# ON THE CONCEPT OF LOCAL TIME

Quantum-mechanical and cosmological perspectives

Miroljub Dugić, Jasmina Jeknić-Dugić, Momir Arsenijević

# OVERVIEW OF THE TALK

- Historical perspective
- The Enss' theorem in many-body scattering
- New reading of the Enss' theorem: Local Time
- Basic elaborations

# HISTORICAL PERSPECTIVE

- Schrodinger's stationary equation
- Constructing the nonstationary equation:

Starts from the wave equation $\Delta\psi - \frac{2(E-V)}{E^2}\frac{\partial^2\psi}{\partial t^2} = 0$

and with implicit use of $i\hbar\frac{\partial\psi}{\partial t} = E\psi$ obtains:

$$-i\hbar\frac{\partial\psi}{\partial t} + \left(-\frac{1}{2\mu}\Delta + V\right)\psi = 0$$

So, which equation to use? Schrodinger's answer: the second one.

Hitoshi Kitada of Tokyo: both, with altering Time.

(H. Kitada, Nuovo Cimento B **109**, 281 (1994))

# THE ENSS' THEOREM

- The Schrodinger law:

$$\psi(t_m) = e^{-\frac{i}{h}Ht_m}\psi(0) \qquad m = 0,1,2,\dots$$

i.e. a one-parameter unitary group $e^{-\frac{i}{h}Ht_m}$

- For arbitrary state, and particularly for a scattering state $\psi$ :

$$\left\|\left(\frac{x}{t_m} - \frac{p}{\mu}\right)e^{-\frac{i}{h}Ht_m}\psi(0)\right\| \xrightarrow{m\to\infty} 0 \qquad (1)$$

# THE ENSS' THEOREM

$$\frac{x}{t_m} \sim \frac{p}{\mu}, \qquad t_m \to \infty, m = 0,1,2, \ldots$$

with the time instants $t_m$

Taking the continuous limit (not implied by the Theorem though):

$$t = \frac{x\mu}{p} \qquad (2)$$

a classical "definition" of time—often used for "time quantization" (e.g., Y. Aharonov, D. Bohm, Phys. Rev. **122** (1961) 1649).

$p = -i\hbar\frac{\partial}{\partial x}$

# NEW READING OF THE ENSS' THEOREM

Let be prejudice-less on the Enss' theorem.

Let's be minimalist!

Just read what is already there in the theorem, purely mathematically, and start over.

Dynamics: $\psi(0) \to \psi(t_1) \to \psi(t_2) \to \cdots \to \psi(t_m) \to \cdots$

No idea about the physics of the parameter $t_m$ yet. There is no a priori time in either x or p.

# NEW READING OF THE ENSS' THEOREM

$$A_m(\psi(0)) \equiv \left(\frac{x}{t_m} - \frac{p}{\mu}\right) e^{-\frac{i}{h}Ht_m}\psi(0)$$

The Enss' theorem asserts for the dynamical chain, $\psi(0) \to \psi(t_1) \to \psi(t_2) \to \cdots \to \psi(t_m) \to \cdots$

$$\|A_0\| > \|A_1\| > \|A_2\| > \cdots > \|A_m\| > ..$$

that is:

$t = \dfrac{x\mu}{p}$ is getting better satisfied down the dynamical chain. No recurrence.

# NEW READING OF THE ENSS' THEOREM

Collecting those above:

$$\psi(0) \to \psi(t_1) \to \psi(t_2) \to \cdots \to \psi(t_m) \to \cdots$$

$$t_0 \neq \frac{x\mu}{p}, t_1 \neq \frac{x\mu}{p}, t_2 \neq \frac{x\mu}{p} \dots t_m \approx \frac{x\mu}{p}$$

the Time is <u>dynamically *born*</u> ($m \to \infty$ ).

The link:

One Hamiltonian (one system) $\leftrightarrow$ one Time

# NEW READING OF THE ENSS' THEOREM

Notice: this link is two-directional, mathematically rigorous:

H. Kitada, J. Jeknic-Dugic, M. Arsenijevic, M. Dugic, Phys. Lett. A **380**, 3970 (2016).

Intuition: one (isolated) system, one Time.

There is not any assumption additional to the Enss' theorem. Minimalist thinking prefers the concept of Local Time.

# BASIC ELABORATIONS

In reality, the limit $m \to \infty$ cannot be reached.

Therefore, for every finite $t_m$ the time is not uniquely determined - uncertainty of Local Time.

That is, instead of the unitary dynamics, for a statistical ensemble there is the

new fundamental QM dynamical law:

$$\sigma(t_0) = \int_{t_0-\Delta t}^{t_0+\Delta t} dt\, \rho(t)\, U(t)\sigma(0)U^+(t) \qquad (3)$$

# BASIC ELABORATIONS

The choice of the very narrow Gaussian density probability $\rho(t)$ leads to a number of relevant results.

J. Jeknic-Dugic, M. Arsenijevic and M. Dugic, *Proc. R. Soc. A 2014* ***470,*** *20140283*

J. Jeknic-Dugic, M. Arsenijevic and M. Dugic, *Proc. R. Soc. A 2016* ***472,*** *20160041*

# BASIC ELABORATIONS

Some basic results:

- Unique "pointer basis" for quantum bipartitions
- Border line between "micro" and "macro"
- A new kind of (non-differentiable) dynamical map
- Derivation of the Luders-von Neumann formula
- Smaller systems' faster "relaxation"
- Dynamical appearance of Markovianity
- *A new interpretation of the Wheeler-DeWitt equation, lack of spacetime quantization etc*.

# THANK YOU FOR YOUR ATTENTION