GENERAL RELATIVITY FROM NON-EQUILIBRIUM THERMODYNAMICS OF QUANTUM INFORMATION

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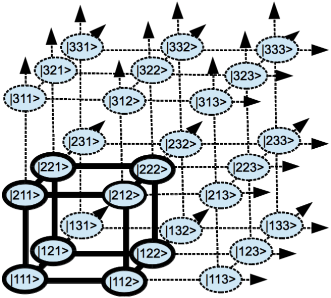


based on arxiv:1603.07982, 1706.02229, 1707.05004 and “shoulders of giants”

Main Idea:

General Relativity emerges from Quantum Mechanics with many d.o.f.

$$\boxed{GR = \lim_{N\to\infty} QM}$$

(just like Thermodynamics emerges from Classical Mechanics with many d.o.f.)

OUTLINE:

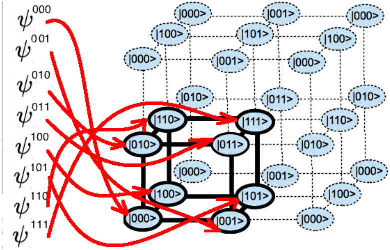
- I. **SPATIAL METRIC** from **QUANTUM INFORMATION**
  - define statistical ensembles using information as constraint
  - derive a spatially covariant description of quantum information
- II. **SPACE-TIME METRIC** from **QUANTUM COMPUTATION**
  - define a dual theory description of computational complexities
  - derive a space-time covariant description of quantum comp.
- III. **GRAVITY** from **NON-EQUILIBRIUM THERMODYNAMICS**
  - define thermodynamic variables in the limit of local equilibrium
  - derive an equation for a non-equilibrium entropy production

WAVE FUNCTION FOR QUBITS, QUTRITS AND QUDITS

- Consider a vector in Hilbert space with preferred t.p. factorization

$$|\psi\rangle = \sum_{X=0}^{2^D-1} \psi^X |X\rangle \equiv \sum_{X=0}^{2^D-1} \psi^X \bigotimes_{i=1}^{D} |X_i\rangle$$

  where $X^i \in \{0, 1\}$ for qubits, $X^i \in \{0, 1, 2\}$ for qutrits, etc.
- Then components ψ^X's define a wave-function representation of $|\psi\rangle$
- For qubits it is useful to think of ψ^X as a function on D dim. lattice



- For qutrits the periodicity of the lattice is 3 (or in general k for *qudits*).
- In all cases it is convenient to replace the discrete X^i with continuous x^i, differences Δ with differentiations ∂_i, sums $\sum$ with integrals $\int$, etc.

Statistical dependence or entanglement

- Question: What is a good measure of entanglement of variables i and j?
- Related Question: What is a good measure of statistical dependence between i and j described by distribution $P(\vec{x}) \equiv \psi^*(\vec{x})\psi(\vec{x})$?
- For statistically dependent random variables we know that

$$P(\vec{x}) \neq P(x^1)P(x^2)...P(x^D) \tag{1}$$

or

$$\log(P(\vec{x})) \neq \log(P(x^1)) + \log(P(x^2)) + ... + \log(P(x^D)).$$

- Then if we expand the left hand side around a global maxima

$$\log(P(\vec{x})) \approx \log(P(\vec{y})) - \frac{1}{2}(x^i - y^i)\Sigma_{ij}(x^i - y^i) + ... \tag{2}$$

then a good measure of statistical dependence is

$$\Sigma_{ij} \equiv -2\left[\frac{\partial^2}{\partial x^i \partial x^j}\log(P(\vec{x}))\right]_{\vec{x}=\vec{y}} \tag{3}$$

FISHER INFORMATION MATRIX

- More generally the Hessian matrix (which is a local quantity)

$$\Sigma_{ij}(\vec{x}) \equiv -2 \frac{\partial^2}{\partial x^i \partial x^j} \log(P(\vec{x})) \tag{4}$$

allows us to approximate the distribution as a sum of Gaussians

$$P(\vec{x}) \propto \sum_m \exp\left(-\frac{1}{2}\left(x^i - y^i_m\right) \Sigma_{ij}(\vec{y}_m) \left(x^j - y^j_m\right)\right) \tag{5}$$

- To obtain a measure of statistical dependence between i's and j's qubits (or subsystems) the quantity must be summed (or integrated) over different values with perhaps different weights. One useful choice is

$$A_{ij} \equiv \frac{1}{4} \int d^N x \; P(\vec{x}) \Sigma_{ij}(\vec{x}) = -\frac{1}{4} \int d^N x \; P(\vec{x}) \frac{\partial^2}{\partial x^i \partial x^j} \log(P(\vec{x}))$$

where the factor of $1/4$ is introduced for future convenience.

- It can be shown that A_{ij} is the so-called Fisher information matrix obtained from shifts of coordinates $\vec{x}$.

FUBINI-STUDY METRIC

- For periodic/vanishing boundary conditions the matrix reduces to

$$A_{ij} = \int d^N x \;\; \frac{\partial \sqrt{P(\vec{x})}}{\partial x^i} \frac{\partial \sqrt{P(\vec{x})}}{\partial x^j} \tag{6}$$

- Then one can try to define information matrix

$$A_{ij} = \int d^N x \frac{\partial |\psi(\vec{x})|}{\partial x^i} \frac{\partial |\psi(\vec{x})|}{\partial x^j} \tag{7}$$

  but it does not measure well certain quantum entanglements.
- A better object is a straightforward generalization, i.e.

$$A_{ij} = \int d^N x \frac{\partial \psi^*(\vec{x})}{\partial x^i} \frac{\partial \psi(\vec{x})}{\partial x^j}. \tag{8}$$

  which is closely related to the so-called Fubini-Study metric.
- We will refer to A_{ij} (for both statistical and quantum systems) as information matrix.

INFOTON FIELD OR UNNORMALIZED WAVE FUNCTION

- Consider a *dual* field $\varphi(\vec{x})$ (we shall call *infoton*) in the sample/configuration space defined (for now) as

$$\varphi(\vec{x}) \propto \psi(\vec{x}) \tag{9}$$

  and then the information matrix is

$$A_{ij} \propto \int d^N x \frac{\partial \varphi^*(\vec{x})}{\partial x^i} \frac{\partial \varphi(\vec{x})}{\partial x^j}. \tag{10}$$

- Next step is to define distributions over $|\psi\rangle$ and so one can think of this as "2^{nd} quantization", i.e. prob. distribution over prob. amplitudes.
- More precisely, we shall construct statistical ensembles $P[\varphi]$ that would define probabilities of pure states

$$P[|\psi\rangle] = \int_{\varphi \propto \psi} \mathcal{D}\varphi \mathcal{D}\varphi^* P[\varphi] \tag{11}$$

- So we are now dealing with mixed states, but instead of density matrices we will work with statistical ensembles described by $P[\varphi]$.

STATISTICAL ENSEMBLE OVER WAVE FUNCTIONS

- What we really want is machinery to define distributions over "microscopic" quantum states subject to "macroscopic" constraints.
- For example, we might want to define a statistical ensemble over infoton φ such that the (expected) information matrix is

$$\langle A_{ij} \rangle = \bar{A}_{ij} \tag{12}$$

  for a given Hermitian matrix $\bar{A}_{ij}$.
- Statistical ensembles are usually defined using partition functions

$$\mathcal{Z} = \int \mathcal{D}\varphi \mathcal{D}\varphi^* \quad \exp\left(-\mathcal{S}[\varphi]\right) \tag{13}$$

- If the theory is local then the (Euclidean) action $\mathcal{S}$ is given by an integral over a local function $\mathcal{L}$ of fields and its derivatives, e.g.

$$\mathcal{S}[\varphi] = \int d^N x \quad \left(g^{ij} \frac{\partial \varphi^*(\vec{x})}{\partial x^i} \frac{\partial \varphi(\vec{x})}{\partial x^j} + \lambda \varphi^*(\vec{x}) \varphi(\vec{x}) \right) \tag{14}$$

  where the values of g^{ij} do not depend on $\vec{x}$ and the "mass-squared" constant λ must be chosen so that the infoton field φ (which is proportional to wave-functions ψ) is on average normalized.

Information tensor

- For more general ensembles (e.g. over sums of Gaussians) g_{ij} can depend on coordinates $\vec{x}$ and thus to play the role of a metric tensor.
- To make the expression covariant we will also add $\sqrt{|g|}$ to the volume integral and replace partial derivatives with covariant derivatives, i.e.

$$\mathcal{S} = \int d^D x \sqrt{|g|} \left(g^{ij}(\vec{x}) \nabla_i \varphi^*(\vec{x}) \nabla_j \varphi(\vec{x}) + \lambda(\vec{x}) \varphi^*(\vec{x}) \varphi(\vec{x}) \right) \tag{15}$$

- Then, we can define a covariant *information tensor* as

$$\mathcal{A}_{ij}(\vec{x}) \equiv \nabla_i \varphi^*(\vec{x}) \nabla_j \varphi(\vec{x}). \tag{16}$$

and a covariant *probability scalar*

$$\mathcal{N}(\vec{x}) \equiv \varphi^*(\vec{x}) \varphi(\vec{x}). \tag{17}$$

- Note that both $\mathcal{A}_{ij}(\vec{x})$ and $\mathcal{N}(\vec{x})$ are local quantities in configuration space.

STRESS TENSOR

- These two quantities can be used to express the stress tensor

$$T_{ij} = \nabla_{(i}\varphi^* \nabla_{j)}\varphi + g_{ij}\left(g^{kl}\nabla_k\varphi^*\nabla_l\varphi + \lambda\varphi^*\varphi\right) \quad (18)$$

$$= 2\mathcal{A}_{(ij)} + g_{ij}\left(g^{kl}\mathcal{A}_{kl} + \lambda\mathcal{N}\right) \quad (19)$$

where $A_{(\mu\nu)} \equiv \frac{1}{2}(A_{\mu\nu} + A_{\mu\nu})$.

- Then for a given expected information tensor $\bar{\mathcal{A}}_{(ij)}$ and probability density $\bar{\mathcal{N}}$, the macroscopic parameters $g_{ij}(\vec{x})$ and $\lambda(\vec{x})$ are to be chosen such that

$$\langle\mathcal{N}\rangle = \bar{\mathcal{N}} \quad (20)$$

and

$$\langle T_{ij}\rangle = 2\bar{\mathcal{A}}_{(ij)} + g_{ij}\left(g^{kl}\bar{\mathcal{A}}_{kl} + \lambda\bar{\mathcal{N}}\right). \quad (21)$$

- Note that the corresponding free energy depends on only "macroscopic" parameters $g_{ij}(\vec{x})$ and $\lambda(\vec{x})$ (as it should) , i.e.

$$\mathcal{F}[g_{ij},\lambda] \equiv -\log(\mathcal{Z}[g_{ij},\lambda]) = -\log\left(\int \mathcal{D}\varphi\mathcal{D}\varphi^* \;\; \exp\left(-\mathcal{S}[\varphi, g_{ij},\lambda]\right)\right)$$

ACTION-COMPLEXITY CONJECTURE

- Once again, consider a quantum system of D qubits.
- All states are points on 2^D dim. unit sphere separated by distance $\mathcal{O}(1)$ if you were allowed to move along geodesics.
- Now imagine that you are only allowed to move in $\mathcal{O}(D^2)$ orthogonal directions out of $\mathcal{O}(2^D)$.
- More precisely, at any point you are allowed to only apply $\mathcal{O}(D)$ of one-qubit gates or $\mathcal{O}(D^2)$ of two- qubit gates.
- This is like playing a very high-dimensional maze with many walls and very few pathways.
- Question: What is the shortest distance (also known as computational complexity) connecting an arbitrary pair of points on the unit sphere?
- **Action-complexity conjecture:** *There exist a dual field theory whose action equals to computational complexity of the shortest quantum circuit connecting any pair of states,*

$$\boxed{\mathcal{C}(|\psi_{out}\rangle, |\psi_{in}\rangle) = S[\varphi]} \tag{22}$$

where φ is a collective notation for all degrees of freedom.

DUAL THEORIES

- Consider dual theories with d.o.f. represented by infoton field, i.e.

$$\mathcal{C}(|\psi_{\rm out}\rangle, |\psi_{\rm in}\rangle) = \int_0^T dt\, \mathcal{L}\left(\varphi^X(t), \frac{d\varphi^X(t)}{dt}\right). \tag{23}$$

- We set initial/final conditions

$$\begin{aligned} |\psi_{\rm in}\rangle &= \sum_X \psi^X_{\rm in}|X\rangle \propto \sum_X \varphi^X(0)|X\rangle \\ |\psi_{\rm out}\rangle &= \sum_X \psi^X_{\rm out}|X\rangle \propto \sum_X \varphi^X(T)|X\rangle, \end{aligned} \tag{24}$$

  and demand that the (yet to be discovered) dual theory satisfies the following symmetries/constrains:
  - States remain (approximately) normalized, i.e.

$$\sum_X \varphi_X(t)\varphi^X(t) \approx 1 \tag{25}$$

  - Theory is invariant under permutations of bits, i.e. interactions depend only on Hamming distance $h(I, J)$ between strings of bits I and J, e.g. $h(0,7) = 3, h(2,6) = 1$.

DUAL LAGRANGIAN

- Then the leading terms of the Lagrangian can be written as

$$\mathcal{L}(\varphi_X, \dot{\varphi}_X) = \alpha \sum_X \dot{\varphi}_X \dot{\varphi}^X + \lambda \sum_X \varphi_X \varphi^X + \sum_{X,Y} f(h(X,Y)) \varphi_X \varphi^Y + \ldots \quad (26)$$

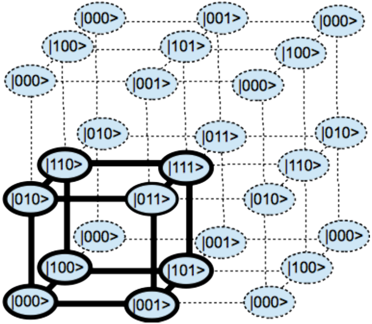
  where $f(h(X,Y))$ is some function of Hamming distance $h(X,Y)$.
- And we arrive at a path integral expression

$$\mathcal{Z}(|\psi_{\text{out}}\rangle, |\psi_{\text{in}}\rangle) = \int_{|\psi_{\text{in}}\rangle = \varphi^X(0)|X\rangle}^{|\psi_{\text{out}}\rangle = \varphi^X(T)|X\rangle} d^{2^D}\varphi^* d^{2^D}\varphi \;\; e^{i \int_0^T dt \left(\alpha \dot{\varphi}_X \dot{\varphi}^X + \lambda \varphi_X \varphi^X + f^X{}_Y \varphi_X \varphi^Y \right)}$$

  where the Einstein summation convention is assumed.
- Note that:
  - $f^X_Y \equiv f(h(X,Y))$ in computational basis and to transform to other basis it must be treated as a rank $(1,1)$ tensor under $U\left(2^D\right)$.
  - Roughly speaking, we expect the function $f(h)$ to quickly vanish for $h > 2$, i.e. penalizing more than two q-bit gates.
  - It will be convenient to denote the three relevant constants as $\beta \equiv f(0)$, $\gamma \equiv f(1)$ and $\delta \equiv f(2)$ (in addition to α defined above).

Lattice Field Theory

The path integral can also be written as a quantum field theory path integral on D dimensional torus with only 2^D lattice points



Then in a continuum limit the path integral would be given by

$$\mathcal{Z}(|\psi_{\text{out}}\rangle, |\psi_{\text{in}}\rangle) = \int D\varphi^* D\varphi \;\; \exp\left(i \int_0^T dt \int d^D x \tilde{\mathcal{L}}(\varphi(x), \partial_\mu \varphi(x)) \right) \tag{27}$$

where tildes denote spacetime quantities and μ labels $D+1$ dimentions.

Lagrangian Density

- After some math we arrive at Klein-Gordon theory

$$\tilde{\mathcal{L}}(\varphi(\vec{x}), \partial_\mu \varphi(\vec{x}) = \tilde{g}^{\mu\nu} \partial_\mu \varphi^*(\vec{x}) \partial_\nu \varphi(\vec{x}) - m^2 \varphi^*(\vec{x}) \varphi(\vec{x}) \tag{28}$$

where the "mass'-squared'

$$m^2 \equiv -\left(\beta + D\gamma + \frac{D(D-1)}{2}\delta\right) l^{-D-1} \tag{29}$$

and the inverse "metric" is

$$\tilde{g}^{00} \equiv \alpha l^{1-D} \tag{30}$$

$$\tilde{g}^{ii} \equiv -\frac{1}{2}\left(\gamma + (D-1)\delta\right) l^{1-D} \tag{31}$$

$$\tilde{g}^{ij} \equiv \frac{1}{2}\delta l^{1-D}, \tag{32}$$

where $i, j \in \{1, ..., D\}$ and $i \neq j$.

- For the path integral to be finite we need the mass squared to be positive and all but one eigenvalues of the metric to be negative, e.g.

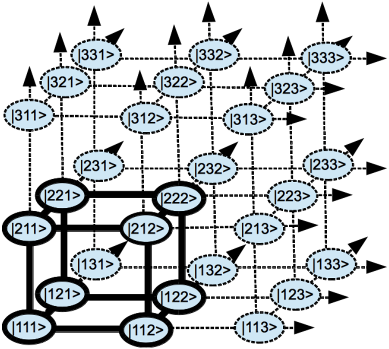
$$\alpha > 0; \qquad \gamma > 0; \qquad \delta > -\frac{\gamma}{D}; \qquad \beta < -D\gamma - \frac{D(D-1)}{2}\delta.$$

LOCAL COMPUTATIONS

- More generally the infoton field theories defined by Lagrangian

$$\tilde{\mathcal{L}}(\varphi(\vec{x}), \partial_\mu \varphi(\vec{x}) = \tilde{g}^{\mu\nu}(\vec{x})\partial_\mu \varphi^*(\vec{x})\partial_\nu \varphi(\vec{x}) - \lambda(\vec{x})\varphi^*(\vec{x})\varphi(\vec{x}) \qquad (33)$$

can give a dual description to the theories of computation of qudits



- Computations in each hypercube are described by one/two qubit gates but these computations share each other's memory on boundaries.
- The qubit computers associated with each hypercube run separately, but exchange information and thus the results of computations.

INFORMATION-COMPUTATION TENSOR

- Now that we have a fully covariant action we can look at a covariant generalization of the information tensor, i.e.

$$\mathcal{A}_{\mu\nu} \equiv \nabla_\nu \varphi^* \nabla_\mu \varphi. \tag{34}$$

- The tensor $\mathcal{A}_{\mu\nu}$ is related to the the energy momentum tensor

$$T_{\mu\nu} = -2\mathcal{A}_{(\mu\nu)} + \tilde{g}_{\mu\nu} \left(\tilde{g}^{\alpha\beta} \mathcal{A}_{\alpha\beta} \right) \tag{35}$$

  which implies that it should satisfy the following equation

$$\nabla^\nu \left(\mathcal{A}_{(\mu\nu)} - \frac{1}{2} \mathcal{A} \tilde{g}_{\mu\nu} \right) = 0 \tag{36}$$

- Space-space components, i.e. ij, provide a good measure of informational dependence between (k-local and x-local) subsystems.
- Space-time component, i.e. $0i$, measures the amount that a given qubit i is contributing to the computations (zero if $\nabla_i \varphi$ or $\nabla_0 \varphi$ vanishes)
- Thus it is useful to think of 00 as a "density of computations" and of $0i$ as a "flux of computations", which together with ij form a generally covariant information-computation tensor $\mathcal{A}_{\mu\nu}$ (defined for a network of parallel computers or on a D dimensional dual space-time).

EMERGENT GRAVITY

- Let us go back to "spatial" partition function

$$\mathcal{Z} = \int \mathcal{D}\varphi\mathcal{D}\varphi^* \quad \exp\left(-\mathcal{S}[\varphi]\right) \tag{37}$$

described by the action with only spatial covariance

$$\mathcal{S} = \int d^D x \sqrt{|g|} \left(g^{ij}(\vec{x})\nabla_i\varphi^*(\vec{x})\nabla_j\varphi(\vec{x}) + \lambda(\vec{x})\varphi^*(\vec{x})\varphi(\vec{x})\right) \tag{38}$$

- The corresponding free energy can be expanded as

$$\begin{aligned} \mathcal{F}[g_{ij}, \lambda, \hbar] \quad &\equiv \quad -\hbar \log(\mathcal{Z}[g_{ij}, \lambda, \hbar]) \approx \\ &\approx \quad \int d^D x \sqrt{|g|} \left(g^{ij} \langle \mathcal{A}_{ij} \rangle + \lambda \langle \mathcal{N} \rangle\right) - S. \end{aligned} \tag{39}$$

- If we turn on a random, but unitary dynamics of wave functions then the infoton field should also evolve accordingly.
- But if we want to keep the form of the ensemble to remain the same, then the macroscopic parameters $g_{ij}(\vec{x})$ and $\lambda(\vec{x})$ must evolve as well.
- And if so, can one describe the emergent dynamics of $g_{ij}(\vec{x})$ and $\lambda(\vec{x})$ using dynamical equations, e.g. Einstein equations, corrections?

Thermodynamic Variables

- We define (local) thermodynamic variables

$$
\begin{aligned}
\text{information tensor} \quad \mathfrak{a}_{ij} &\equiv \langle \mathcal{A}_{ij} \rangle \\
\text{metric tensor} \quad \mathfrak{g}^{ij} &\equiv g^{ij} \\
\text{particle number scalar} \quad \mathfrak{n} &\equiv \langle \mathcal{N} \rangle \\
\text{chemical potential scalar} \quad \mathfrak{m} &\equiv \lambda \\
\text{entropy scalar} \quad \mathfrak{s} &\equiv \frac{S}{\int d^D x \sqrt{|g|}} \\
\text{free energy scalar} \quad \mathfrak{f} &\equiv \left(\mathfrak{g}^{ij} \mathfrak{a}_{ij} + \mathfrak{m}\mathfrak{n} \right) - \mathfrak{s} \qquad (40)
\end{aligned}
$$

- Equation (40) together with the First Law of thermodynamics

$$0 = \mathfrak{m} d\mathfrak{n} + \mathfrak{g}^{ij} d\mathfrak{a}_{ij} - d\mathfrak{s} \qquad (41)$$

gives us the Gibbs-Duhem Equation

$$d\mathfrak{f} = \mathfrak{n} d\mathfrak{m} + \mathfrak{a}_{ij} d\mathfrak{g}^{ij}. \qquad (42)$$

Onsager Tensor

- Non-equilibrium entropy production (which is to be extremized)

$$\mathcal{S}[\mathfrak{g}, \varphi] \equiv \int d^{D+1}x \sqrt{|\mathfrak{g}|} \left(\mathcal{L}(\varphi, \mathfrak{g}) - \frac{1}{2\kappa}\mathfrak{R}(\mathfrak{g}) + \Lambda \right) \tag{43}$$

- By following the standard prescription we expand entropy production

$$\frac{1}{2\kappa}\mathfrak{R} = \mathfrak{g}_{\alpha\beta,\mu}\mathfrak{J}^{\mu\alpha\beta} \tag{44}$$

where the generalized forces are taken to be

$$\mathfrak{g}_{\alpha\beta,\mu} \equiv \frac{\partial \mathfrak{g}_{\alpha\beta}}{\partial x^{\nu}} \tag{45}$$

and fluxes are expanded to the linear order in generalized forces

$$\mathfrak{J}^{\mu\alpha\beta} = \mathfrak{L}^{\mu\nu\ \alpha\beta\ \gamma\delta}\mathfrak{g}_{\gamma\delta,\nu}. \tag{46}$$

and thus

$$\frac{1}{2\kappa}\mathfrak{R} = \mathfrak{L}^{\mu\nu\ \alpha\beta\ \gamma\delta}\mathfrak{g}_{\alpha\beta,\mu}\mathfrak{g}_{\gamma\delta,\nu}. \tag{47}$$

where $\mathfrak{L}^{\mu\nu\ \alpha\beta\ \gamma\delta}$ is the Onsager tensor.

ONSAGER RECIPROCITY RELATIONS

- Onsager relations force us to only consider Onsager tensors that are symmetric under exchange $(\mu, \alpha, \beta) \leftrightarrow (\nu, \gamma, \delta)$, i.e.

$$\mathfrak{L}^{\mu\nu\ \alpha\beta\ \gamma\delta} = \mathfrak{L}^{\nu\mu\ \beta\alpha\ \delta\gamma} \tag{48}$$

- To illustrate the procedure, let us first consider a tensor

$$\mathfrak{L}^{\mu\nu\ \alpha\beta\ \gamma\delta} = \frac{1}{2\kappa}\left(\mathfrak{g}^{\alpha\nu}\mathfrak{g}^{\beta\delta}\mathfrak{g}^{\mu\gamma} + \mathfrak{g}^{\alpha\gamma}\mathfrak{g}^{\beta\nu}\mathfrak{g}^{\mu\delta} - \mathfrak{g}^{\alpha\gamma}\mathfrak{g}^{\beta\delta}\mathfrak{g}^{\mu\nu}\right) \tag{49}$$

for which the flux can be rewritten as

$$\mathfrak{J}^{\mu\alpha\beta} = \frac{1}{\kappa}\mathfrak{g}^{\alpha\gamma}\mathfrak{g}^{\beta\delta}\Gamma^{\mu}{}_{\gamma\delta} \tag{50}$$

where $\Gamma^{\mu}{}_{\gamma\delta}$ are Christoffel symbols and κ is some constant.

- If we inset it back into the entropy functional we get

$$\int d^{D+1}x\sqrt{|\mathfrak{g}|}\frac{1}{2\kappa}\mathfrak{R} = \frac{1}{\kappa}\int d^{D+1}x\sqrt{|\mathfrak{g}|}\mathfrak{g}^{\mu\nu}\left(\Gamma^{\alpha}{}_{\mu\nu,\alpha} + \Gamma^{\beta}{}_{\mu\nu}\Gamma^{\alpha}{}_{\alpha\beta}\right) \tag{51}$$

- Note quite what one needs for GR to emerge.

GENERAL RELATIVITY

- The overall space of Onsager tensors is pretty large, but it turns out that a very simple choice leads to general relativity, i.e.

$$L^{\mu\nu\ \alpha\beta\ \gamma\delta} = \frac{1}{8\kappa}\left(\mathfrak{g}^{\alpha\nu}\mathfrak{g}^{\beta\delta}\mathfrak{g}^{\mu\gamma} + \mathfrak{g}^{\alpha\gamma}\mathfrak{g}^{\beta\nu}\mathfrak{g}^{\mu\delta} - \mathfrak{g}^{\alpha\gamma}\mathfrak{g}^{\beta\delta}\mathfrak{g}^{\mu\nu} - \mathfrak{g}^{\alpha\beta}\mathfrak{g}^{\gamma\delta}\mathfrak{g}^{\mu\nu}\right).$$

- It has a lot more symmetries and as a result of these symmetries we are led to a fully covariant theory of general relativity

$$\int d^{D+1}x\sqrt{|\mathfrak{g}|}\frac{1}{2\kappa}\mathfrak{R} = \frac{1}{\kappa}\int d^{D+1}x\sqrt{|\mathfrak{g}|}\mathfrak{g}^{\mu\nu}\left(\Gamma^{\alpha}{}_{\nu[\mu,\alpha]} + \Gamma^{\beta}{}_{\nu[\mu}\Gamma^{\alpha}{}_{\alpha]\beta}\right)$$

- By varying the full action with respect to metric (what is equivalent to minimization of entropy production) we arrive at the Einstein equations

$$\mathfrak{R}_{\mu\nu} - \frac{1}{2}\mathfrak{R}\mathfrak{g}_{\mu\nu} + \Lambda\mathfrak{g}_{\mu\nu} = \kappa\langle T_{\mu\nu}\rangle \qquad (52)$$

  where the Ricci tensor is

$$\mathfrak{R}_{\mu\nu} \equiv 2\left(\Gamma^{\alpha}{}_{\nu[\mu,\alpha]} + \Gamma^{\beta}{}_{\nu[\mu}\Gamma^{\alpha}{}_{\alpha]\beta}\right) \qquad (53)$$

- Of course, this result is expected to break down far away from equilibrium (dark matter?) What about inflation and dark energy?

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