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Towards noncommutative SUSY field theories

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Outline

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  -SUSY transformations
- Deformed (noncommutative):
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- Deformed Wess-Zumino Lagrangian
  -construction
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Undeformed superspace

Generated by (anti)commuting coordinates

$$[x^m, x^n] = 0, \quad \{\theta^\alpha, \theta^\beta\} = 0, \quad \{\bar\theta_{\dot\alpha}, \bar\theta_{\dot\beta}\} = 0, \qquad (1)$$

with $m = 0, \dots 3$ and $\alpha, \beta, \dot\alpha, \dot\beta = 1, 2$.
Derivatives consistent with this algebra are given by

$$[\partial_m, x^n] = \delta_m^n, \quad \{\partial_\alpha, \theta^\beta\} = \delta_\alpha^\beta, \quad \{\bar\partial^{\dot\alpha}, \bar\theta_{\dot\beta}\} = \delta_{\dot\alpha}^{\dot\beta} \qquad (2)$$

and

$$[\partial_m, \partial_n] = 0, \quad \{\partial_\alpha, \partial_\beta\} = \{\bar\partial^{\dot\alpha}, \bar\partial^{\dot\beta}\} = 0, \dots . \qquad (3)$$

Superfiled $F(x, \theta, \bar\theta)$ can be expanded in powers of θ and $\bar\theta$

$$\begin{aligned} F(x,\theta,\bar\theta) \ = \ & f(x) + \theta\phi(x) + \bar\theta\bar\chi(x) + \theta\theta m(x) + \bar\theta\bar\theta n(x) \\ & + \theta\sigma^m\bar\theta v_m + \theta\theta\bar\theta\bar\lambda(x) + \bar\theta\bar\theta\theta\varphi(x) + \theta\theta\bar\theta\bar\theta d(x). \end{aligned} \qquad (4)$$

Under infinitesimal SUSY transformations

$$\delta_\xi F \;=\; \big(\xi Q + \bar\xi\bar Q\big)F(x), \tag{5}$$

$$Q_\alpha = \partial_\alpha - i\sigma^m{}_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}\partial_m\,, \quad \bar Q^{\dot\alpha} = \bar\partial^{\dot\alpha} - i\theta^\alpha\sigma^m{}_{\alpha\dot\beta}\varepsilon^{\dot\beta\dot\alpha}\partial_m. \tag{6}$$

Also $\xi^a, \bar\xi_{\dot\alpha} = const.$ and $\{\xi^\alpha, \xi^\beta\} = \{\xi^\alpha, \bar\xi_{\dot\alpha}\} = \{\bar\xi_{\dot\alpha}, \bar\xi_{\dot\beta}\} = 0$.

Transformations (5) close in the algebra

$$[\delta_\xi, \delta_\eta] = -2i\big(\eta\sigma^m\bar\xi - \xi\sigma^m\bar\eta\big)\partial_m. \tag{7}$$

Leibniz rule is undeformed

$$\begin{aligned}\delta_\xi\big(F\cdot G\big) \;&=\; (\delta_\xi F)\cdot G + F\cdot(\delta_\xi G)\\ &=\; \big(\xi Q + \bar\xi\bar Q\big)\big(F\cdot G\big).\end{aligned} \tag{8}$$

Hopf algebra of undeformed SUSY transformations

- algebra

$$\{Q_\alpha, Q_\beta\} = \{\bar{Q}_{\dot{\alpha}}, \bar{Q}_{\dot{\beta}}\} = 0, \quad \{Q_\alpha, \bar{Q}_{\dot{\beta}}\} = 2i\sigma^m{}_{\alpha\dot{\beta}}\partial_m,$$
$$[\partial_m, \partial_n] = [\partial_m, Q_\alpha] = [\partial_m, \bar{Q}_{\dot{\alpha}}] = 0. \tag{9}$$

- coproduct

$$\Delta Q_\alpha = Q_\alpha \otimes 1 + 1 \otimes Q_\alpha, \quad \Delta\bar{Q}_{\dot{\alpha}} = \bar{Q}_{\dot{\alpha}} \otimes 1 + 1 \otimes \bar{Q}_{\dot{\alpha}},$$
$$\Delta\partial_m = \partial_m \otimes 1 + 1 \otimes \partial_m. \tag{10}$$

- counit

$$\varepsilon(Q_\alpha) = \varepsilon(\bar{Q}_{\dot{\alpha}}) = \varepsilon(\partial_m) = 0. \tag{11}$$

- antipod

$$S(Q_\alpha) = -Q_\alpha, \quad S(\bar{Q}_{\dot{\alpha}}) = -\bar{Q}_{\dot{\alpha}}, \quad S(\partial_m) = -\partial_m. \tag{12}$$

From (5) transformation laws of component fields follow

$$\delta_\xi f = \xi^\alpha \phi_\alpha + \bar\xi_{\dot\alpha}\bar\chi^{\dot\alpha}, \tag{13}$$

$$\delta_\xi \phi_\alpha = 2\xi_\alpha m + \sigma^m{}_{\alpha\dot\alpha}\bar\xi^{\dot\alpha}\big(v_m + i(\partial_m f)\big), \tag{14}$$

$$\delta_\xi \bar\chi^{\dot\alpha} = 2\bar\xi^{\dot\alpha} n + \bar\sigma^{m\dot\alpha\alpha}\xi_\alpha\big(-v_m + i(\partial_m f)\big), \tag{15}$$

$$\delta_\xi m = \bar\xi_{\dot\alpha}\bar\lambda^{\dot\alpha} + \frac{i}{2}\bar\xi_{\dot\alpha}\bar\sigma^{m\dot\alpha\alpha}(\partial_m\phi_\alpha), \tag{16}$$

$$\delta_\xi n = \xi^\alpha\varphi_\alpha + \frac{i}{2}\xi^\alpha\sigma^m{}_{\alpha\dot\alpha}(\partial_m\bar\chi^{\dot\alpha}), \tag{17}$$

$$\begin{aligned}\sigma^m{}_{\alpha\dot\alpha}\delta_\xi v_m &= -i(\partial_m\phi_\alpha)\xi^\beta\sigma^m{}_{\beta\dot\alpha} + 2\xi_\alpha\bar\lambda_{\dot\alpha} \\ &\quad + i\sigma^m{}_{\alpha\dot\beta}\bar\xi^{\dot\beta}(\partial_m\bar\chi_{\dot\alpha}) + 2\varphi_\alpha\bar\xi_{\dot\alpha},\end{aligned} \tag{18}$$

$$\delta_\xi\bar\lambda^{\dot\alpha} = 2\bar\xi^{\dot\alpha} d + i\bar\sigma^{l\dot\alpha\alpha}\xi_\alpha(\partial_l m) + \frac{i}{2}\bar\sigma^{l\dot\alpha\alpha}\sigma^m{}_{\alpha\dot\beta}\bar\xi^{\dot\beta}(\partial_m v_l), \tag{19}$$

$$\delta_\xi\varphi_\alpha = 2\xi_\alpha d + i\sigma^l{}_{\alpha\dot\alpha}\bar\xi^{\dot\alpha}(\partial_l n) - \frac{i}{2}\sigma^l{}_{\alpha\dot\alpha}\bar\sigma^{m\dot\alpha\beta}\xi_\beta(\partial_m v_l), \tag{20}$$

$$\delta_\xi d = \frac{i}{2}\xi^\alpha\sigma^m{}_{\alpha\dot\alpha}(\partial_m\bar\lambda^{\dot\alpha}) - \frac{i}{2}(\partial_m\varphi^\alpha)\sigma^m{}_{\alpha\dot\alpha}\bar\xi^{\dot\alpha}. \tag{21}$$

Deformed SUSY transformations

We choose twist $\mathcal{F}$

$$\mathcal{F} = e^{\frac{1}{2}C^{\alpha\beta}\partial_\alpha\otimes\partial_\beta + \frac{1}{2}\bar{C}_{\dot\alpha\dot\beta}\bar\partial^{\dot\alpha}\otimes\bar\partial^{\dot\beta}}, \quad C^{\alpha\beta} = C^{\beta\alpha} \in \mathbb{C}. \tag{22}$$

Hopf algebra of deformed SUSY transformations

$$\{Q_\alpha, Q_\beta\} = \{\bar{Q}_{\dot\alpha}, \bar{Q}_{\dot\beta}\} = 0, \quad \{Q_\alpha, \bar{Q}_{\dot\beta}\} = 2i\sigma^m_{\ \alpha\dot\beta}\partial_m, \dots, \tag{23}$$

$$\begin{aligned}
\Delta_{\mathcal{F}}(Q_\alpha) &= \mathcal{F}\Big(Q_\alpha\otimes 1 + 1\otimes Q_\alpha\Big)\mathcal{F}^{-1} \\
&= Q_\alpha\otimes 1 + 1\otimes Q_\alpha \\
&\quad -\frac{i}{2}\bar{C}_{\dot\alpha\dot\beta}\Big(\sigma^m_{\alpha\dot\gamma}\varepsilon^{\dot\gamma\dot\alpha}\partial_m\otimes\bar\partial^{\dot\beta} + \bar\partial^{\dot\alpha}\otimes\sigma^m_{\ \alpha\dot\gamma}\varepsilon^{\dot\gamma\dot\beta}\partial_m\Big), \\
\Delta_{\mathcal{F}}(\bar{Q}_\alpha) &= \bar{Q}_{\dot\alpha}\otimes 1 + 1\otimes\bar{Q}_{\dot\alpha} \\
&\quad +\frac{i}{2}C^{\alpha\beta}\Big(\sigma^m_{\alpha\dot\alpha}\partial_m\otimes\partial_\beta + \partial_\alpha\otimes\sigma^m_{\ \beta\dot\alpha}\partial_m\Big), \\
\varepsilon(Q_\alpha) &= \varepsilon(\bar{Q}_{\dot\alpha}) = 0, \qquad S(Q_\alpha) = -Q_\alpha, \quad S(\bar{Q}_{\dot\alpha}) = -\bar{Q}_{\dot\alpha}.
\end{aligned} \tag{24}$$

$\star$-product

Inverse of $\mathcal{F}$ introduces a $\star$-product on the superspace as

$$
\begin{aligned}
F\star G &= \mu\{\mathcal{F}^{-1}\,F\otimes G\}\\
&= F\cdot G-\frac{1}{2}(-1)^{|F|}C^{\alpha\beta}(\partial_\alpha F)\cdot(\partial_\beta G)-\frac{1}{2}(-1)^{|F|}\bar{C}_{\dot\alpha\dot\beta}(\bar\partial^{\dot\alpha}F)(\bar\partial^{\dot\beta}G)\\
&\quad -\frac{1}{8}C^{\alpha\beta}C^{\gamma\delta}(\partial_\alpha\partial_\gamma F)\cdot(\partial_\beta\partial_\delta G)-\frac{1}{8}\bar{C}_{\dot\alpha\dot\beta}\bar{C}_{\dot\gamma\dot\delta}(\bar\partial^{\dot\alpha}\bar\partial^{\dot\gamma}F)(\bar\partial^{\dot\beta}\bar\partial^{\dot\delta}G)\\
&\quad -\frac{1}{4}C^{\alpha\beta}\bar{C}_{\dot\alpha\dot\beta}(\partial_\alpha\bar\partial^{\dot\alpha}F)(\partial_\beta\bar\partial^{\dot\beta}G)\\
&\quad +\frac{1}{16}(-1)^{|F|}C^{\alpha\beta}C^{\gamma\delta}\bar{C}_{\dot\alpha\dot\beta}(\partial_\alpha\partial_\gamma\bar\partial^{\dot\alpha}F)(\partial_\beta\partial_\delta\bar\partial^{\dot\beta}G)\\
&\quad +\frac{1}{16}(-1)^{|F|}C^{\alpha\beta}\bar{C}_{\dot\alpha\dot\beta}\bar{C}_{\dot\gamma\dot\delta}(\partial_\alpha\bar\partial^{\dot\alpha}\bar\partial^{\dot\gamma}F)(\partial_\beta\bar\partial^{\dot\beta}\bar\partial^{\dot\delta}G)\\
&\quad +\frac{1}{64}C^{\alpha\beta}C^{\gamma\delta}\bar{C}_{\dot\alpha\dot\beta}\bar{C}_{\dot\gamma\dot\delta}(\partial_\alpha\partial_\gamma\bar\partial^{\dot\alpha}\bar\partial^{\dot\gamma}F)(\partial_\beta\partial_\delta\bar\partial^{\dot\beta}\bar\partial^{\dot\delta}G),
\end{aligned}
\tag{25}
$$

where $|F|=1$ if F is odd and $|F|=0$ if F is even.

Under complex conjugation

$$(F \star G)^* = G^* \star F^*. \tag{26}$$

Special examples

$$\{\theta^\alpha \stackrel{\star}{,} \theta^\beta\} = C^{\alpha\beta}, \quad \{\bar\theta_{\dot\alpha} \stackrel{\star}{,} \bar\theta_{\dot\beta}\} = \bar C_{\dot\alpha\dot\beta}, \quad [x^m \stackrel{\star}{,} x^n] = 0. \tag{27}$$

Non(anti)commutative space; non(anti)commutativity is encoded in terms of the $\star$-product (27).
Derivatives consistent with (28) and (29) are just the usual derivatives (2).

Usual integral is the "good" integral.

Deformed SUSY transformation is defined as

$$\delta^\star_\xi F(x,\theta,\bar\theta) = \big(\xi Q + \bar\xi\bar Q\big) F(x,\theta,\bar\theta). \tag{28}$$

As a consequence of (24) the $\star$-product of two superfields is again a superfield

$$\begin{aligned}\delta^\star_\xi (F\star G) &= \big(\xi Q + \bar\xi\bar Q\big)(F\star G), \\ &\neq (\delta_\xi F)\star G + \underbrace{F\star(\delta_\xi G)}_{F\star\left((\xi Q+\bar\xi\bar Q)G\right)}.\end{aligned} \tag{29}$$

Leibniz rule is deformed

$$\begin{aligned}\delta^\star_\xi (F\star G) &= (\delta_\xi F)\star G + F\star(\delta_\xi G) \\ &+ \frac{i}{2}C^{\alpha\beta}\Big(\bar\xi^{\dot\gamma}\sigma^m_{\alpha\dot\gamma}(\partial_m F)\star(\partial_\beta G) + (\partial_\alpha F)\star\bar\xi^{\dot\gamma}\sigma^m_{\beta\dot\gamma}(\partial_m G)\Big) \\ &- \frac{i}{2}\bar C_{\dot\alpha\dot\beta}\Big(\xi^\alpha\sigma^m_{\alpha\dot\gamma}\varepsilon^{\dot\gamma\dot\alpha}(\partial_m F)\star(\bar\partial^{\dot\beta}G) + (\bar\partial^{\dot\alpha}F)\star\xi^\alpha\sigma^m_{\alpha\dot\gamma}\varepsilon^{\dot\gamma\dot\beta}(\partial_m G)\Big).\end{aligned} \tag{30}$$

Chiral field Φ fulfils $\bar{D}_{\dot\alpha}\Phi = 0$, with $\bar{D}_{\dot\alpha} = -\bar{\partial}_{\dot\alpha} - i\theta^\alpha\sigma^m_{\ \alpha\dot\alpha}\partial_m$

$$\begin{aligned}\Phi(x) \;=\; & A(x) + \sqrt{2}\theta^\alpha\psi_\alpha(x) + \theta\theta\mathsf{F}(x) + i\theta\sigma^l\bar\theta(\partial_l A(x)) \\ & -\frac{i}{\sqrt{2}}\theta\theta(\partial_m\psi^\alpha(x))\sigma^m_{\ \alpha\dot\alpha}\bar\theta^{\dot\alpha} + \frac{1}{4}\theta\theta\bar\theta\bar\theta(\Box A(x)). \qquad (31)\end{aligned}$$

The $\star$-product of two, three,... chiral superfields is not chiral

$$\begin{aligned}\Phi\star\Phi \;=\; & A^2 - \frac{C^2}{2}\mathsf{F}^2 + \frac{1}{4}C^{\alpha\beta}\bar{C}^{\dot\alpha\dot\beta}\sigma^m_{\ \alpha\dot\alpha}\sigma^l_{\ \beta\dot\beta}(\partial_m A)(\partial_l A) + \frac{1}{64}C^2\bar{C}^2(\Box A)^2 \\ & +\theta^\alpha\Big(2\sqrt{2}\psi_\alpha A - \frac{1}{\sqrt{2}}C^{\gamma\beta}\bar{C}^{\dot\alpha\dot\beta}\varepsilon_{\gamma\alpha}(\partial_m\psi^\rho)\sigma^m_{\ \rho\dot\beta}\sigma^l_{\ \beta\dot\alpha}(\partial_l A)\Big) \\ & -\frac{i}{\sqrt{2}}C^2\bar\theta_{\dot\alpha}\bar\sigma^{m\dot\alpha\alpha}(\partial_m\psi_\alpha)\mathsf{F} + \theta\theta\Big(2A\mathsf{F} - \psi\psi\Big) \\ & +\bar\theta\bar\theta\Big(-\frac{C^2}{4}\big(\mathsf{F}\Box A - \frac{1}{2}(\partial_m\psi)\sigma^m\bar\sigma^l(\partial_l\psi)\big)\Big) \\ & +\theta\sigma^m\bar\theta\Big(i(\partial_m A^2) + \frac{i}{4}C^{\alpha\beta}\bar{C}^{\dot\alpha\dot\beta}\sigma_{m\alpha\dot\alpha}\sigma^l_{\ \beta\dot\beta}(\Box A)(\partial_l A)\Big) \\ & +i\sqrt{2}\theta\theta\bar\theta_{\dot\alpha}\bar\sigma^{m\dot\alpha\alpha}\big(\partial_m(\psi_\alpha A)\big) + \frac{1}{4}\theta\theta\bar\theta\bar\theta(\Box A^2), \qquad (32)\end{aligned}$$

where $\quad C^2 = C^{\alpha\beta}C^{\gamma\delta}\varepsilon_{\alpha\gamma}\varepsilon_{\beta\delta}$ and $\bar{C}^2 = \bar{C}_{\dot\alpha\dot\beta}\bar{C}_{\dot\gamma\dot\delta}\varepsilon^{\dot\alpha\dot\gamma}\varepsilon^{\dot\beta\dot\delta}$.

We project out chiral, antichiral and transverse component of $\Phi\star\Phi$ and $\Phi\star\Phi\star\Phi$ by using projectors P_1, P_2 and P_T

$$P_1=\frac{1}{16}\frac{D^2\bar{D}^2}{\Box},\quad P_2=\frac{1}{16}\frac{\bar{D}^2D^2}{\Box},\quad P_T=-\frac{1}{8}\frac{D\bar{D}^2D}{\Box}, \tag{33}$$

$$f(x)\frac{1}{\Box}g(x)=f(x)\int \mathrm{d}^4y\ G(x-y)g(y).$$

$$\begin{aligned}
P_2\big(\Phi\star\Phi\big) \;=\; & A^2-\frac{C^2}{8}\mathsf{F}^2\\
&+\frac{1}{16}C^{\alpha\beta}\bar{C}^{\dot\alpha\dot\beta}\sigma^m{}_{\alpha\dot\alpha}\sigma^l{}_{\beta\dot\beta}\Big((\partial_mA)(\partial_lA)+\frac{2}{\Box}\partial_m\big((\Box A)(\partial_lA)\big)\Big)\\
&+\sqrt{2}\theta^\alpha\Big(2\psi_\alpha A-\frac{1}{4}C^{\gamma\beta}\bar{C}^{\dot\alpha\dot\beta}\varepsilon_{\gamma\alpha}(\partial_m\psi^\rho)\sigma^m{}_{\rho\dot\beta}\sigma^l{}_{\beta\dot\alpha}(\partial_lA)\Big)\\
&+\theta\theta\Big(2A\mathsf{F}-\psi\psi\Big)+i\theta\sigma^k\bar\theta\partial_k\Big(A^2-\frac{C^2}{8}\mathsf{F}^2\\
&+\frac{1}{16}C^{\alpha\beta}\bar{C}^{\dot\alpha\dot\beta}\sigma^m{}_{\alpha\dot\alpha}\sigma^l{}_{\beta\dot\beta}\Big((\partial_mA)(\partial_lA)+\frac{2}{\Box}\partial_m\big((\Box A)(\partial_lA)\big)\Big)\Big)\\
&+i\sqrt{2}\theta\theta\bar\theta_{\dot\alpha}\bar\sigma^{k\dot\alpha\alpha}\partial_k\Big(\psi_\alpha A-\frac{1}{8}C^{\gamma\beta}\bar{C}^{\dot\alpha\dot\beta}\varepsilon_{\gamma\alpha}(\partial_m\psi^\rho)\sigma^m{}_{\rho\dot\beta}\sigma^l{}_{\beta\dot\alpha}(\partial_lA)\Big)\\
&+\frac{1}{4}\theta\theta\bar\theta\bar\theta\Box\Big(A^2-\frac{C^2}{8}\mathsf{F}^2+\frac{1}{16}C^{\alpha\beta}\bar{C}^{\dot\alpha\dot\beta}\sigma^m{}_{\alpha\dot\alpha}\sigma^l{}_{\beta\dot\beta}\\
&\quad\Big((\partial_mA)(\partial_lA)+\frac{2}{\Box}\partial_m\big((\Box A)(\partial_lA)\big)\Big)\Big)+\mathcal{O}(C^3).
\end{aligned} \tag{34}$$

Wess-Zumino Lagrangian

Undeformed Wess-Zumino Lagrangian

$$\mathcal{L} = \Phi^+ \cdot \Phi\Big|_{\theta\theta\bar\theta\bar\theta} + \Big(\frac{m}{2}\Phi\cdot\Phi\Big|_{\theta\theta} + \frac{\lambda}{3}\Phi\cdot\Phi\cdot\Phi\Big|_{\theta\theta} + \text{ c.c. }\Big), \tag{35}$$

with m and λ real constants.

Deformation

$$\Phi^+\cdot\Phi \quad\to\quad \Phi^+\star\Phi,$$

$$\Phi\cdot\Phi \quad\to\quad P_2\big(\Phi\star\Phi\big),$$

$$\Phi\cdot\Phi\cdot\Phi \quad\to\quad \begin{cases} P_2\Big(\Phi\star P_2\big(\Phi\star\Phi\big)\Big), \\ P_2\Big(P_2\big(\Phi\star\Phi\big)\star\Phi\Big), \\ P_2\Big(\Phi\star\Phi\star\Phi\Big). \end{cases}$$

Comments I

- kinetic term

$$\Phi^+ \star \Phi\Big|_{\theta\theta\bar\theta\bar\theta} = \mathsf{F}^*\mathsf{F} + \frac{1}{4}A^*\Box A + \frac{1}{4}A\Box A^* \tag{36}$$
$$-\frac{1}{2}(\partial_m A^*)(\partial^m A) + \frac{i}{2}(\partial_m\bar\psi)\bar\sigma^m\psi - \frac{i}{2}\bar\psi\bar\sigma^m(\partial_m\psi),$$
$$\delta^\star_\xi\Big(\Phi^+ \star \Phi\Big|_{\theta\theta\bar\theta\bar\theta}\Big) = \partial_m(\dots).$$

remains undeformed.

- mass term

$$P_2\big(\Phi\star\Phi\big)\Big|_{\theta\theta} + \text{c.c.} = 2A\mathsf{F} - \psi\psi + 2A^*\mathsf{F}^* - \bar\psi\bar\psi,$$
$$\delta^\star_\xi\Big(P_2\big(\Phi\star\Phi\big)\Big|_{\theta\theta} + \text{ c.c. }\Big) = \partial_m(\dots). \tag{37}$$

remains undeformed.

- interaction term

$$
\begin{aligned}
1)\quad & P_2\Big(\Phi\star P_2\big(\Phi\star\Phi\big)\Big)\Big|_{\theta\theta} + \text{c.c.} \;= 3\big(A^2\mathsf{F} - (\psi\psi)A\big)\\
& -\frac{1}{4}K^{ab}K_{ab}\mathsf{F}^3\\
& +\frac{1}{2}K^m{}_aK^{*na}\mathsf{F}\Big((\partial_mA)(\partial_nA) + \frac{2}{\Box}\partial_m\big((\Box A)(\partial_nA)\big)\Big)\\
& -\Big(K^m{}_aK^{*na}\psi(\partial_n\psi) - 2K^m{}_aK^{*n}{}_c(\partial_n\psi)\sigma^{ca}\psi\Big)(\partial_mA) \qquad (38)\\
& +\frac{1}{2}K^*_{ab}(\bar\sigma^{ab}\bar\sigma^{lm})^{\dot\beta}{}_{\dot\beta}(\partial_mA)\partial_l\Big[A^2 - \frac{1}{4}K^{ab}K_{ab}\mathsf{F}^2\\
& +\frac{1}{2}K^m{}_aK^{*na}\mathsf{F}\Big((\partial_mA)(\partial_nA) + \frac{2}{\Box}\partial_m\big((\Box A)(\partial_nA)\big)\Big)\Big]\\
& + \text{c.c.} \;+\mathcal{O}(K^4),
\end{aligned}
$$

where we introduced the following notation

$$
C_{\alpha\beta} = K_{ab}(\sigma^{ab}\varepsilon)_{\alpha\beta},\quad \bar C_{\dot\alpha\dot\beta} = K^*_{ab}(\varepsilon\bar\sigma^{ab})_{\dot\alpha\dot\beta}. \qquad (39)
$$

$2)$

$$P_2\Big(P_2\big(\Phi\star\Phi\big)\star\Phi\Big)\Big|_{\theta\theta}\neq P_2\Big(\Phi\star P_2\big(\Phi\star\Phi\big)\Big)\Big|_{\theta\theta},$$

but the difference (under the integral) is a surface term

$$+\frac{1}{2}K^*_{ab}(\bar\sigma^{ab}\bar\sigma^{lm})^{\dot\beta}{}_{\dot\beta}\partial_l\Big[(\partial_m A)\Big(A^2-\frac{C^2}{8}\mathsf{F}^2$$
$$+\frac{1}{2}K^m{}_aK^{*na}\mathsf{F}\Big((\partial_m A)(\partial_n A)+\frac{2}{\Box}\partial_m\big((\Box A)(\partial_n A)\big)\Big)\Big)\Big].$$

$3)$ Term $P_2\Big(\Phi\star\Phi\star\Phi\Big)\Big|_{\theta\theta}+$ c.c. has "wrong" transformation law

$$\delta^\star_\xi\Big(P_2\Big(\Phi\star\Phi\star\Phi\Big)\Big|_{\theta\theta}+\text{ c.c. }\Big)\neq\partial_m(\dots)$$

and would not lead to a SUSY invariant action.

Finally, the transformation law of (38) is given by

$$\delta^\star_\xi\Big(P_2\big(\Phi\star P_2(\Phi\star\Phi)\big)\Big|_{\theta\theta}+\text{ c.c. }\Big)$$
$$=i\sqrt{2}\bar\xi_{\dot\alpha}\bar\sigma^{l\dot\alpha\alpha}\partial_l\Big(K^m{}_aK^{*na}\psi_\alpha\frac{1}{\Box}\partial_m(\partial_nA\Box A)+\text{ local terms }\Big).$$

Interaction term is deformed and nonlocal, but it is still SUSY invariant and has good classical limit

$$P_2\big(\Phi\star P_2(\Phi\star\Phi)\big)\Big|_{\theta\theta}+\text{ c.c. }\to 3\big(A^2\mathsf{F}-(\psi\psi)A+\text{ c.c. }\big),$$

when $C,\bar C\to 0$.

Deformed Wess-Zumino Lagrangian

$$
\begin{aligned}
\mathcal{L} &= \Phi^{+} \star \Phi \Big|_{\theta\theta\bar{\theta}\bar{\theta}} \\
&\quad + \Big(\frac{m}{2} P_2\big(\Phi \star \Phi\big)\Big|_{\theta\theta} + \frac{\lambda}{3} P_2\Big(\Phi \star P_2\big(\Phi \star \Phi\big)\Big)\Big|_{\theta\theta} + \text{ c.c. } \Big) \\
&= A^{*}\Box A + i(\partial_m \bar{\psi})\bar{\sigma}^{m}\psi + \mathsf{F}^{*}\mathsf{F} \\
&\quad + \Big[\frac{m}{2}\Big(2A\mathsf{F} - \psi\psi\Big) + \lambda\Big(\mathsf{F}A^{2} - A\psi\psi\Big) \\
&\quad - \frac{\lambda}{3}\Big(K^{m}{}_{a}K^{*na}\psi(\partial_n\psi) - 2K^{m}{}_{a}K^{*n}{}_{b}(\partial_n\psi)\sigma^{ba}\psi\Big)(\partial_m A) \\
&\quad + \frac{\lambda}{6}K^{m}{}_{a}K^{*na}\mathsf{F}\Big((\partial_m A)(\partial_n A) + \frac{2}{\Box}\partial_m\big((\partial_n A)\Box A\big)\Big) \qquad (40) \\
&\quad + \text{ c.c. } \Big] + \mathcal{O}(K^{4}).
\end{aligned}
$$

Equations of motion for fields F and F^*

$$\mathsf{F}^* + mA + \lambda A^2 - \frac{\lambda}{4}K^{ab}K_{ab}\mathsf{F}^2 + \frac{\lambda}{6}K^{m}{}_{a}K^{*na}(\partial_m A)(\partial_n A)$$
$$+\frac{\lambda}{3}K^{m}{}_{a}K^{*na}\frac{1}{\Box}\partial_m\Big((\partial_n A)\Box A\Big) + \mathcal{O}(K^4) = 0, \quad (41)$$

$$\mathsf{F} + mA^* + \lambda(A^*)^2 - \frac{\lambda}{4}K^{*ab}K^*_{ab}(\mathsf{F}^*)^2$$
$$+\frac{\lambda}{6}K^{m}{}_{a}K^{*na}(\partial_m A)^*(\partial_n A)^*$$
$$+\frac{\lambda}{3}K^{m}{}_{a}K^{*na}\frac{1}{\Box}\partial_m\Big((\partial_n A)^*\Box A^*\Big) + \mathcal{O}(K^4) = 0 \quad (42)$$

are solved perturbatively and solutions inserted in the Lagrangian (40). That gives

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_2 + \mathcal{O}(K^4)\ , \quad (43)$$

with

$$
\begin{aligned}
\mathcal{L}_0 \;=\;& A^*\Box A + i(\partial_m\bar{\psi})\bar{\sigma}^m\psi - \lambda A^*\bar{\psi}\bar{\psi} - \lambda A\psi\psi - \frac{m}{2}\big(\psi\psi + \bar{\psi}\bar{\psi}\big) \\
& - m^2A^*A - m\lambda A(A^*)^2 - m\lambda A^*A^2 - \lambda^2A^2(A^*)^2, \qquad (44) \\
\mathcal{L}_2 \;=\;& \frac{\lambda}{3}K^m{}_aK^{*na}\Big(m(\partial_mA) + 2\lambda A(\partial_mA)\Big)\frac{1}{\Box}\big((\partial_nA^*)\Box A^*\big) \\
& + \frac{\lambda}{3}K^m{}_aK^{*na}\Big(m(\partial_mA^*) + 2\lambda A^*(\partial_mA^*)\Big)\frac{1}{\Box}\big((\partial_nA)\Box A\big) \\
& + \frac{\lambda}{12}K^{ab}K_{ab}\Big(mA^* + \lambda(A^*)^2\Big)^3 + \frac{\lambda}{12}K^{*ab}K^*_{ab}\Big(mA + \lambda A^2\Big)^3 \\
& - \frac{\lambda}{6}K^m{}_aK^{*na}\Big((mA + \lambda A^2)(\partial_mA^*)(\partial_nA^*) \\
& + (mA^* + \lambda(A^*)^2)(\partial_mA)(\partial_nA)\Big) \\
& - \frac{\lambda}{3}\Big(K^m{}_aK^{*na}\psi(\partial_n\psi) - 2K^m{}_aK^{*n}{}_b(\partial_n\psi)\sigma^{ba}\psi\Big)(\partial_mA) \qquad (45) \\
& - \frac{\lambda}{3}\Big(K^m{}_aK^{*na}\bar{\psi}(\partial_n\bar{\psi}) - 2K^m{}_aK^{*n}{}_b\bar{\psi}\bar{\sigma}^{ab}(\partial_n\bar{\psi})\Big)(\partial_mA^*).
\end{aligned}
$$

Comments II

- we constructed a deformation of the Wess-Zumino Lagrangian with the good classicl limit
- non-local interaction, but still SUSY invariant
- work in progress:
  -renormalisability of the model
  -renormalisation might restrict the choice of deformation (we might be forced to choose $C^{\alpha\beta}$ such that $K^{ab}K_{ab} = 0$)
- future work
  -gauge theories
  -...
  -deformed SUSY Standard Model

Comments III

What has been done on this subject so far in the literature

- the choice $\mathcal{F} = e^{-\frac{i}{2}\theta^{mn}\partial_m \otimes \partial_n}$ leads to $[x^m \stackrel{\star}{,} x^n] = i\theta^{mn}$, analysed in hep-th/9912153 and hep-th/0002084 .
- the choice $\mathcal{F} = e^{\frac{1}{2}C^{\alpha\beta}\partial_\alpha \otimes \partial_\beta}$ analysed by Seiberg in hep-th/0305248; $\star$-product non-hermitian, chirality preserved.
- the choice $\mathcal{F} = e^{\frac{1}{2}C^{\alpha\beta}Q_\alpha \otimes Q_\beta}$ analysed by Zupnik in hep-th/0506043, Ihl and Sämann in hep-th/0506057, . . . ; chirality preserved, $\star$-product non-hermitian.
- some combinations of $\mathcal{F} = e^{\frac{1}{2}C^{\alpha\beta}D_\alpha \otimes D_\beta}$ analysed by Ferrara et al. in hep-th/0307039; . . . , some remarks on SUSY invariant Lagrangians.

Deformed Lorentz transformations

Under infinitesimal Lorentz transformations coordinates of the superspace transform as follows

$$\begin{aligned}
\delta_\omega x^m &= \omega^m{}_n x^n = -\frac{1}{2}\omega^{rs}L_{rs}x^m, \\
\delta_\omega \theta_\alpha &= \omega^{mn}(\sigma_{mn})_\alpha^{\ \beta}\theta_\beta = -\frac{1}{2}\omega^{mn}L_{mn}\theta_\alpha, \\
\delta_\omega \bar\theta^{\dot\alpha} &= \omega^{mn}(\bar\sigma_{mn})^{\dot\alpha}_{\ \dot\beta}\bar\theta^{\dot\beta} = -\frac{1}{2}\omega^{mn}L_{mn}\bar\theta^{\dot\alpha},
\end{aligned} \tag{46}$$

$$\begin{aligned}
L_{mn} &= x_m\partial_n - x_n\partial_m - (\sigma_{mn}\varepsilon)_{\alpha\beta}(\theta^\alpha\partial^\beta + \theta^\beta\partial^\alpha) \\
&\quad -(\varepsilon\bar\sigma_{mn})_{\dot\alpha\dot\beta}(\bar\theta^{\dot\alpha}\bar\partial^{\dot\beta} + \bar\theta^{\dot\beta}\bar\partial^{\dot\alpha}).
\end{aligned} \tag{47}$$

General superfield F is a scalar under (46),

$$\begin{aligned}
F'(x',\theta',\bar\theta') &= F(x,\theta,\bar\theta), \\
\delta_\omega F &= \frac{1}{2}\omega^{mn}L_{mn}F.
\end{aligned} \tag{48}$$

The Hopf algebra of undeformed Lorentz transformations is given by

$$\begin{aligned} [\delta_\omega, \delta_{\omega'}] &= \delta_{[\omega,\omega']}, \\ \Delta(\delta_\omega) &= \delta_\omega \otimes 1 + 1 \otimes \delta_\omega, \\ \varepsilon(\delta_\omega) &= 0, \qquad S(\delta_\omega) = -\delta_\omega. \end{aligned} \quad (49)$$

Using the twist $\mathcal{F}$ (22) we (as usual) twist this Hopf algebra, . . .
The deformed Leibniz rule tells us that $F \star G$ is a scalar field again. . .

Simple example: transformation law of $\theta^\alpha \star \theta^\beta$
In the undeformed case we have

$$\delta_\omega(\theta^\alpha \cdot \theta^\beta) = -\frac{1}{2}\omega^{mn} L_{mn}(\theta^\alpha \cdot \theta^\beta).$$

Therefore we say that also

$$\begin{aligned}\delta_\omega(\theta^\alpha\star\theta^\beta) &= -\frac{1}{2}\omega^{mn}L_{mn}(\theta^\alpha\star\theta^\beta)\\ &= \frac{1}{2}\omega^{mn}(\sigma_{mn}\varepsilon)_{\gamma\delta}(\theta^\gamma\partial^\delta+\theta^\delta\partial^\gamma)\big(\theta^\alpha\theta^\beta+\frac{1}{2}C^{\alpha\beta}\big)\\ &= -\omega^{mn}\Big((\sigma_{mn})_\gamma^{\ \alpha}\theta^\gamma\theta_\beta+(\sigma_{mn})_\gamma^{\ \beta}\theta_\alpha\theta^\gamma\Big).\end{aligned}\tag{50}$$

The deformed Leibniz on the other hand gives

$$\begin{aligned}\delta_\omega(\theta^\alpha\star\theta^\beta) &= (\delta_\omega\theta^\alpha)\star\theta^\beta+\theta^\alpha\star(\delta_\omega\theta^\beta)\\ &\quad -\frac{1}{2}C^{\rho\sigma}\omega^{mn}\Big((\partial_\rho\theta^\alpha)\star(\sigma_{mn}\varepsilon)_{\sigma\gamma}(\partial^\gamma\theta^\beta)\\ &\quad +(\sigma_{mn}\varepsilon)_{\rho\gamma}(\partial^\gamma\theta^\alpha)\star(\partial_\sigma\theta^\beta)\Big)\\ &= -\omega^{mn}\Big((\sigma_{mn})_\gamma^{\ \alpha}\theta^\gamma\theta_\beta+(\sigma_{mn})_\gamma^{\ \beta}\theta_\alpha\theta^\gamma\Big).\end{aligned}\tag{51}$$

Results (50) and (51) are the same.