LSS with Lagrangian perturbation theory: including the stream crossing

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with:

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Structure Formation and Evolution

CMB: $\Delta\rho/\rho \sim 10^{-6}$

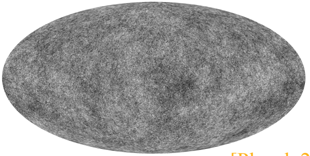
z=1100

LSS: $\Delta\rho/\rho \sim 10^{0}$

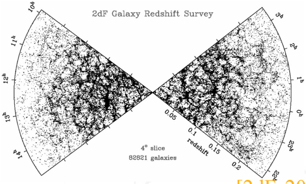
z=2

Galaxies: $\Delta\rho/\rho \sim 10^{6}$

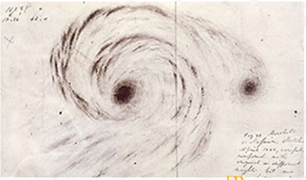
z=0



[Planck,2013]



[2dF, 2002]



[Parsons, 1845]

LSS: motivations and observations

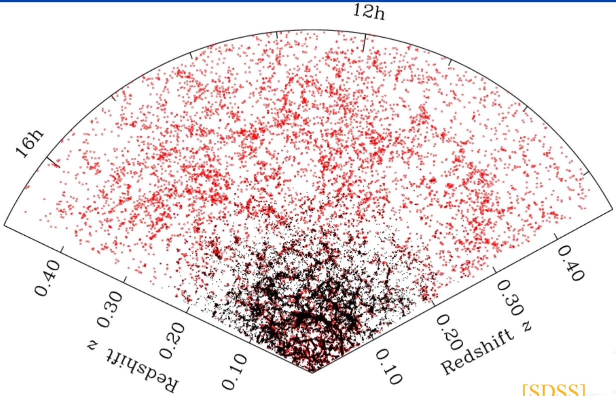
Theoretical motivations:

- Inflation - origin of structures
- Expansion history
- Composition of the universe
- Nature of dark energy and dark matter
- Neutrino mass and number of species
- Test of GR and modifications of gravity

Current and future observations:

- SDSS and SDSS3/4: Sloan Digital Sky Survey
- BOSS: the Baryon Oscillation Spectroscopic Survey
- DES: the Dark Energy Survey
- LSST: the large synoptic survey telescope.
- Euclid: the ESA mission to map the geometry of the dark Universe
- DESI: Dark Energy Spectroscopic Instrument
- SPHEREx: An All-Sky Spectral Survey (?)

Galaxy clustering



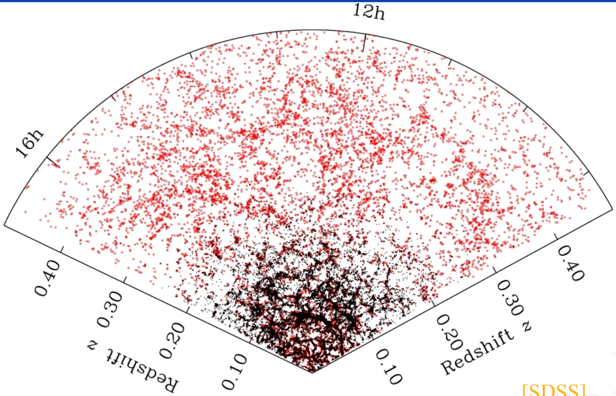
[SDSS]

- Measured $3D$ distribution $\Rightarrow$ much more modes than projected quantities (shear from weak lensing, etc.)
- Redshift surveys measure: θ, ϕ, redshift z

overdensity: $\delta = (n - \bar{n})/\bar{n}$,

power spectrum: $P(k) \sim \langle \delta(\mathbf{k}) | \delta(\mathbf{k}) \rangle$

Galaxy clustering



[SDSS]

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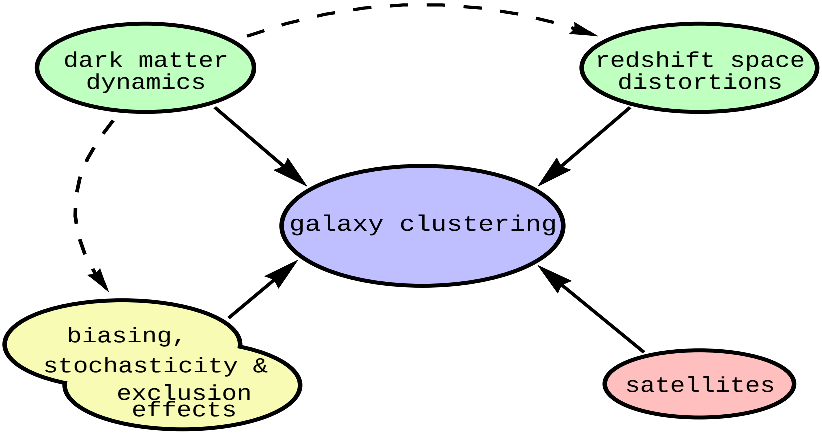
Generalization is the multi-spectra:

$$\langle \delta(\mathbf{k}_1) \dots \delta(\mathbf{k}_N) \rangle_c \sim P_N(\mathbf{k}_1, \dots, \mathbf{k}_N)$$

Galaxy clustering scheme

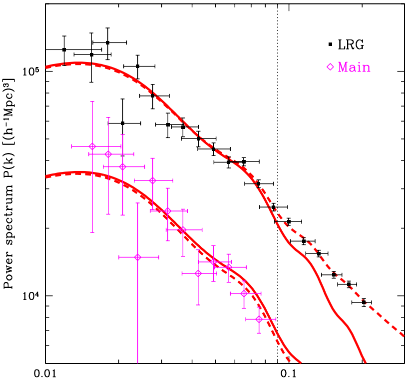


+ others: baryons, assembly bias, neutrinos, (clustering) dark energy, GR effects, multiple d.m. species ...

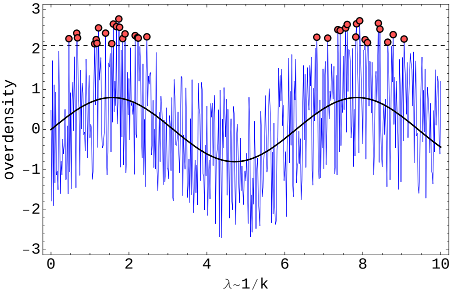
Galaxies and biasing of dark matter halos

Galaxies form at high density peaks of initial matter density:

- rare peaks exhibit higher clustering!

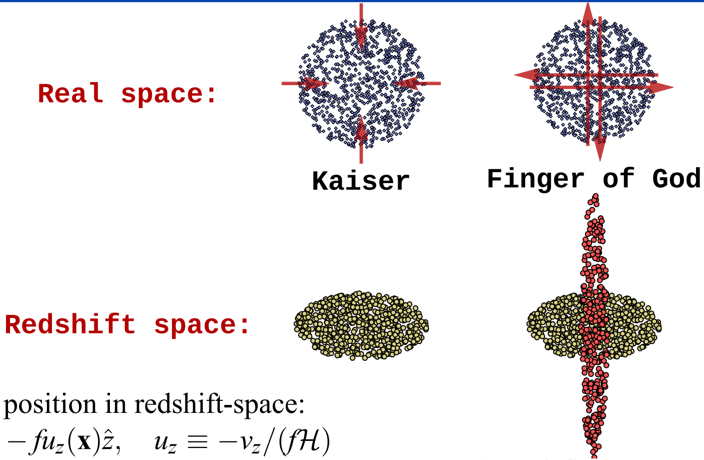


[Tegmark et al, 2006]



- Tracer detriments the amplitude: $P_g(k) = b^2 P_m(k) + \ldots$
- Understanding bias is crucial for understanding the galaxy clustering

Redshift space distortions (RSD)



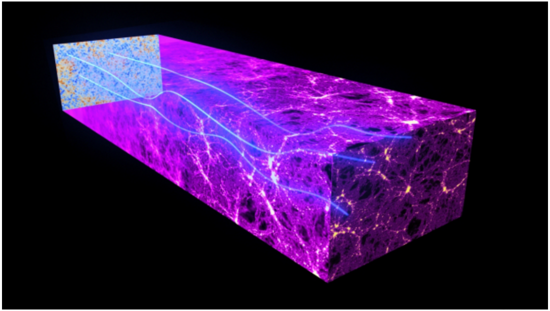
Object position in redshift-space:

$$\mathbf{s} = \mathbf{x} - f u_z(\mathbf{x})\hat{z}, \quad u_z \equiv -v_z/(f\mathcal{H})$$

Density in redshift-space:

$$\delta_s(\mathbf{s}) = \int_r \delta(\boldsymbol{r})\delta^D(\mathbf{s} - \boldsymbol{x} - f u_z(\mathbf{x})\hat{z}).$$

Weak gravitational lensing



- sensitivity on lensing potential; both in CMB and galaxy lensing
- e.g. convergence power spectrum

$$C_\kappa(l) = \int_0^{\chi_s} d\chi\, w^2(\chi)/\chi^2 P_\delta(l/\chi)$$

where χ is comoving angular-diameter distance, and w is weight function.

Nonlinear dynamics of dark matter

Key messages:

- Nonlinear effects of dark matter clustering allow analytic investigation (UV complete) - including *shell crossing*.
- In a regime of largest possible scales (BAO) further simplifications can be achieved in the *EFT* framework – natural extensions to galaxies.

Why perturbative approach?

- Goal is the high precision at large scales (in scope of next gen. surveys), as well as to push to small scales.
- This problem is also amenable to direct simulation.
  - Though the combination of volume, mass and force resolution and numerical accuracy is very demanding - in scope of next gen. surveys.
  - PT is a viable alternative as well as a guide what range of k, M_h, scales are necessary and what statistics are needed.
  - N-body can be used to test PT for ‘fiducial’ models.
- However PT can be used to search a large parameter space efficiently, and find what kinds of effects are most important.
  - Can be much more flexible/inclusive, especially for biasing schemes.
  - It is much easier to add new physics, especially if the effects are small (e.g. neutrinos, clustering dark energy, non-Gaussianity)
- Gaining insights!
- Complementarity reason; if we can, we should.

Gravitational clustering of dark matter

Evolution of collisionless particles - Vlasov equation:

$$\frac{df}{d\tau} = \frac{\partial f}{\partial \tau} + \frac{1}{m}\mathbf{p} \cdot \nabla f - am\nabla\phi \cdot \nabla_p f = 0,$$

and $\nabla^2\phi = 3/2\mathcal{H}\Omega_m\delta$.

Integral moments of the distribution function:

mass density field & mean streaming velocity field

$$\rho(\mathbf{x}) = ma^{-3}\int d^3p\, f(\boldsymbol{x}, \boldsymbol{p}), \qquad v_i(\mathbf{x}) = \frac{\int d^3p\, \frac{p_i}{am} f(\boldsymbol{x}, \boldsymbol{p})}{\int d^3p\, f(\boldsymbol{x}, \boldsymbol{p})},$$

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Eulerian framework - fluid approximation:

$$\frac{\partial \delta}{\partial \tau} + \nabla \cdot [(1+\delta)\mathbf{v}] = 0$$
$$\frac{\partial v_i}{\partial \tau} + \mathcal{H}v_i + \mathbf{v} \cdot \nabla v_i = -\nabla_i\phi - \frac{1}{\rho}\nabla_i(\rho\sigma_{ij}),$$

where σ_{ij} is the velocity dispersion.

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Eulerian framework - **pressureless perfect fluid** approximation:

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Irrotational fluid: $\theta = \nabla \cdot \mathbf{v}$.

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EFT approach introduces a tress tensor for the long-distance fluid:

$$\frac{\partial \delta}{\partial \tau} + \nabla \cdot [(1+\delta)\mathbf{v}] = 0$$
$$\frac{\partial v_i}{\partial \tau} + \mathcal{H}v_i + \mathbf{v} \cdot \nabla v_i = -\nabla_i\phi - \frac{1}{\rho}\nabla_j(\tau_{ij}),$$

with given as $\tau_{ij} = p_0\delta_{ij} + c_s^2\delta\rho\delta_{ij} + O(\partial^2\delta, \ldots)$
-derived by smoothing the short scales in the fluid with
the smoothing filter $W(\Lambda)$, where $\Lambda \propto 1/k_{\rm NL}$.
[Baumann et al 2010, Carrasco et al 2012]

Gravitational clustering of dark matter

Evolution of collisionless particles - Vlasov equation:

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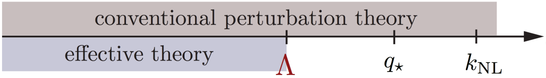

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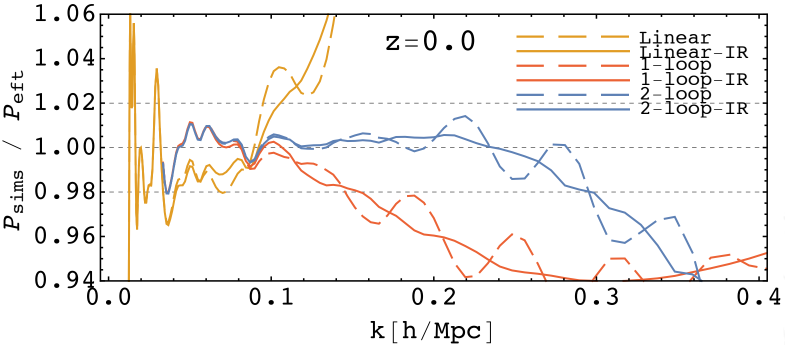
Power spectrum, correlation function & BAO

$$P_{\text{EFT-1-loop}} = P_{11} + P_{\text{1-loop}} - 2(2\pi)c^2_{s(1)}\frac{k^2}{k^2_{\text{NL}}}P_{11}$$

$$P_{\text{EFT-2-loop}} = P_{11} + P_{\text{1-loop}} + P_{\text{2-loop}} - 2(2\pi)c^2_{s(1)}\frac{k^2}{k^2_{\text{NL}}}(P_{11} + P_{\text{1-loop}}) + \text{c.t.}$$



[Carrasco et al, '12/'13, Senatore et al '14, Baldauf et al '15, Foreman et al '15, Vlah et al '15]

- Well defined/convergent expansion in k/k_{NL} (one parameter).
- Six c. t. for two-loop - approximate degeneracy! [Zaldarriaga et al, '15]

Lagrangian vs Eulerian framework

Eulerian:

Lagrangian:

Coordinate of a (t)racer particle at a given moment in time $\mathbf{r}$

$$\mathbf{r}(\mathbf{q},\tau) = \mathbf{q} + \psi(\mathbf{q},\tau),$$

is given in terms of Lagrangian displacement.

Continuity equation:

$$(1+\delta(\mathbf{r}))\, d^3r = d^3q \quad \text{vs.} \quad 1+\delta(\mathbf{r}) = \int_q \delta^D\left(\mathbf{r} - \mathbf{q} - \psi(\mathbf{q})\right),$$

Shell crossing

$$(1+\delta(\mathbf{r}))\, d^3r = \sum_{shells} d^3q \quad \text{vs.} \quad 1+\delta(\mathbf{r}) = \int_q \delta^D\left(\mathbf{r} - \mathbf{q} - \psi(\mathbf{q})\right),$$

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Fourier space

$$(2\pi)^3 \delta^D(\mathbf{k}) + \delta(\mathbf{k}) = \int_q e^{i\mathbf{k}\cdot\mathbf{q}} \exp\left(i\mathbf{k} \cdot \psi\right),$$

Lagrangian dynamics and EFT

Fluid element at position $\boldsymbol{q}$ at time t_0, moves due to gravity:

The evolution of ψ is governed by

$$\ddot{\psi} + \mathcal{H}\dot{\psi} = -\nabla\Phi(\boldsymbol{q} + \psi(\boldsymbol{q})).$$

Integrating out short modes (using filter $W_R(\boldsymbol{q}, \boldsymbol{q}')$) system is splitting tnto $L-$long and $S-$short wavelength modes, e.g.

$$\psi_L(\boldsymbol{q}) = \int_q W_R(\boldsymbol{q}, \boldsymbol{q}')\psi(\boldsymbol{q}'), \quad \psi_S(\boldsymbol{q}, \boldsymbol{q}') = \psi(\boldsymbol{q}') - \psi_L(\boldsymbol{q}).$$

This defines δ_L as the long-scale component of the density perturbation corresponding to ψ_L and also Φ_L as the gravitational potential $\nabla^2\Phi_L \sim \delta_L$.
E.o.m. for long displacement: [Vlah et al, '15]

$$\ddot{\psi}_L + \mathcal{H}\dot{\psi}_L = -\nabla\Phi_L(\boldsymbol{q} + \psi_L(\boldsymbol{q})) + \boldsymbol{a}_S\big(\boldsymbol{q}, \psi_L(\boldsymbol{q})\big),$$

and $\boldsymbol{a}_S(\boldsymbol{q}) = -\nabla\Phi_S(\boldsymbol{q} + \psi_L(\boldsymbol{q})) - \frac{1}{2}Q_L^{ij}(\boldsymbol{q})\nabla\nabla_i\nabla_j\Phi_L(\boldsymbol{q} + \psi_L(\boldsymbol{q})) + \ldots,$
Similar formalism was also derived in [Porto et al, '14].

Lagrangian dynamics and EFT

The correlation function and power spectrum can now be defined through the cumulants of the displacement, e.g.

$$P(k) = \int_q e^{i\boldsymbol{q}\cdot\boldsymbol{k}} \left[\left\langle e^{i\boldsymbol{k}\cdot\boldsymbol{\Delta}(\boldsymbol{q})} \right\rangle - 1 \right].$$

For one loop power spectrum results, keeping linear modes resumed:

$$P(k) = \int_q e^{i\boldsymbol{k}\cdot\boldsymbol{q}} \exp\left[-\frac{1}{2} k_i k_j \left\langle \Delta_i \Delta_j \right\rangle_c + \frac{i}{6} k_i k_j k_k \left\langle \Delta_i \Delta_j \Delta_k \right\rangle_c + \cdots \right]$$

Final results equivalent to the Eulerian scheme. [Sugiyama '14, Vlah et al, '14 & '15]
Allows for the insight in the counter term structure and IR resummation schemes (in particular one leads to the scheme in [Senatore&Zaldarriaga, '14]).
Simple IR scheme was suggested also in [Baldauf et al, '15].

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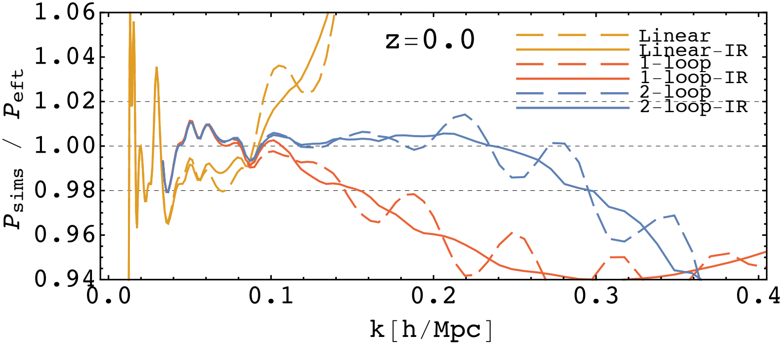

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- IR resummation (Lagrangian approach) - BAO peak! [Vlah et al '15]
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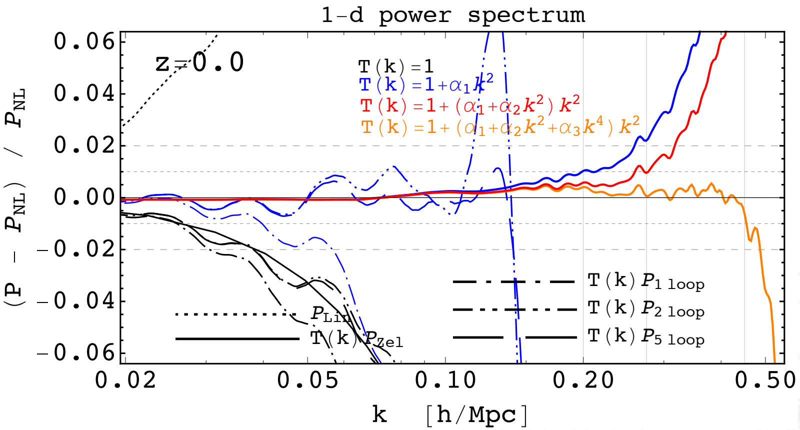
Clustering in 1D - collapsing shells

1D case studied recently in: [McQuinn&White, '15, Vlah et al, '15]



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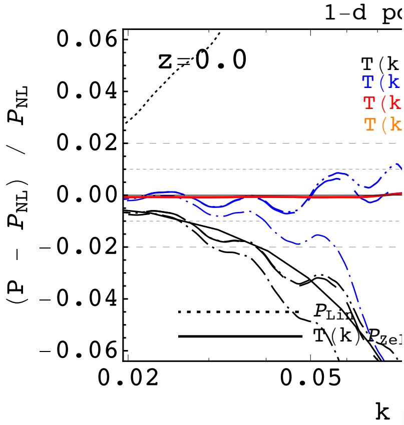
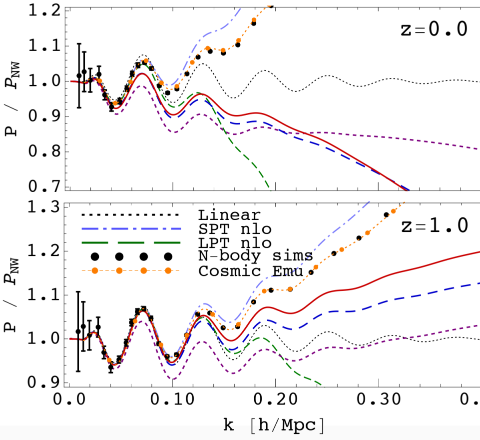




Path integrals and going beyond shell crossing

- as we saw the Lagrangian framework includes shell crossing
- Lagrangian dynamics can be compactly written using

$$\boldsymbol{L}_0 \phi + \boldsymbol{\Delta}_0(\phi) = \boldsymbol{\epsilon},$$

where:

$$\phi \equiv (\psi, \upsilon)\,, \quad [\boldsymbol{L}_0]_{i_2 i_1} = \begin{pmatrix} \frac{\partial}{\partial \eta_2} & -1 \\ -\frac{3}{2} & \frac{\partial}{\partial \eta_2} + \frac{1}{2} \end{pmatrix}, \quad \boldsymbol{\Delta}_0(\phi) = \frac{3}{2}\left(0, \partial_{\boldsymbol{x}} \partial_{\boldsymbol{x}}^{-2} \delta + \psi\right).$$

Statistics of interest given by generating function

$$Z(\boldsymbol{j}) \equiv \int d\boldsymbol{\epsilon}\; e^{-\frac{1}{2}\boldsymbol{\epsilon} \boldsymbol{N}^{-1} \boldsymbol{\epsilon} + \boldsymbol{j}\phi[\boldsymbol{\epsilon}]} \;\text{ and }\; \langle \phi_{i_1} \phi_{i_2} \rangle = \frac{\partial^2}{\partial j_{i_1} \partial j_{i_2}} Z(\boldsymbol{j}) \Big|_{\boldsymbol{j}=0},$$

which after the variable change becomes

$$Z(\boldsymbol{j}) \equiv \int d\phi\; e^{-S(\phi) + \boldsymbol{j}\phi},$$

with $S(\phi) = 1/2\,[\boldsymbol{L}_0 \phi + \boldsymbol{\Delta}_0(\phi)]\, \boldsymbol{N}^{-1}\, [\boldsymbol{L}_0 \phi + \boldsymbol{\Delta}_0(\phi)]\,.$ [McDonald&Vlah, ’17]

Path integrals and going beyond shell crossing

We can organize our perturbation theory as:

$$S = S_g + S_p, \quad \text{where then we do} \quad \exp(-S) = \exp(-S_g)(1 - S_p + S_p^2/2 + ...)$$

where we can choose what the "Gaussian part" will be, i.e.

$$S_g \equiv 1/2\chi \boldsymbol{N} \chi + i\chi[W^{-1}\boldsymbol{L}_0]\phi \equiv 1/2\chi \boldsymbol{N}\chi + i\chi \boldsymbol{L}\phi$$

and

$$S_p \equiv i\chi \boldsymbol{\Delta}_0(\phi) + i\chi[(1 - W^{-1})\boldsymbol{L}_0]\phi \equiv i\chi \boldsymbol{\Delta}(\phi),$$

where χ is the auxiliary field from the Hubbard-Stratonovich transformation.

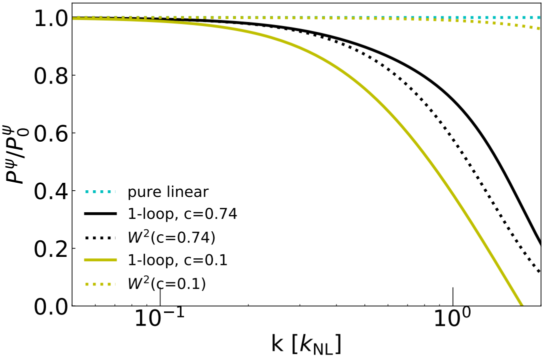
Perturbation theory result : $Z_0(\boldsymbol{j}) = Z_0(\boldsymbol{j}) + Z_1(\boldsymbol{j}) + \ldots$

Leading order result: truncate Zel'dovich dynamics!!!

$$Z_0 = e^{\frac{1}{2}\boldsymbol{j}.\boldsymbol{C}.\boldsymbol{j}} \quad \text{and} \quad P(k) = \int d^3q \, e^{i\boldsymbol{q}\cdot\boldsymbol{k}} e^{-\frac{1}{2}k_i k_j A_{ij}^W}$$

higher orders more complicated, build in renormalization! [McDonald&Vlah, '17]

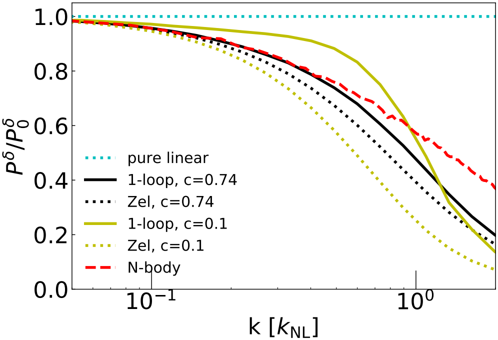
Path integral approach; going beyond shell crossing



Significance and connection EFT formalism:

- no need of EFT free parameters, i.e. counter terms are predicted
- CMB lensing: direct information on baryonic and neutrinos physics
- reduction of degeneracy in galaxy bias coefficients
- possible connection to the EFT formalism by matching the $k \to 0$ limit

Path integrals and going beyond shell crossing



Significance and connection EFT formalism: - goal is 3D!

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Summary

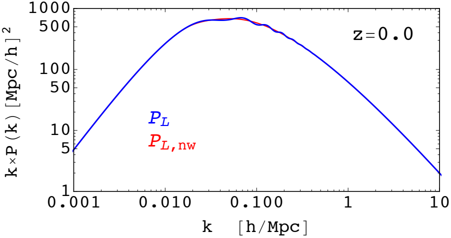
Key points:

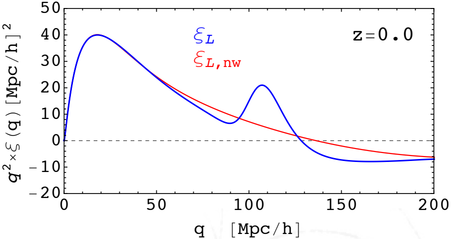
- Shell crossing can be consistently added to the perturbative Lagrangian scheme.
- Perturbative Lagrangian approach is a viable and UV-complete approach in the study of structure formation.
- Applications in LSS statistics, amongst which the most direct is weak lensing.
- EFT framework offers further simplifications on largest scales & Lagrangian setting is a natural for the study of BAO effects in LSS statistics..

Baryon acoustic oscillation

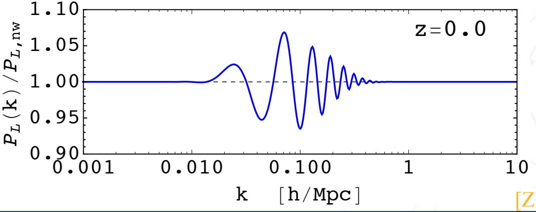
Linear power spectrum, correlation function & BAO

Linear power spectrum $P_{\rm L}$: obtained form Boltzmann codes (CAMB, Class). Formally we can divide it into smooth part $P_{\rm L,nw}$ and wiggle part $P_{\rm L,w}$ so that: $$P_{\rm L} = P_{\rm L,nw} + P_{\rm L,w}$$





Wiggle power spectrum: $P_{\rm L,w} \quad \rightarrow \quad \sigma_n = \int_a q^{-n} P_{\rm L,w}(q) = 0$ for $n = \{0, 2\}$.



[Z.V. et al, '14 & '15]

Redshift space distortions (RSD)

Power spectrum in RSD

$$P_s(\boldsymbol{k}) = \int d^3r\, e^{i\boldsymbol{k}\cdot\boldsymbol{r}} \mathcal{M}\left(k_{\parallel}\hat{z}, \boldsymbol{r}\right) = \int d^3r\, e^{i\boldsymbol{k}\cdot\boldsymbol{r}} \mathcal{M}\left(\boldsymbol{J} = \boldsymbol{k}.\boldsymbol{R}, \boldsymbol{r}\right),$$

where $\boldsymbol{r} = \boldsymbol{x}_2 - \boldsymbol{x}_1$ and

$$1 + \mathcal{M}\left(\boldsymbol{J}, \boldsymbol{r}\right) = \left\langle \left(1 + \delta(\boldsymbol{x})\right)\left(1 + \delta(\boldsymbol{x}')\right) e^{i\boldsymbol{J}\cdot\Delta\boldsymbol{u}} \right\rangle,$$

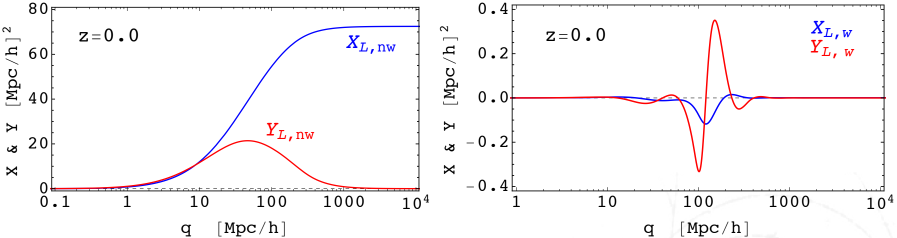
is the generating funciton for the moments of pairwise velocity, where $\Delta\boldsymbol{u} = \boldsymbol{u}(\boldsymbol{x}_2) - \boldsymbol{u}(\boldsymbol{x}_1)$. RSD families :

- Streaming approach: cumulant expansion theorem, $\mathcal{M}$ to $\mathcal{Z}$. [Peebles, White, ...]
- Distribution function approach: $\mathcal{M}$ is expanded moments that can be individually evaluated. [SPT, Seljak&McDonald, ...]
- Scoccimarro approach: Cumulant expansion theorem on individual contributions. [Scoccimarro, TNS ...]
- Direct Lagrangian approach: $\mathcal{M}$ is transformed into Lagrangian coordinates [Matsubara, White ...]

Resummation of IR modes: simple scheme

Separating the wiggle and non-wiggle part $A^{ij}_{\rm L}(\boldsymbol{q}) = A^{ij}_{\rm L,nw}(\boldsymbol{q}) + A^{ij}_{\rm L,w}(\boldsymbol{q})$;

$$P = P_{\rm nw} + \int_q e^{i\boldsymbol{k}\cdot\boldsymbol{q} - (1/2)k_ik_jA^{ij}_{\rm L,nw}} \left[-\frac{k_ik_j}{2} A^{ij}_{\rm L,w} + \cdots \right] \simeq P_{\rm nw} + e^{-k^2\Sigma^2} P_{\rm L,w} + \ldots$$



IR-SPT resummation model with $\Sigma^2 = \int \frac{dp}{3\pi^2}(1 - j_0(q_{\rm max}k))P_L(p)$:

$$P_{\rm dm}(k) = P_{\rm nw,L}(k) + P_{\rm nw,SPT,1-loop}(k) + \alpha_{\rm SPT,1-loop,IR}(k)k^2 P_{\rm nw,L}(k)$$
$$+ e^{-k^2\Sigma^2}\Big(\Delta P_{\rm w,SPT,1-loop}(k) + \left(1 + (\alpha_{\rm SPT,1-loop,IR} + \Sigma^2)k^2\right) \Delta P_{\rm w,L}(k) \Big).$$

Alternative derivation in: [Baldauf et al, 2015]

Wiggle residuals in our schemes: BAO