(S)$

Reduction of D-dim YM-Dirac Lagrangian

$4-\mathrm{dim}$ *Chiral fermions I*

For $D = 4n + 2$ (**odd**) spaces a **chirality operator** Γ^{D+1} can be defined

$$\Gamma^{D+1} = i^{(D-2)/2}\Gamma^0\Gamma^1 \cdots \Gamma^{D-1} = \gamma^5 \otimes \gamma^{d+1},$$
$$\gamma^{d+1} = -i^{D/2}\gamma^1\gamma^2 \cdots \gamma^d$$

and $\exists$ **Weyl spinors** ψ_+, ψ_-

$$\Gamma^{D+1}\psi_\pm = \pm\psi_\pm \qquad \text{(Weyl condition)}$$

Start with **Weyl Spinors** in **higher dimensions**, decomposed us

$$SO(D) \supset SU(2) \times SU(2) \times SO(d)$$
$$\sigma_D = (\mathbf{2}, \mathbf{1}, \sigma_d) + (\mathbf{1}, \mathbf{2}, \overline{\sigma}_d)$$

(Γ^{D+1} commutes with Lorentz generators)

Reduction of D-dim YM-Dirac Lagrangian

$4-\mathrm{dim}$ *Chiral fermions II*

From **CSDR rules**

$$SO(d) \supset R \quad \sigma_d = \sum \sigma_k, \quad \overline{\sigma}_d = \sum \overline{\sigma}_k \tag{9}$$

$$G \supset R_G \times H \quad F = \sum (t_i, h_i) + \sum (\bar{t}_i, \bar{h}_i) \quad \textbf{(vector-like rep)} \tag{10}$$

$\forall$ (t_i, σ_k) pair: $t_i \equiv \sigma_k \Rightarrow \psi_L \in f_L = \sum h_k^L$

$\forall$ $(\bar{t}_i, \overline{\sigma}_k)$ pair: $\bar{t}_i \equiv \overline{\sigma}_k \Rightarrow \overline{\psi}_R \in f_R = \sum \overline{h}_k^R$

F **vector-like rep** $\Rightarrow \overline{h}_k^R \sim h_k^L$

$6-\dim$ Symmetric Cosets

SO(6) Vector & Spinor Content

Table 1: **Six-dimensional symmetric cosets spaces.** *We note their vector and fermionic content; the available discrete symmetries are also shown.*

Case	6D Coset Spaces	$\mathbf{Z}(S)$	W	V	F
a	$\frac{SO(7)}{SO(6)}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbf{6}\leftrightarrow\mathbf{6}$	$\mathbf{4}\leftrightarrow\overline{\mathbf{4}}$
b	$\frac{SU(4)}{SU(3)\times U(1)}$	$\mathbb{Z}_4$	$\mathbf{1}$	$\mathbf{6}=\mathbf{3}_{(-2)}+\overline{\mathbf{3}}_{(2)}$ –	$\mathbf{4}=\mathbf{1}_{(3)}+\mathbf{3}_{(-1)}$ –
c	$\frac{Sp(4)}{(SU(2)\times U(1))_{max}}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbf{6}=\mathbf{3}_{(-2)}+\mathbf{3}_{(2)}$ $\mathbf{3}_{(-2)}\leftrightarrow\mathbf{3}_{(2)}$	$\mathbf{4}=\mathbf{1}_{(3)}+\mathbf{3}_{(-1)}$ $\mathbf{1}_{(3)}\leftrightarrow\mathbf{1}_{(-3)}$ $\mathbf{3}_{(-1)}\leftrightarrow\mathbf{3}_{(1)}$
d	$\left(\frac{SU(3)}{SU(2)\times U(1)}\right)\times\left(\frac{SU(2)}{U(1)}\right)$	$\mathbb{Z}_2\times\mathbb{Z}_3$	$\mathbb{Z}_2$	$\mathbf{6}=\mathbf{1}_{(0,2a)}+\mathbf{1}_{(0,-2a)}$ $+\mathbf{2}_{(b,0)}+\mathbf{2}_{(-b,0)}$ $\mathbf{1}_{(0,2a)}\leftrightarrow\mathbf{1}_{(0,-2a)}$	$\mathbf{4}=\mathbf{2}_{(0,a)}+\mathbf{1}_{(b,-a)}+\mathbf{1}_{(-b,-a)}$ $\mathbf{2}_{(0,a)}\leftrightarrow\mathbf{2}_{(0,-a)}$ $\mathbf{1}_{(b,-a)}\leftrightarrow\mathbf{1}_{(b,a)}$ $\mathbf{1}_{(-b,-a)}\leftrightarrow\mathbf{1}_{(-b,a)}$
e	$\left(\frac{Sp(4)}{SU(2)\times SU(2)}\right)\times\left(\frac{SU(2)}{U(1)}\right)$	$(\mathbb{Z}_2)^2$	$(\mathbb{Z}_2)^2$	$\mathbf{6}=(\mathbf{2},\mathbf{2})_{(0)}+(\mathbf{1},\mathbf{1})_{(2)}+(\mathbf{1},\mathbf{1})_{(-2)}$ ($\mathbb{Z}_2$ of $SU(2)/U(1)$) $(\mathbf{1},\mathbf{1})_{(2)}\leftrightarrow(\mathbf{1},\mathbf{1})_{(-2)}$	$\mathbf{4}=(\mathbf{2},\mathbf{1})_{(1)}+(\mathbf{1},\mathbf{2})_{(-1)}$ $(\mathbf{2},\mathbf{1})_{(1)}\leftrightarrow(\mathbf{2},\mathbf{1})_{(-1)}$ $(\mathbf{1},\mathbf{2})_{(1)}\leftrightarrow(\mathbf{1},\mathbf{2})_{(-1)}$
f	$\left(\frac{SU(2)}{U(1)}\right)^3$	$(\mathbb{Z}_2)^3$	$(\mathbb{Z}_2)^3$	$\mathbf{6}=(2a,0,0)+(0,2b,0)+(0,0,2c)$ $+(-2a,0,0)+(0,-2b,0)+(0,0,-2c)$ each $\mathbb{Z}_2$ changes the sign of a,b, c	$\mathbf{4}=(a,b,c)+(-a,-b,c)$ $+(-a,b,-c)+(a,-b,-c)$ each $\mathbb{Z}_2$ changes the sign of a,b, c

$6-\dim$ Non-symmetric Cosets

$SO(6)$ Vector & Spinor Content

Table 2: **Six-dimensional non-symmetric cosets spaces.** *We note their vector and fermionic content; the available discrete symmetries are also shown.*

Case	$6D$ Coset Spaces	$\mathbf{Z}(S)$	W	V	F
$\mathbf{a}'$	$\frac{G_2}{SU(3)}$	$\mathbf{1}$	$\mathbb{Z}_2$	$\mathbf{6}=\mathbf{3}+\overline{\mathbf{3}}$ $\mathbf{3}\leftrightarrow\overline{\mathbf{3}}$	$\mathbf{4}=\mathbf{1}+\mathbf{3}$ $\mathbf{1}\leftrightarrow\mathbf{1}$ $\mathbf{3}\leftrightarrow\overline{\mathbf{3}}$
$\mathbf{b}'$	$\frac{Sp(4)}{(SU(2)\times U(1))_{nonmax}}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbf{6}=\mathbf{1}_{(2)}+\mathbf{1}_{(-2)}+\mathbf{2}_{(1)}+\mathbf{2}_{(-1)}$ $\mathbf{1}_{(2)}\leftrightarrow\mathbf{1}_{(-2)}$ $\mathbf{2}_{(1)}\leftrightarrow\mathbf{2}_{(-1)}$	$\mathbf{4}=\mathbf{1}_{(0)}+\mathbf{1}_{(2)}+\mathbf{2}_{(-1)}$ $\mathbf{1}_{(2)}\leftrightarrow\mathbf{1}_{(-2)}$ $\mathbf{1}_{(0)}\leftrightarrow\mathbf{1}_{(0)}$ $\mathbf{2}_{(1)}\leftrightarrow\mathbf{2}_{(-1)}$
$\mathbf{c}'$	$\frac{SU(3)}{U(1)\times U(1)}$	$\mathbb{Z}_3$	$\mathbf{S}_3$	$\mathbf{6}=(a,c)+(b,d)+(a+b,c+d)$ $+(-a,-c)+(-b,-d)+(-a-b,-c-d)$	$\mathbf{4}=(0,0)+(a,c)+(b,d)+(-a-b,-c-d)$
			$\mathbb{Z}_2$	$(b,d)\leftrightarrow(-b,-d)$ $(a+b,c+d)\leftrightarrow(a,c)$ $(-a,-c)\leftrightarrow(-a-b,-c-d)$	$(b,d)\leftrightarrow(-b,-d)$ $(a,c)\leftrightarrow(a+b,c+d)$ $(-a-b,-c-d)\leftrightarrow(-a,-c)$
			$\mathbb{Z}_2$	$(b,d)\leftrightarrow(a+b,c+d)$ $(a,c)\leftrightarrow(-a,-c)$ $(-b,-d)\leftrightarrow(-a-b,-c-d)$	$(b,d)\leftrightarrow(a+b,c+d)$ $(a,c)\leftrightarrow(-a,-c)$ $(-a-c,-b-d)\leftrightarrow(-b,-d)$
			$\mathbb{Z}_2$	$(b,d)\leftrightarrow(-a,-c)$ $(a+b,c+d)\leftrightarrow(-a-b,-c-d)$ $(a,c)\leftrightarrow(-b,-d)$	$(b,d)\leftrightarrow(-a,-c)$ $(a,c)\leftrightarrow(-b,-d)$ $(-a-b,-c-d)\leftrightarrow(a+b,c+d)$

The $4-\mathrm{dim}$ theory I

Taking in account

- **metric** of $M^4 \times S/R$ **compactification**

$$g^{MN} = \begin{pmatrix} \eta^{\mu\nu} & 0 \\ 0 & -g^{ab} \end{pmatrix}$$

  where $\eta^{\mu\nu} = diag(1, -1, -, 1, -1)$ and g^{ab} the coset space metric,

- **solutions** of the **CSDR constraints** *and*
- **integrating out** the **extra coords**. . .

The $4-\mathrm{dim}$ theory II

...the **higher-dim YM-Dirac action** (3) result in the $4-\mathrm{dim}$ theory

$$A = C\int d^4x\Big[-\frac{1}{4}F^{(t)}_{\mu\nu}F^{(t)\mu\nu} + \frac{1}{2}(D_\mu\phi_\alpha)^{(t)}(D^\mu\phi_\alpha)^{(t)} + V(\phi) + \frac{i}{2}\overline{\psi}\Gamma^\mu D_\mu\psi - \frac{i}{2}\overline{\psi}\Gamma^a D_a\psi\Big] \quad (11)$$

C: coset space volume

$D_\mu = \partial_\mu - A_\mu$

$D_a = \partial_a - \theta_a - \phi_a, \quad \theta_a = \frac{1}{2}\theta_{abc}\Sigma^{bc}$

$$V(\phi) = -\frac{1}{4}\mathrm{Tr}(F_{ab}F^{ab}) \quad \textbf{(potential)} \quad (12)$$

$$F_{ab} = f_{ab}{}^C\phi_C - [\phi_a, \phi_b]$$

The $4-\dim$ *theory III*

Remarks on $4-\dim$ *Potential*

ϕ_a must satisfy the **constraints**

$$f_{ai}{}^{c}\phi_c - [\phi_a, \phi_i] = 0$$

with ϕ_i generating R_G.

- **Minimization** of **potential**: a difficult problem
- In case S have an **isomorphic image** $S_G \hookrightarrow G$ such as $S_G \supset R$
  - $K = C_G(S_G)$ ($4-\dim$ **gauge group**)
  - the $4-\dim$ **chiral fermions** become **massive**
- **dimensional reduction** over **symmetric coset spaces** $\Rightarrow$ potential of **spontaneous symmetry breaking** form

An $G = E_8$ over $SO(7)/SO(6)$ example

Let the **decomposition chain**

$$E_8 \supset SO(16) \supset SO(6) \times SO(10)$$
$$\mathbf{248} = (\mathbf{1}, \mathbf{45}) + (\mathbf{15}, \mathbf{1}) + (\mathbf{6}, \mathbf{10}) + (\mathbf{4}, \mathbf{16}) + (\overline{\mathbf{4}}, \overline{\mathbf{16}})$$

(Decomp. Chain 1)

$4-\mathrm{dim}$ **gauge group**: $H = C_{E_8}(SO(6)) = SO(10)$

$4-\mathrm{dim}$ **scalars**: $\mathbf{10}$ of H

$4-\mathrm{dim}$ **chiral fermions**: $\mathbf{16}_L$ and $\overline{\mathbf{16}}_R$ of H

CSDR scalar rules
Symmetric Cosets
CSDR fermion rules

Wilson Flux Breaking Mechanism

Preliminaries

- **CSDR** result in *interesting* $4-\dim$ **GUTs**
- **SM** *cannot* be *obtained* by a **Higgs mechanism** *(scalars not in adjoint or higher reps)*.
- *Way-out proposal*:
  - use of **non-trivial topological properties** of **extra dimensions**
  - Further breaking of the GUT gauge symmetry by means of **topological breaking mechanism**

Wilson Flux Breaking Mechanism

The Method I

- Consider gauge theory over $M^4 \times B$, $B = B_0/F^{S/R}$
  $B_0 = S/R$: a coset space
  $F^{S/R}$: a **freely acting discrete symmetry** of B_0
- B becomes **multiply connected**
- $\forall\ g \in F^{S/R}\ \exists\ U_g \in H$ which **generates** an **homomorphic image** of $F^{S/R}$, $T^H = \{U_g\}$
- $4-\mathrm{dim}$ **gauge group**

$$K = C_H(T^H) \tag{13}$$

- **surviving** $4-\mathrm{dim}$ **field content**:
  invariant under

$$F^{S/R} \oplus T^H \tag{14}$$

Wilson Flux Breaking Mechanism

The Method II

Freely-acting discrete symmetries on $B_0 = S/R$

- The center of S, $\mathrm{Z}(S)$
- The $\mathrm{W} = \mathrm{W}_S/\mathrm{W}_R$
  (W_S and W_R Weyl groups of S and R)

Symmetric Cosets
Non-symmetric Cosets

Wilson Flux Breaking Mechanism

Some first conclusions

▸ Symmetric Cosets
▸ Non-symmetric Cosets

- **Surviving** $4-\mathrm{dim}$ **fields** invariant under $F^{S/R} \oplus T^H$
- $\mathbb{Z}_n$ discrete symmetries induces $T^H = \{U_g\}$ (U_g *diagonal matrices of phase exponentials*)
- $\mathrm{Z}(S)$ alone: **not interesting**
- $\mathbb{Z}_n$, $n > 2$: **not interesting**
- Only the **symmetric cosets** '**a**', '**d**' and '**e**' have **surviving scalars** in $4-\mathrm{dim}$
- . . . **with** R *only* **embeddable** in $\mathcal{R} = SO(6) \sim SU(4)$ or $SU(5)$

The $G = E_8$ over $SO(7)/SO(6)$ example revisited I

- *Recall*: $4-\mathrm{dim}$ **gauge group** $H = SO(10)$ (**CSDR**)
- $F^{S/R}$: $\mathrm{W} = \mathbb{Z}_2$ and $\mathrm{Z}(S) = \mathbb{Z}_2$
- $\mathbb{Z}_2 \hookrightarrow SU(5)$ of

$$SO(10) \supset SU(5) \times U(1)$$

- **realized** by

$$U_g = diag(-1,-1,1,1,1) \text{ for } \mathbb{Z}_2$$

- result in $4-\mathrm{dim}$ **gauge group**

$$K' = C_H(T^H) = SU^a(2) \times SU^b(2) \times SU(4)$$

The $G = E_8$ over $SO(7)/SO(6)$ example revisited II

…under the **branching rule**

$$
\begin{aligned}
SO(10) &\supset SU^a(2) \times SU^b(2) \times SU(4) \\
\mathbf{10} &= \underbrace{(\mathbf{2},\mathbf{2},\mathbf{1})}_{(-1)} + \underbrace{(\mathbf{1},\mathbf{1},\mathbf{6})}_{(+1)} \\
\mathbf{16} &= \underbrace{(\mathbf{2},\mathbf{1},\mathbf{4})}_{(-1)} + \underbrace{(\mathbf{1},\mathbf{2},\overline{\mathbf{4}})}_{(+1)} \\
\mathbf{45} &= \underbrace{(\mathbf{3},\mathbf{1},\mathbf{1})}_{(+1)} + \underbrace{(\mathbf{1},\mathbf{3},\mathbf{1})}_{(+1)} + \underbrace{(\mathbf{1},\mathbf{1},\mathbf{15})}_{(+1)} + \underbrace{(\mathbf{2},\mathbf{2},\mathbf{6})}_{(-1)}
\end{aligned}
\tag{15}
$$

[fields invariant under $F^{S/R} \oplus T^H$]

$4-\mathrm{dim}$ **gauge group**: $K' = SU_L(2) \times SU_R(2) \times SU(4)$

$4-\mathrm{dim}$ **scalars**: $(\mathbf{1},\mathbf{1},\mathbf{6})$ [$(\mathbf{2},\mathbf{2},\mathbf{1})$ for $\mathbb{Z}_2 \times \mathbb{Z}_2$]

$4-\mathrm{dim}$ **fermions**: $(\mathbf{2},\mathbf{1},\mathbf{4})_L \mp (\mathbf{2},\mathbf{1},\mathbf{4})'_L$, $(\mathbf{1},\mathbf{2},\mathbf{4})_R \pm (\mathbf{1},\mathbf{2},\mathbf{4})'_R$

CSDR scalar rules
Symmetric Cosets
CSDR fermion rules
previous CSDR results

The $G = E_8$ over $SO(7)/SO(6)$ example revisited III

Proof of $SO(10) \to PS$ breaking

$\mathbb{Z}_2 \hookrightarrow SU(5)$ is **realized** by $U_g = diag(-1,-1,1,1,1)$ and **breaks** $SU(5)$ as

$$
\begin{aligned}
SU(5) &\supset SU(2) \times SU(3) \times U(1) \\
\mathbf{5} &= \underbrace{(\mathbf{2},\mathbf{1})_{(3)}}_{(-1)} + \underbrace{(\mathbf{1},\mathbf{3})_{(-2)}}_{(+1)} \\
\mathbf{10} &= \underbrace{(\mathbf{1},\mathbf{1})_{(6)}}_{(+1)} + \underbrace{(\mathbf{1},\overline{\mathbf{3}})_{(-4)}}_{(+1)} + \underbrace{(\mathbf{2},\mathbf{3})_{(1)}}_{(-1)} \\
\mathbf{24} &= \underbrace{(\mathbf{1},\mathbf{1})_{(0)}}_{(+1)} + \underbrace{(\mathbf{3},\mathbf{1})_{(0)}}_{(+1)} + \underbrace{(\mathbf{1},\mathbf{8})_{(0)}}_{(+1)} + \underbrace{(\mathbf{2},\mathbf{3})_{(-5)}}_{(-1)} + \underbrace{(\mathbf{2},\overline{\mathbf{3}})_{(5)}}_{(-1)} .
\end{aligned}
$$

Entering the above in the $SO(10) \supset SU(5) \times U(1)$ branching rule
& **collecting terms** of **equal phase exponentials**

$\Rightarrow$ the $SO(10) \to PS$ **breaking** follows.

The $G = E_8$ over $SO(7)/SO(6)$ example revisited IV

- S has an **isomorphic image** $S_G \hookrightarrow E_8$

$$\begin{array}{ccc} E_8 \supset SO(7) \times & SO(9) & \\ \cup & \cap & \\ SO(6) \times & SO(10). & \end{array}$$

- $4-\dim$ **gauge group** with **no topological breaking**

$$\mathcal{H} = C_{E_8}(SO(7)) = SO(9)$$

- $4-\dim$ **gauge group** with **CSDR & Wilson Flux Breaking**

$$K = SU(2)^{diag} \times SU(4)$$

Summary and Results

- We *considered* **YM-Dirac** Lagrangian with $G = E_8$ over $M^4 \times S/R$ ($6-\mathrm{dim}$ cosets)
- *performed* **CSDR** and *classified* the particle physics models (interesting $R \hookrightarrow E_8$)
- *broke* further the **GUT's gauge symmetry** (**Wilson Flux breaking mechanism**)
- *calculated* **surviving** $4-\mathrm{dim}$ **field content.**
- 3 of the **models** found to be **more interesting** ($E_8 \supset SO(6) \times SO(10)$ case):
    - one with $B_0 = SO(7)/SO(6)$ (1 chiral fermion family) and two with $B_0 = CP^2 \times S^2$ (3 chiral fermion families)
    - all resulting in Pati-Salam gauge group in $4-\mathrm{dim}$

Outlook

- **Scalars** in **higher dimensions** & **diverse reps** of **gauge group**
  - *probably justified* by *embedding* the **CSDR framework** *in* a more **fundamental theory**
  - *could* **provide** the necessary **symmetry breaking patterns** *towards* **SM**
- *Implementation* of **gravity** *provides* **further breaking** of the **higher dimensional gauge group** G
  - **A. Chatzistavrakidis, P. Manousselis, N. Prezas and G. Zoupanos**, *Work in progress*.

$\mathcal{R}$ embedding in $SO(6)$ tangent vector space

The **splitting** $\mathcal{S} = \mathcal{R} \oplus \mathcal{K}$ is characterized by

$$S \supset R \tag{16}$$
$$\mathrm{adj}\mathcal{S} = \mathrm{adj}\mathcal{R} + \mathrm{v}$$
$$\mathrm{v} : \mathcal{K} \text{ coset generators}$$

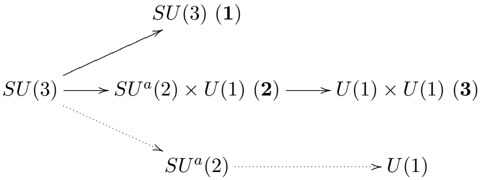
the v form a **tangent vector** on the **coset space.**
Requiring v to **transform** as the **vector rep** **6** of $SO(6)$ (**6** $= \mathrm{v}$) a $R \hookrightarrow SO(6)$ **embedding** is **defined**:

$$T_i = -\frac{1}{2} f_{iab} \Sigma^{ab}$$

with Σ^{ab}, $SO(6)$ generators.

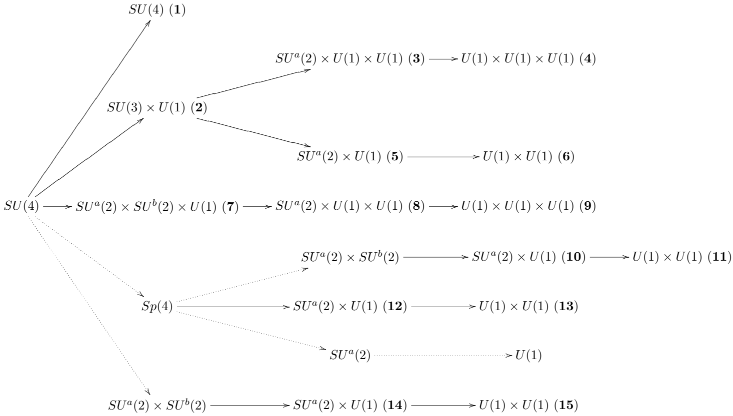
$R \hookrightarrow SU(3)$

Figure 1: $SU(3)$ **decomposition table.**



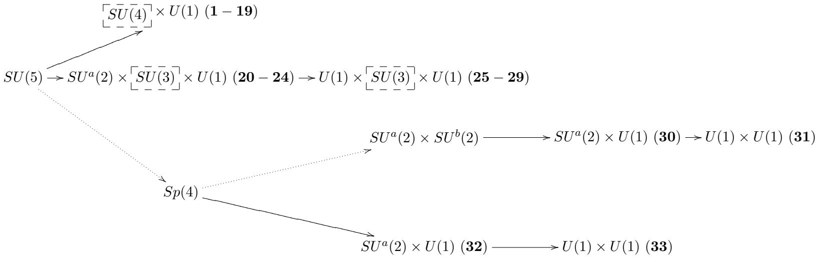
$R \hookrightarrow SU(4)$

Figure 2: $SU(4)$ **decomposition table.**



$R \hookrightarrow SU(5)$

Figure 3: $SU(5)$ **decomposition table.**



For Further Reading

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