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Boundary effects for magnetized quantum matter in particle and astroparticle physics

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Outline

- Confining boundary condition for quantized spinor matter.
- Impact of magnetized matter on the Casimir effect.
- Hot dense magnetized matter in particle and astroparticle physics.

Confining boundary condition

A quest for boundary conditions ensuring the confinement of the quantized spinor matter was initiated in the context of a model description of hadrons as composite systems with their internal structure being associated with quark-gluon constituents (A.Chodos, R.L.Jaffe, K.Johnson, C.B.Thorn and V.Weisskopf, 1974). If an hadron is an extended object occupying spatial region Ω bounded by surface $\partial\Omega$, then the condition that the quark matter field be confined inside the hadron is formulated as

$$\boldsymbol{n} \cdot \boldsymbol{J}(\mathbf{r})|_{\mathbf{r}\in\partial\Omega} = 0,$$

where $\boldsymbol{n}$ is the unit normal to the boundary surface, and $\boldsymbol{J}(\mathbf{r}) = \psi^\dagger(\mathbf{r})\boldsymbol{\alpha}\psi(\mathbf{r})$ with $\psi(\mathbf{r})$ ($\mathbf{r} \in \Omega$) being the quark matter field (α^1, α^2, α^3 and β are the generating elements of the Dirac-Clifford algebra); an appropriate condition is also formulated for the gluon matter field.

The concept of confined matter fields is quite familiar in the context of condensed matter physics: collective excitations (e.g., spin waves and phonons) exist only inside material objects and do not spread outside. Moreover, in the context of quantum electrodynamics, if one is interested in the effect of a classical background magnetic field on the vacuum of the quantized electron-positron matter, then the latter should be considered as confined to the spatial region between the sources of the magnetic field, as long as collective quasielectronic excitations inside a magnetized material differ from electronic excitations in the vacuum. It should be noted in this respect that the study of the effect of the background electromagnetic field on the vacuum of quantized charged matter has begun already eight decades ago (W.Heisenberg and H.Euler, 1936; V.S.Weisskopf, 1936).

However, the case of a background field filling the whole (infinite) space is hard to be regarded as realistic. The case of both the background and quantized fields confined to a bounded spatial region with boundaries serving as sources of the background field looks much more physically plausible, it can even be regarded as realizable in laboratory. Moreover, there is no way to detect the energy density that is induced in the vacuum in the first case, whereas the pressure from the vacuum onto the boundaries, resulting in the second case, is in principle detectable.

In view of the above, an issue of a choice of boundary conditions for the quantized matter fields gains a crucial significance, and condition for the current should be resolved to take the form of a boundary condition that is linear in $\psi(\mathbf{r})$. An immediate way of such a resolution is known as the MIT bag boundary condition (K.Johnson, 1975),

$$(I + \mathrm{i}\beta \boldsymbol{n} \cdot \boldsymbol{\alpha})\psi(\mathbf{r})|_{\mathbf{r}\in\partial\Omega} = 0,$$

but it is needless to say that this way is not a unique one.

The most general boundary condition is provided by the condition of the self-adjointness of the differential operator of one-particle energy in first-quantized theory (Dirac hamiltonian operator in the case of relativistic spinor matter). The self-adjointness of operators of physical observables is required by general principles of comprehensibility and mathematical consistency, see, e.g.,

J.von Neumann, *Mathematische Grundlagen der Quantummechanik* (Springer, Berlin, 1932).

To put it simply, a multiple action is well defined for a self-adjoint operator only, allowing for the construction of functions of the operator, such as resolvent, evolution, heat kernel and zeta-function operators, with further implications upon second quantization.

Self-adjointness of the Dirac hamiltonian operator

Defining a scalar product as $(\tilde{\chi}, \chi) = \int\limits_{\Omega} \mathrm{d}^3 r \, \tilde{\chi}^\dagger \chi$,

we get, using integration by parts,

$$(\tilde{\chi}, H\chi) = (H^\dagger \tilde{\chi}, \chi) - \mathrm{i} \int\limits_{\partial\Omega} \mathrm{d}\boldsymbol{\sigma} \cdot \mathbf{J}[\tilde{\chi}, \chi],$$

where

$$H = H^\dagger = -\mathrm{i}\boldsymbol{\alpha} \cdot \boldsymbol{\nabla} + \beta m$$

is the formal expression for the Dirac hamiltonian operator ($\boldsymbol{\nabla}$ is a covariant derivative) and

$$\mathbf{J}[\tilde{\chi}, \chi] = \tilde{\chi}^\dagger \boldsymbol{\alpha} \chi.$$

Operator H is Hermitian,

$$(\tilde{\chi}, H\chi) = (H^\dagger \tilde{\chi}, \chi),$$

if

$$\int\limits_{\partial\Omega} \mathrm{d}\boldsymbol{\sigma} \cdot \mathbf{J}[\tilde{\chi}, \chi] = 0 \qquad \text{or} \qquad \mathbf{n} \cdot \mathbf{J}[\tilde{\chi}, \chi]|_{\mathbf{r}\in\partial\Omega} = 0.$$

It is almost evident that the latter condition can be satisfied by imposing different boundary conditions for χ and $\tilde{\chi}$. But, a nontrivial task is to find a possibility that a boundary condition for $\tilde{\chi}$ is the same as that for χ; then operator H is self-adjoint. We impose the same boundary condition for χ and $\tilde{\chi}$ in the form

$$\chi|_{\mathbf{r}\in\partial\Omega} = K\chi|_{\mathbf{r}\in\partial\Omega}, \quad \tilde{\chi}|_{\mathbf{r}\in\partial\Omega} = K\tilde{\chi}|_{\mathbf{r}\in\partial\Omega},$$

where K is a matrix (element of the Clifford algebra) which is determined by two conditions:

$$K^2 = I$$

and

$$K^\dagger \boldsymbol{n}\cdot\boldsymbol{\alpha}\, K = -\boldsymbol{n}\cdot\boldsymbol{\alpha}.$$

It should be noted that, in addition, the following combination of χ and $\tilde{\chi}$ is also vanishing at the boundary:

$$\tilde{\chi}^\dagger \boldsymbol{n}\cdot\boldsymbol{\alpha}\, K\chi|_{\mathbf{r}\in\partial\Omega} = \tilde{\chi}^\dagger K^\dagger \boldsymbol{n}\cdot\boldsymbol{\alpha}\,\chi|_{\mathbf{r}\in\partial\Omega} = 0.$$

Explicit form is (Yu. A. S., 2015)

$$K = \left[\beta e^{i\varphi\gamma^5} \cos\theta + (\alpha^1 \cos\varsigma + \alpha^2 \sin\varsigma) \sin\theta\right] e^{i\tilde{\varphi}\boldsymbol{n}\cdot\boldsymbol{\alpha}},$$

where $\gamma^5 = i\alpha^1\alpha^2\alpha^3$, matrices α^1 and α^2 are chosen to obey condition

$$[\alpha^1, \boldsymbol{n}\cdot\boldsymbol{\alpha}]_+ = [\alpha^2, \boldsymbol{n}\cdot\boldsymbol{\alpha}]_+ = [\alpha^1, \alpha^2]_+ = 0,$$

and the boundary parameters are chosen to vary as

$$-\frac{\pi}{2} < \varphi \leq \frac{\pi}{2}, \quad -\frac{\pi}{2} \leq \tilde{\varphi} < \frac{\pi}{2}, \quad 0 \leq \theta < \pi, \quad 0 \leq \varsigma < 2\pi.$$

The MIT bag boundary condition (K.Johnson, 1975),

$$(I + i\beta\boldsymbol{n}\cdot\boldsymbol{\alpha})\chi(\mathbf{r})\,|_{\mathbf{r}\in\partial\Omega} = 0,$$

is obtained at $\varphi = \theta = 0$, $\tilde{\varphi} = -\pi/2$.

Quantum matter in extremal conditions:

- hot and dense
- in strong magnetic field

Physical systems in:

- relativistic heavy-ion collisions
- compact astrophysical objects (neutron stars and magnetars)
- the early universe
- novel materials (the Dirac and Weyl semimetals) Cd_3As_2, Na_3Bi, K_3Bi, Rb_3Bi, $TaAs$, $BaAuBi$, $BaCuBi$, $BaAgBi$, Bi_2Se_3, $TlBiSe_2$, ...

Ultrarelativistic (chiral) effects

$$|eB| >> m^2, \quad T >> m, \quad \mu >> m$$

chiral magnetic effect
(A. Vilenkin, 1980; K. Fukushima, D. E. Kharzeev, and H. J. Warringa, 2008):

$$\mathbf{J} = -\frac{e\mathbf{B}}{2\pi^2}\mu_5$$

chiral separation effect
(M. A. Metlitski and A. R. Zhitnitsky, 2005):

$$\mathbf{J}^5 = -\frac{e\mathbf{B}}{2\pi^2}\mu$$

in unbounded (infinite) medium

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in unbounded (infinite) medium

Role of boundaries?

Operator of quantized spinor field in a static background

$$\hat{\Psi}(\mathbf{r}, t) = \sum_{E_\lambda > 0} e^{-\mathrm{i}E_\lambda t} \langle \mathbf{r}|\lambda\rangle \, \hat{a}_\lambda + \sum_{E_\lambda < 0} e^{-\mathrm{i}E_\lambda t} \langle \mathbf{r}|\lambda\rangle \, \hat{b}^\dagger_\lambda, \qquad (1)$$

where $\hat{a}^\dagger_\lambda$ and $\hat{a}_\lambda$ ($\hat{b}^\dagger_\lambda$ and $\hat{b}_\lambda$) are the spinor particle (antiparticle) creation and destruction operators satisfying anticommutation relations,

$$\left[\hat{a}_\lambda, \, \hat{a}^\dagger_{\lambda'}\right]_+ = \left[\hat{b}_\lambda, \, \hat{b}^\dagger_{\lambda'}\right]_+ = \left\langle \lambda|\lambda' \right\rangle, \qquad (2)$$

and $\langle \mathbf{r}|\lambda\rangle$ is the solution to the stationary Dirac equation,

$$H \langle \mathbf{r}|\lambda\rangle = E_\lambda \langle \mathbf{r}|\lambda\rangle \,, \qquad (3)$$

H is the Dirac hamiltonian, λ is the set of parameters (quantum numbers) specifying a one-particle state, E_λ is the energy of the state. Ground state $|\mathtt{vac}>$ is defined by condition

$$\hat{a}_\lambda|\mathtt{vac}> = \hat{b}_\lambda|\mathtt{vac}> = 0. \qquad (4)$$

Operators of dynamical variables (physical observables) in second-quantized theory are defined as bilinears of the fermion field operator (1).

Fermion number operator

$$\hat{N} = \frac{1}{2}\int\limits_{\Omega} \mathrm{d}^3 r(\hat{\Psi}^\dagger\hat{\Psi} - \hat{\Psi}^T\hat{\Psi}^{\dagger T}) = \sum \left[\hat{a}^\dagger_\lambda \hat{a}_\lambda - \hat{b}^\dagger_\lambda \hat{b}_\lambda - \frac{1}{2}\mathrm{sgn}(E_\lambda)\right], \tag{5}$$

where $\mathrm{sgn}(\pm u) = \pm 1$ at $u > 0$.

Energy (temporal component of the energy-momentum vector) operator

$$\hat{P}^0 = \frac{1}{2}\int\limits_{\Omega} \mathrm{d}^3 r(\hat{\Psi}^\dagger H\hat{\Psi} - \hat{\Psi}^T H^T\hat{\Psi}^{\dagger T}) = \sum |E_\lambda| \left(\hat{a}^\dagger_\lambda \hat{a}_\lambda + \hat{b}^\dagger_\lambda \hat{b}_\lambda - \frac{1}{2}\right). \tag{6}$$

Partition function

$$Z(T,\mu) = \mathrm{Sp}\exp\left[-(\hat{P}^0 - \mu\hat{N})/T\right]. \tag{7}$$

Average of operator $\hat{U}$ over the grand canonical ensemble

$$\left\langle \hat{U} \right\rangle_{T,\mu} = Z^{-1}(T,\mu)\,\mathrm{Sp}\,\hat{U}\exp\left[-(\hat{P}^0 - \mu\hat{N})/T\right]. \tag{8}$$

In particular, one can compute averages

$$\left\langle \hat{a}^\dagger_\lambda \hat{a}_\lambda \right\rangle_{T,\mu} = \{\exp[(E_\lambda - \mu)/T] + 1\}^{-1}, \quad E_\lambda > 0 \tag{9}$$

and

$$\left\langle \hat{b}^\dagger_\lambda \hat{b}_\lambda \right\rangle_{T,\mu} = \{\exp[(-E_\lambda + \mu)/T] + 1\}^{-1}, \quad E_\lambda < 0. \tag{10}$$

$$\hat{U} = \frac{1}{2}\left(\hat{\Psi}^{\dagger}\Upsilon\hat{\Psi} - \hat{\Psi}^{T}\Upsilon^{T}\hat{\Psi}^{\dagger T}\right), \tag{11}$$

where Υ is an element of the Dirac-Clifford algebra. Using (9) and (10), we obtain

$$\left\langle \hat{U} \right\rangle_{T,\mu} = -\frac{1}{2}\mathrm{tr}\left\langle \mathbf{r} | \Upsilon \tanh[(H - \mu I)(2T)^{-1}] | \mathbf{r} \right\rangle. \tag{12}$$

Vector current density

$$\mathbf{J} = \left\langle \hat{U} \right\rangle_{T,\mu}\Big|_{\Upsilon=\gamma^0\boldsymbol{\gamma}}. \tag{13}$$

Axial current density

$$\mathbf{J}^5 = \left\langle \hat{U} \right\rangle_{T,\mu}\Big|_{\Upsilon=\gamma^0\boldsymbol{\gamma}\gamma^5}. \tag{14}$$

Axial charge density

$$J^{05} = \left\langle \hat{U} \right\rangle_{T,\mu}\Big|_{\Upsilon=\gamma^5}, \tag{15}$$

where $\gamma^5 = -\mathrm{i}\gamma^0\gamma^1\gamma^2\gamma^3$.

Owing to the presence of chiral symmetry,

$$[H, \gamma^5]_- = 0, \tag{16}$$

one can define axial charge operator

$$\hat{N}^5 = \frac{1}{2} \int_\Omega \mathrm{d}^3 r (\hat{\Psi}^\dagger \gamma^5 \hat{\Psi} - \hat{\Psi}^T \gamma^{5T} \hat{\Psi}^{\dagger T}) \tag{17}$$

and modified partition function

$$\tilde{Z}(T, \mu_5) = \mathrm{Sp} \exp \left[-(\hat{P}^0 - \mu_5 \hat{N}^5)/T \right], \tag{18}$$

where μ_5 is the chiral chemical potential.

Average of operator $\hat{U}$ over the modified grand canonical ensemble

$$\left\langle \hat{U} \right\rangle_{T,\mu_5} = \tilde{Z}^{-1}(T, \mu_5)\, \mathrm{Sp}\, \hat{U} \exp \left[-(\hat{P}^0 - \mu_5 \hat{N}^5)/T \right]. \tag{19}$$

$$\left\langle \hat{U} \right\rangle_{T,\mu_5} = -\frac{1}{2}\mathrm{tr}\left\langle \mathbf{r} | \Upsilon \tanh[(H - \mu_5\gamma^5)(2T)^{-1}] | \mathbf{r} \right\rangle . \tag{20}$$

Vector current density

$$\mathbf{J} = \left\langle \hat{U} \right\rangle_{T,\mu_5} \Big|_{\Upsilon = \gamma^0 \boldsymbol{\gamma}} . \tag{21}$$

Axial current density

$$\mathbf{J}^5 = \left\langle \hat{U} \right\rangle_{T,\mu_5} \Big|_{\Upsilon = \gamma^0 \boldsymbol{\gamma} \gamma^5} . \tag{22}$$

Axial charge density

$$J^{05} = \left\langle \hat{U} \right\rangle_{T,\mu_5} \Big|_{\Upsilon = \gamma^5} . \tag{23}$$

In the case of a disconnected boundary consisting of two simply-connected components, $\partial\Omega = \partial\Omega^{(+)} \bigcup \partial\Omega^{(-)}$, there are in general 8 boundary parameters: φ_+, $\tilde{\varphi}_+$, θ_+ and ς_+ corresponding to $\partial\Omega^{(+)}$ and φ_-, $\tilde{\varphi}_-$, θ_- and ς_- corresponding to $\partial\Omega^{(-)}$. If spatial region Ω has the form of a slab bounded by parallel planes, $\partial\Omega^{(+)}$ and $\partial\Omega^{(-)}$, separated by distance *a*, then the boundary condition takes form

$$\left(I - K^{(\pm)}\right) \chi(\mathbf{r}) \,|_{z=\pm a/2} = 0, \tag{24}$$

where

$$K^{(\pm)} = \gamma^0 \left[\mathrm{e}^{\mathrm{i}\varphi_\pm \gamma^5} \cos\theta_\pm + (\gamma^1 \cos\varsigma_\pm + \gamma^2 \sin\varsigma_\pm) \sin\theta_\pm \right] \mathrm{e}^{\pm \mathrm{i}\tilde{\varphi}_\pm \gamma^0 \gamma^z}, \tag{25}$$

coordinates $\mathbf{r} = (x,\, y,\, z)$ are chosen in such a way that x and y are tangential to the boundary, while z is normal to it, and the position of $\partial\Omega^{(\pm)}$ is identified with $z = \pm a/2$.

The confinement of matter inside the slab means that the vector bilinear, $\chi^\dagger(\mathbf{r})\gamma^0\gamma^z\chi(\mathbf{r})$, vanishes at the slab boundaries,

$$\chi^\dagger(\mathbf{r})\gamma^0\gamma^z\chi(\mathbf{r}) \,|_{z=\pm a/2} = 0, \qquad (26)$$

and this is ensured by condition (24). As to the axial bilinear, $\chi^\dagger(\mathbf{r})\gamma^0\gamma^z\gamma^5\chi(\mathbf{r})$, it vanishes at the slab boundaries,

$$\chi^\dagger(\mathbf{r})\gamma^0\gamma^z\gamma^5\chi(\mathbf{r}) \,|_{z=\pm a/2} = 0, \qquad (27)$$

in the case of $\theta_+ = \theta_- = \pi/2$ only, that is due to relation

$$[K^{(\pm)} \,|_{\theta_\pm=\pi/2}, \gamma^5]_- = 0. \qquad (28)$$

However, there is a symmetry with respect to rotations around a normal to the slab, and the cases differing by values of ς_+ and ς_- are physically indistinguishable, since they are related by such a rotation. The only way to avoid the unphysical degeneracy of boundary conditions with different values of ς_+ and ς_- is to fix $\theta_+ = \theta_- = 0$. Then $\chi^\dagger(\mathbf{r})\gamma^0\gamma^z\gamma^5\chi(\mathbf{r})$ is nonvanishing at the slab boundaries, and the boundary condition takes form

$$\left\{ I - \gamma^0 \exp\left[\mathrm{i}\left(\varphi_\pm \gamma^5 \pm \tilde{\varphi}_\pm \gamma^0 \gamma^z \right) \right] \right\} \chi(\mathbf{r}) \mid_{z=\pm a/2} = 0. \qquad (29)$$

Condition (29) determines the spectrum of the wave number vector in the z-direction, k_l. The requirement that this spectrum be real and unambiquous yields constraint

$$\varphi_+ = \varphi_- = \varphi, \quad \tilde{\varphi}_+ = \tilde{\varphi}_- = \tilde{\varphi}; \qquad (30)$$

then the k_l-spectrum is determined implicitly from relation (Yu. A. S., 2015)

$$k_l \sin\tilde{\varphi} \cos(k_l a) + (E_{...l} \cos\tilde{\varphi} - m \cos\varphi) \sin(k_l a) = 0, \qquad (31)$$

where $E_{...l}$ is the energy of the one-particle state.

Casimir effect in a magnetic field

$$T = 0, \quad \mu = 0. \tag{32}$$

The Dirac Hamiltonian takes form

$$H = -\mathrm{i}\gamma^0 \boldsymbol{\gamma} \cdot (\boldsymbol{\partial} - \mathrm{i}e\mathbf{A}) + \gamma^0 m \tag{33}$$

and the one-particle energy spectrum is

$$E_{nl} = \pm\omega_{nl}, \quad \omega_{nl} = \sqrt{2n|eB| + k_l^2 + m}, \quad n = 0, 1, 2, \ldots, \tag{34}$$

where B is the value of the magnetic field strength, $\mathbf{B} = \boldsymbol{\partial} \times \mathbf{A}$, in the direction of the z-axis, n labels the Landau levels, and k_l is the value of the wave number vector along the magnetic field; the set of the k_l values is determined by condition

$$k_l \sin\tilde{\varphi} \cos(k_l a) + (E_{nl} \cos\tilde{\varphi} - m \cos\varphi) \sin(k_l a) = 0. \tag{35}$$

Casimir force (Yu.A.S., 2015):

$$F = -\frac{1}{8\pi^2}\int_0^\infty \frac{\mathrm{d}\tau}{\tau}\mathrm{e}^{-\tau}\left[\frac{eBm^2}{\tau}\coth\left(\frac{eB\tau}{m^2}\right) - \frac{m^4}{\tau^2} - \frac{1}{3}e^2B^2\right] + \Delta_{\varphi,\tilde{\varphi}}(a,|eB|,m), \quad (36)$$

Usually, the Casimir effect is validated experimentally for the macroscopic separation of plates: $a > 10^{-8}\,\mathrm{m}$. So, even if one takes the lightest massive particle, electron (Compton wavelength $m^{-1} = 3.86 \times 10^{-13}\,\mathrm{m}$), then all the dependence on the distance between the plates and boundary conditions is damped exponentially as a consequence of

$$\Delta_{\varphi,\tilde{\varphi}}(a,|eB|,m) \sim \mathrm{e}^{-2ma}, \quad ma \gg 1. \quad (37)$$

Thus, the Casimir effect is repulsive, in particular,

$$F = \frac{e^2B^2}{24\pi^2}\ln\frac{2|eB|}{m^2}, \quad m^2 \ll |eB| \quad (38)$$

and

$$F = \frac{1}{360\pi^2}\frac{e^4B^4}{m^4}, \quad m^2 \gg |eB|, \quad (39)$$

Slab of spinor matter in extremal conditions

$$|eB| >> m^2, \quad T >> m, \quad \mu >> m. \tag{40}$$

The Dirac Hamiltonian takes form ($m = 0$)

$$H = -\mathrm{i}\gamma^0 \boldsymbol{\gamma} \cdot (\boldsymbol{\partial} - \mathrm{i}e\mathbf{A}) \tag{41}$$

and the one-particle energy spectrum is

$$E_{nl} = \pm\omega_{nl}, \quad \omega_{nl} = \sqrt{2n|eB| + k_l^2}, \quad n = 0, 1, 2, \ldots, \tag{42}$$

where B is the value of the magnetic field strength, $\mathbf{B} = \boldsymbol{\partial} \times \mathbf{A}$, in the direction of the z-axis, n labels the Landau levels, and k_l is the value of the wave number vector along the magnetic field; the set of the k_l values is determined by condition

$$k_l \sin\tilde{\varphi} \cos(k_l a) + E_{nl} \cos\tilde{\varphi} \sin(k_l a) = 0, \tag{43}$$

depending on one parameter only, although the boundary condition depends on two parameters:

$$\left\{ I - \gamma^0 \exp\left[\mathrm{i}\left(\varphi\gamma^5 \pm \tilde{\varphi}\gamma^0\gamma^z\right)\right]\right\} \chi(\mathbf{r})|_{z=\pm a/2} = 0. \tag{44}$$

Chiral effects

$$\mathbf{J} = J^{05} = 0, \quad J^{x5} = J^{y5} = 0. \tag{45}$$

As to the component of the axial current density, which is along the magnetic field, only the lowest Landau level ($n = 0$) contributes to it. The spectrum of the wave number vector along the magnetic field is determined from (43) at $n = 0$, i.e.

$$k_l^{(\pm)} = (l\pi \mp \tilde{\varphi})/a, \quad l \in \mathbb{Z}, \quad k_l^{(\pm)} > 0, \tag{46}$$

where the upper (lower) sign corresponds to $E_{0l} > 0$ ($E_{0l} < 0$) and $\mathbb{Z}$ is the set of integer numbers. Hence, the z-component of the axial current density is

$$J^{z5} = \frac{eB}{4\pi a}\left\{\sum_{k_l^{(+)}>0} \tanh[(k_l^{(+)} - \mu)(2T)^{-1}] - \sum_{k_l^{(-)}>0} \tanh[(k_l^{(-)} + \mu)(2T)^{-1}]\right\}, \tag{47}$$

(Yu.A.S., 2016)

$$J^{z5} = -\frac{eB}{2\pi a}\left\{ \mathrm{sgn}(\mu) F\left(|\mu| a + \mathrm{sgn}(\mu)\left[\tilde{\varphi} - \mathrm{sgn}(\tilde{\varphi})\pi/2\right]; Ta \right) - \frac{1}{\pi}\left[\tilde{\varphi} - \mathrm{sgn}(\tilde{\varphi})\pi/2\right] \right\}, \quad (48)$$

where

$$F(s;t) = \frac{s}{\pi} - \frac{1}{\pi}\int\limits_0^\infty \mathrm{d}v \frac{\sin(2s)\sinh(\pi/t)}{[\cos(2s) + \cosh(2v)][\cosh(\pi/t) + \cos(v/t)]} + \frac{\sinh\{[\arctan(\tan s)]/t\}}{\cosh[\pi/(2t)] + \cosh\{[\arctan(\tan s)]/t\}}. \quad (49)$$

In the case of a magnetic field filling the whole (infinite) space we obtain the known resut:

$$\lim_{a\to\infty} J^{z5} = -\frac{eB}{2\pi^2}\mu. \quad (50)$$

Asymptotics at small and large temperatures:

$$\lim_{T\to 0} J^{z5} = -\frac{eB}{2\pi a}\left[\mathrm{sgn}(\mu)\left[\!\left[\frac{|\mu|a+\mathrm{sgn}(\mu)\tilde{\varphi}}{\pi}+\Theta(-\mu\tilde{\varphi})\right]\!\right]-\frac{\tilde{\varphi}}{\pi}+\frac{1}{2}\mathrm{sgn}(\tilde{\varphi})\right] \tag{51}$$

and

$$\lim_{T\to\infty} J^{z5} = -\frac{eB}{2\pi^2}\mu; \tag{52}$$

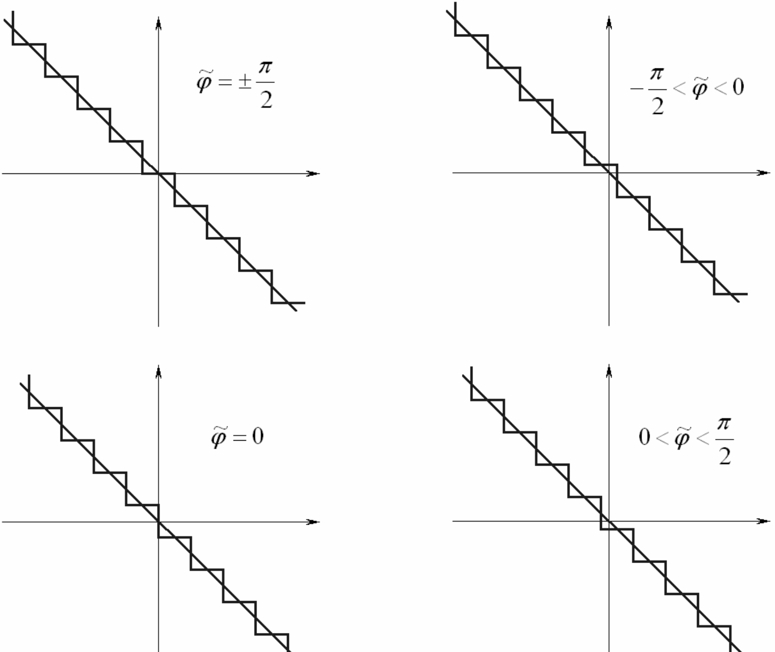
here $[\![u]\!]$ denotes the integer part of quantity u, and $\Theta(u)=\frac{1}{2}[1+\mathrm{sgn}(u)]$ is the step function.

The chiral separation effect can be nonvanishing even at zero chemical potential:

$$J^{z5}|_{\mu=0} = -\frac{eB}{2\pi a}\left\{F(\tilde{\varphi}-\mathrm{sgn}(\tilde{\varphi})\pi/2; Ta)-\frac{1}{\pi}\left[\tilde{\varphi}-\mathrm{sgn}(\tilde{\varphi})\pi/2\right]\right\}; \tag{53}$$

the latter vanishes in the limit of infinite temperature,

$$\lim_{T\to\infty} J^{z5}|_{\mu=0} = 0. \tag{54}$$



$\tilde{\varphi} = \pm\frac{\pi}{2}$
$-\frac{\pi}{2} < \tilde{\varphi} < 0$
$\tilde{\varphi} = 0$
$0 < \tilde{\varphi} < \frac{\pi}{2}$

The trivial boundary condition, $\tilde{\varphi} = -\pi/2$, gives spectrum $k_l = (l + \frac{1}{2})\frac{\pi}{a} \quad (l = 0, 1, 2, \ldots)$, and the axial current density at zero temperature for this case was obtained earlier (E. V. Gorbar et al, 2015)

$$J^{z5} \,|_{T=0,\ \tilde{\varphi}=-\pi/2} = -\frac{eB}{2\pi a}\mathrm{sgn}(\mu)[\![|\mu|a/\pi + 1/2]\!]. \qquad (55)$$

The "bosonic-type" spectrum, $k_l = l\frac{\pi}{a}$ $(l = 0, 1, 2, \ldots)$, is given by $\tilde{\varphi} = 0$; note that the axial current density is continuous at this point:

$$\lim_{\tilde{\varphi}\to 0_+} J^{z5} - \lim_{\tilde{\varphi}\to 0_-} J^{z5} = 0. \qquad (56)$$

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Conclusion

The significant role of boundaries for chiral effects in hot dense magnetized spinor matter:

- dependence on both temperature and chemical potential,
- dependence on the boundary parameter,
- the boundary condition can serve as a source that is additional to the spinor matter density.

Thank you for your attention!