Astrophysical Aspects of Weyl Gravity

Cemsinan Deliduman

Mimar Sinan Fine Arts University, Department of Physics, İstanbul

NAOJ, Department of Theoretical Astronomy, Mitaka, Tokyo

together with Oğuzhan Kaşıkçı and Barış Yapışkan

Outline

- Galaxy rotation curves.
- Einstein-Weyl theory of gravity.
- Geometry of outer region of galaxies.
- Gravitational lensing.
- Future directions.

Dark Matter

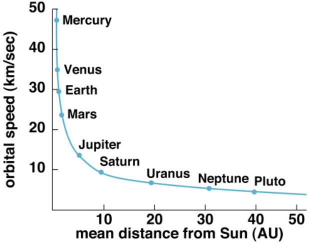
Discovery of Neptune

Observation

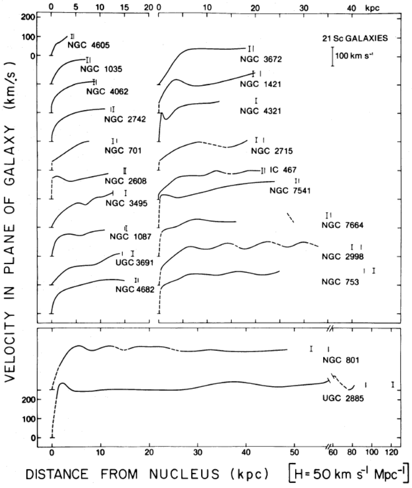
- Alexis Bouvard (1767–1843): Observed irregularities in the motion of Uranus.
- Urbain Le Verrier (1811–1877): Predicted the existence and position of Neptune.
- Johann Galle & Heinrich d'Arrest (1846): They observed Neptune within 1^o of prediction.

Mathematics

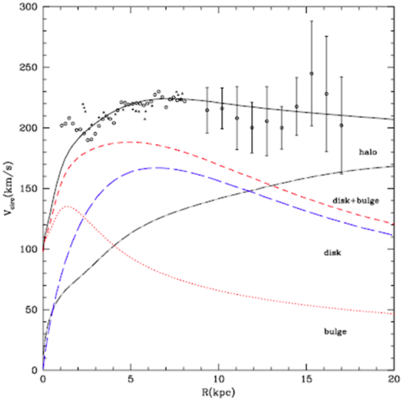
$$F = \frac{GM_\odot m}{r^2} = \frac{mv^2}{r} \rightarrow v = \sqrt{\frac{GM_\odot}{r}}$$



Many galactic rotation curves



- Rubin, Ford and Thonnard, ApJ 238 (1980) 471

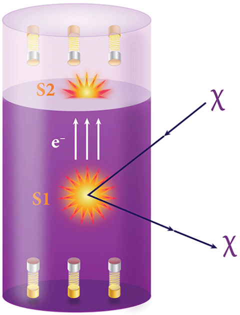


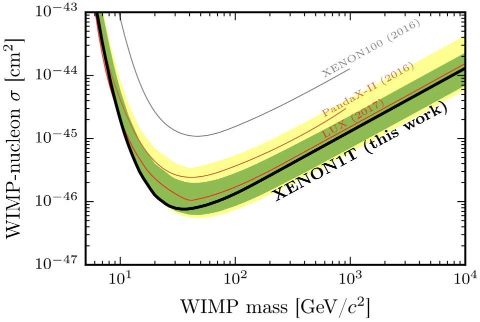
- Klypin, Zhao, Somerville, Astrophys.J. 573 (2002) 597

Darker still

Many direct and indirect detection experiments

CDMS, CRESST, EDELWEISS, EURECA, ZEPLIN, XENON, DEAP, ArDM, WARP, DarkSide, PandaX, LUX, SIMPLE, PICASSO, DAMA/NaI, DAMA/LIBRA, DMTPC, DRIFT, Newage, MIMAC, AMANDA, IceCube, ANTARES, EGRET, PAMELA, AMS, LHC, ADMX, DARWIN





- Cline, Phys. Scripta 91 (2016) 033008
- XENON Coll., PRL 119 (2017) 181301

Gravity

How to modify the General Relativity

Conformal gravity

H. Weyl, Math. Zeit. 2 (1918) 384 :

Assume gravity has an additional symmetry beyond coordinate invariance:

$g_{\mu\nu}(x) \rightarrow e^{-2\varphi(x)} g_{\mu\nu}(x)$, which is the conformal symmetry. There is one and only one action which is invariant under the local conformal transformations:

$$S = -\zeta \int d^4x \sqrt{-g} C^{\mu\nu\rho\sigma} C_{\mu\nu\rho\sigma} \overset{mod\ GB}{=} -2\zeta \int d^4x \sqrt{-g} [-\frac{1}{3} R^2 + R_{\mu\nu} R^{\mu\nu}]$$

Semiclassical corrections

Utiyama and De Witt, JMP 3 (1962) 608; Utiyama, PRD 125 (1962) 1727 :

The mean value of the stress-energy tensor $T_{\mu\nu}$ of a set of quantized fields interacting with a classical geometry is plagued with infinities. In order to make it finite, cosmological constant and Einstein's constant are renormalized, and a counterterm must be introduced in Lagrangian:

$$\Delta L = \sqrt{-g} [\alpha R^2 + \beta R_{\mu\nu} R^{\mu\nu}]$$

How to modify the General Relativity

Holdom and Ren, *A QCD analogy for quantum gravity*, PRD 93 (2016) 124030 :

Ultraviolet Modification

"Quadratic gravity presents us with a renormalizable, asymptotically free theory of quantum gravity. When its couplings grow strong at some scale, as in QCD, then this strong scale sets the Planck mass:"

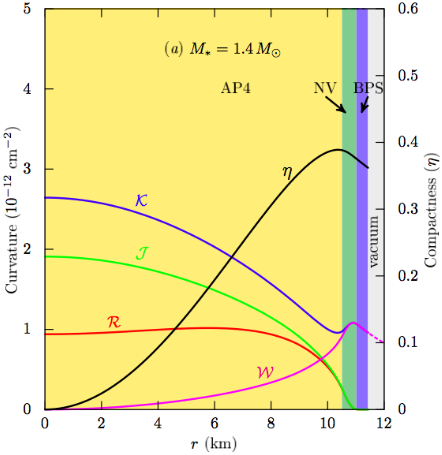
$$S = \int d^4x\sqrt{-g}\left(\frac{1}{2}M^2 R - \frac{1}{3f_0^2}R^2 - \frac{1}{2f_2^2}C^{\mu\nu\rho\sigma}C_{\mu\nu\rho\sigma}\right)$$

Infrared Modification

"Similar to the QCD chiral Lagrangian, the IR physics is expected to be described by a derivative expansion of the curvature tensors with a leading Einstein-Hilbert term:"

$$S = \int d^4x\sqrt{-g}\left(\frac{1}{2}M_{Pl}^2 R + c_1 R^2 + c_2 C^{\mu\nu\rho\sigma}C_{\mu\nu\rho\sigma} + \dots\right)$$

Relative Importance of Terms



- Compactness: $\eta(r) \equiv \frac{2Gm(r)}{rc^2}$
- Scalar curvature:

  $\mathcal{R}(r) = \kappa(\rho c^2 - 3P)$ with $\kappa = \frac{8\pi G}{c^4}$
- Ricci scalar:

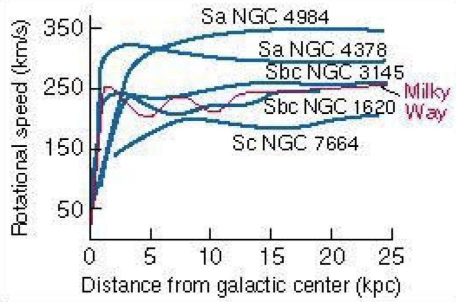
  $\mathcal{J}^2 \equiv R_{\mu\nu} R^{\mu\nu} = \kappa^2[(\rho c^2)^2 + 3P^2]$
- Kretschmann scalar:

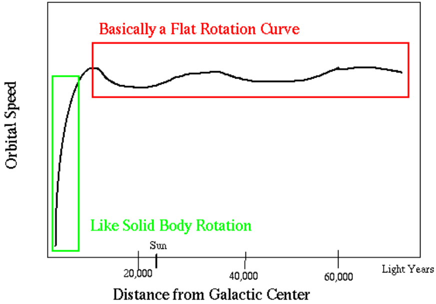
  $\mathcal{K}^2 \equiv R_{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma}$
- Weyl tensor contraction:

  $\mathcal{W}^2 \equiv C^{\mu\nu\rho\sigma} C_{\mu\nu\rho\sigma} = \frac{4}{3}\left(\frac{6Gm(r)}{c^2 r^3} - \kappa\rho c^2\right)$

- Ekşi, Güngör and Türkoğlu, PRD 89 (2014) 063003

Einstein–Weyl Gravity





Basic idea

- Action:

$$S = \frac{M_P^2}{2} \int d^4x \sqrt{-g} \left[R + \zeta \, C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} \right] .$$

- Einstein-Hilbert part dominates in the "like solid body rotation" region.
- Weyl part dominates in the "basically a flat rotation curve" region.

See also: • Psaltis, Living Rev. Rel. 11 (2008) 9; • Maeder, Astrophys. J. 849 (2017) 158.

Weyl Gravity

Action

$$S = -\zeta \int d^4x\sqrt{-g}\,C^{\mu\nu\rho\sigma}C_{\mu\nu\rho\sigma} \overset{mod\_GB}{=} -2\zeta \int d^4x\sqrt{-g}[-\frac{1}{3}R^2 + R_{\mu\nu}R^{\mu\nu}]$$

Equations of motion

Bach tensor:

$$B_{\mu\nu} = -\frac{1}{3}H_{\mu\nu} + K_{\mu\nu} = 0\,,$$

$$H_{\mu\nu} = 2R\left(R_{\mu\nu} - \frac{1}{4}Rg_{\mu\nu}\right) + 2\left(g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu\right)R\;,$$

$$K_{\mu\nu} = \Box\left(R_{\mu\nu} + \frac{1}{2}g_{\mu\nu}R\right) - \nabla_\lambda\nabla_\mu R^\lambda_\nu - \nabla_\lambda\nabla_\nu R^\lambda_\mu + 2R_{\mu\lambda}R^\lambda_\nu - \frac{1}{2}g_{\mu\nu}R^{\alpha\beta}R_{\alpha\beta}\;.$$

Solution

Spherical Symmetry

Metric ansatz:

$$ds^2 = -A(r)dt^2 + \frac{dr^2}{B(r)} + r^2 d\Omega_k^2 \quad \text{with} \quad d\Omega_k^2 \equiv \frac{1}{1-kx^2}dx^2 + (1-kx^2)dy^2$$

Killing vectors:

$$K = \frac{\partial}{\partial t} \quad \text{and} \quad L = \frac{\partial}{\partial \phi}$$

Conserved quantities:

$$A(r)\dot{t} = E \quad \text{and} \quad r^2\dot{\phi} = L$$

Equation of motion:

$$1 = A\dot{t}^2 - \frac{\dot{r}^2}{B(r)} - r^2\dot{\phi}^2$$

Tangential velocity:

$$v_c = \frac{dl}{ds} = \frac{r d\phi}{\sqrt{-g_{00}}dt} \quad \Rightarrow \quad v_c^2 = \frac{r}{2}\frac{A'}{A} \equiv w$$

Metric:

$$ds^2 = -\left(\frac{r}{r_c}\right)^{2w} dt^2 + \frac{dr^2}{B(r)} + r^2 d\Omega_k^2$$

Solutions

"rr" field equation

$$BB''r^2 + wBB'r - \frac{1}{4}B'^2r^2 - B^2(w-1)^2 + \frac{k^2}{(w-1)^2} = 0\,,$$

[1] Trivial solution

$$B(r) = C = \frac{k}{(w-1)^2}$$

[2] Solution specific to torus T^2 ($k = 0$) geometry

Transformation: $B(r) = [b(r)]^n \quad \Rightarrow \quad b[b''r^2 + wb'r - \frac{1}{n}(1-w)^2 b] + (\frac{3n}{4} - 1)(b')^2 r^2 = 0\,.$

Metric:

$$ds^2 = -\left(\frac{r}{r_c}\right)^{2w} dt^2 + \frac{r^{2(w-1)}}{(C_1 + C_2 r^{2(w-1)})^{\frac{4}{3}}} dr^2 + r^2(dx^2 + dy^2)\,.$$

Solutions

[3] Solution valid for S^2 ($k = 1$) and H^2 ($k = -1$) geometries

Transformation:

$$B(r) = \frac{2k}{(1-w)^2} r^{2(1-w)} F(r)$$

and a change of variable:

$$r = z^{1/(1-w)}$$

Solution:

$$B(r) = \frac{3kr^{2(1-w)}}{8(1-w)^2 C_1} (v(r) - 1)^2 \, ,$$

with

$$v(r) = \Big[\Big(\frac{C_1}{r^{2(1-w)}} + C_2 + \sqrt{(\frac{C_1}{r^{2(1-w)}} + C_2)^2 - 1} \Big)^{\frac{1}{3}} + \Big(\frac{C_1}{r^{2(1-w)}} + C_2 - \sqrt{(\frac{C_1}{r^{2(1-w)}} + C_2)^2 - 1} \Big)^{\frac{1}{3}} \Big]^2 .$$

Stability of circular orbits

Metric ansatz:

$$ds^2 = -A(r)dt^2 + \frac{1}{B(r)}dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2$$

Equation for the radial coordinate (at $\theta = \pi/2$):

$$\dot{r}^2 = B(r)\left(\frac{E^2}{A(r)} - \frac{L^2}{r^2} + \epsilon\right)$$

Geodesic equation:

$$\ddot{r} = \frac{B'}{2B}\dot{r}^2 + B\left(\frac{L^2}{r^3} - \frac{A'E^2}{2A^2}\right)$$

Effective potential:

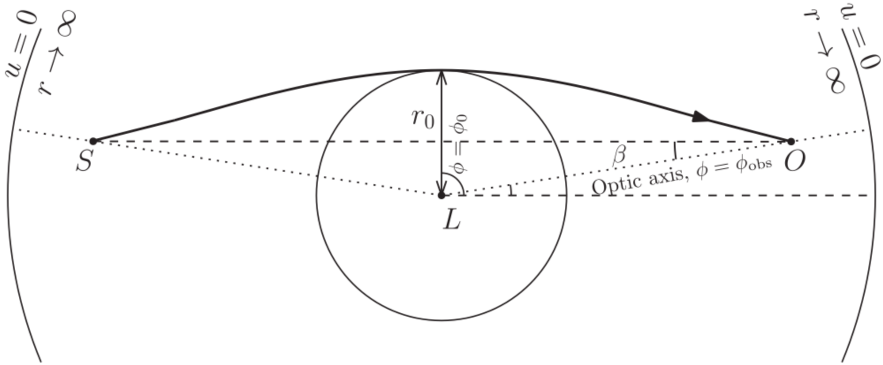
$$V_{eff} \equiv B(r)\left(\frac{E^2}{A(r)} - \frac{L^2}{r^2} + \epsilon\right)$$

Small perturbation to the circular orbit:

$$r = R + \delta \quad \Rightarrow \quad \ddot{\delta} = \frac{V''_{eff}}{2}\delta \quad \Rightarrow \quad V''_{eff} = \frac{-2B\epsilon}{A(2A - RA')}(2A'^2 - AA'' - 3A'A/R) < 0$$

Gravitational Lensing

Strong Lensing : Geometry



Deflection angle: $\triangle\alpha = 2\,(\phi_S - \phi_0) - \pi$

Null Geodesic and Deflection Angle

Metric:

$$ds^2 = -\left(\frac{r}{r_c}\right)^{2w} dt^2 + \frac{1}{B(r)} dr^2 + r^2 d\theta^2 + r^2 \sin\theta d\phi^2 \,,$$

$$\text{with} \qquad B(r) = \frac{3r^{2(1-w)}}{8(1-w)^2\Delta_2}(1 + e^{i2\pi/3}h^2 + e^{i4\pi/3}h^{-2})^2 \,,$$

$$\text{where} \qquad h \equiv \left[A + \sqrt{A^2 - 1}\right]^{1/3}, \; A \equiv [\Delta_1 - \frac{\Delta_2}{r^{2(1-w)}}],$$

$$\Delta_1 \equiv (1 + 3m\gamma)\sqrt{1 - 6m\gamma} - 54m^2 k, \;\; \Delta_2 \equiv \frac{54m^2}{r_c^{2w}}.$$

For a null geodesic in the $\theta = \pi/2$ plane:

$$\frac{d\zeta}{d\phi} = \sqrt{(1-\zeta^2)B(\zeta)}, \qquad \zeta = \frac{r_0}{r} \quad \textit{and} \quad r_0 = b \equiv \frac{L}{E}$$

Deflection angle:

$$\Delta\alpha = 2\int_0^1 \frac{d\zeta}{\sqrt{(1-\zeta^2)B(\zeta)}} - \pi \,.$$

Strong lensing in Weyl Geometry

- Expand the integrand in m and evaluate the integral.
- First expand the result in γ.
- Then expand the result in k.

Deflection angle:

$$\alpha = 4m_0 - 2\sqrt{\frac{\Lambda_0}{3}} - 2m_0\sqrt{\frac{\Lambda_0}{3}} + m_0^2\left(\frac{15\pi}{4} - 4 - 3\sqrt{\frac{\Lambda_0}{3}}\right) + \gamma_0\left(2m_0 + \sqrt{\frac{\Lambda_0}{3}}\right)$$

$$\text{with} \quad m_0 \equiv \frac{m}{r_0}, \ \gamma_0 \equiv \gamma r_0 \ , k_0 \equiv k r_0^2 \equiv \sqrt{\frac{\Lambda_0}{3}} \ , \Lambda_0 \equiv \Lambda r_0^2 \ .$$

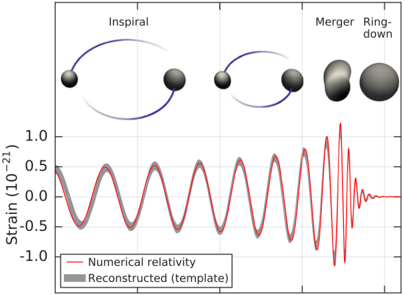
- Batic, Nelson and Nowakowski, PRD 91 (2015) 104015.
- Rindler and Ishak, PRD 76 (2007) 043006; • Arakida and Kasai, PRD 85 (2012) 023006.
- Potapov at el., PRD 93 (2016) 124070; • Lim and Wang, PRD 95 (2017) 024004.

Future directions

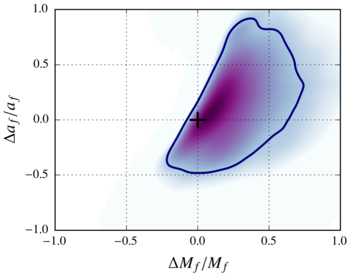
1. Matching Scwarzschild geometry of inner region and Weyl geometry of the outer region at the onset of flat rotation curve behavior.
2. Investigate possible effect of Einstein–Hilbert term on soft breaking of the scale invariance in the outer regions of the galaxies.
3. Explaining gravitational lensing data of elliptical galaxies.
4. Gravitational lensing by clusters.
5. Black holes and ultra compact objects in Einstein–Weyl gravity. Determining gravitational wave profile.
6. Cosmological problems: the Einstein–Weyl gravity in the far infrared, cosmic singularity problem, anisotropic solutions, BBN mechanism, accelerated expansion, etc.

Extra

Gravitational Waves



- LIGO and Virgo Collaborations, PRL 116 (2016) 061102



- LIGO and Virgo Collaborations, PRL 116 (2016) 221101

Konoplya, Zhidenko, PLB 756 (2016) 350 :

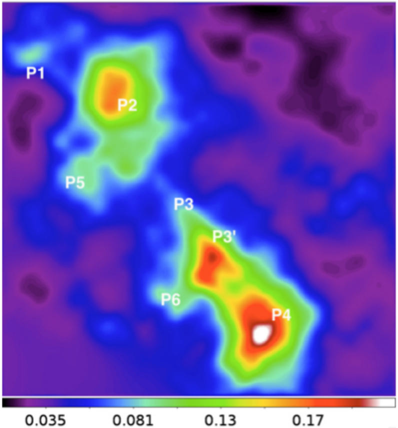
- "The last stages of formation of a single black hole and consequent quasinormal ringing represent intrinsic characteristics of a theory of gravity."
- "There might exist a strongly deformed Kerr-like black hole, corresponding to an alternative theory of gravity, such that its behavior in the post-Newtonian regime is quite similar to Kerr black hole, while its near-horizon behavior is different."

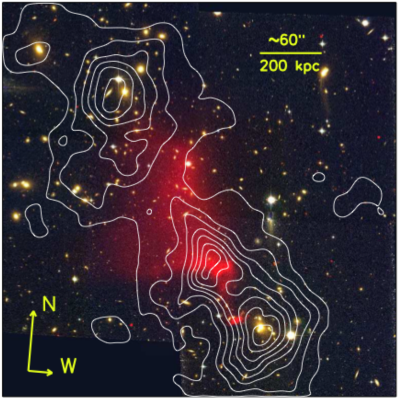
Dark Core in Train Wreck Cluster

The Astrophysical Journal, 783:78 (18pp), 2014 March 10
doi:10.1088/0004-637X/783/2/78

HUBBLE SPACE TELESCOPE/ADVANCED CAMERA FOR SURVEYS CONFIRMATION OF THE DARK SUBSTRUCTURE IN A520*

M. J. Jee[1], H. Hoekstra[2], A. Mahdavi[3], and A. Babul[4,5]





Bullet Cluster from ΛCDM Cosmology

THE ASTROPHYSICAL JOURNAL, 718:60–65, 2010 July 20 doi:10.1088/0004-637X/718/1/60

BULLET CLUSTER: A CHALLENGE TO ΛCDM COSMOLOGY

JOUNGHUN LEE[1] AND EIICHIRO KOMATSU[2]
[1] Department of Physics and Astronomy, FPRD, Seoul National University, Seoul 151-747, Republic of Korea; jounghun@astro.snu.ac.kr
[2] Texas Cosmology Center and Department of Astronomy, The University of Texas at Austin, 1 University Station, C1400 Austin, TX 78712, USA
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ABSTRACT

To quantify how rare the bullet-cluster-like high-velocity merging systems are in the standard Λ cold dark matter (CDM) cosmology, we use a large-volume ($27\,h^{-3}$ Gpc^3) cosmological *N*-body MICE simulation to calculate the distribution of infall velocities of subclusters around massive main clusters. The infall velocity distribution is given at $(1\text{–}3)R_{200}$ of the main cluster (where R_{200} is similar to the virial radius), and thus it gives the distribution of realistic initial velocities of subclusters just before collision. These velocities can be compared with the initial velocities used by the non-cosmological hydrodynamical simulations of 1E0657-56 in the literature. The latest parameter search carried out by Mastropietro & Burkert has shown that an initial velocity of 3000 km s^{-1} at about $2R_{200}$ is required to explain the observed shock velocity, X-ray brightness ratio of the main and subcluster, X-ray morphology of the main cluster, and displacement of the X-ray peaks from the mass peaks. We show that such a high infall velocity at $2R_{200}$ is incompatible with the prediction of a ΛCDM model: the probability of finding 3000 km s^{-1} in $(2\text{–}3)R_{200}$ is between 3.3×10^{-11} and 3.6×10^{-9}. A lower velocity, 2000 km s^{-1} at $2R_{200}$, is also rare, and moreover, Mastropietro & Burkert have shown that such a low initial velocity does not reproduce the X-ray brightness ratio of the main and subcluster or morphology of the main cluster. Therefore, we conclude that the existence of 1E0657-56 is incompatible with the prediction of a ΛCDM model, unless a lower infall velocity solution for 1E0657-56 with $\lesssim$1800 km s^{-1} at $2R_{200}$ is found.

Mon. Not. R. Astron. Soc. **383,** 417–423 (2008) doi:10.1111/j.1365-2966.2007.12403.x

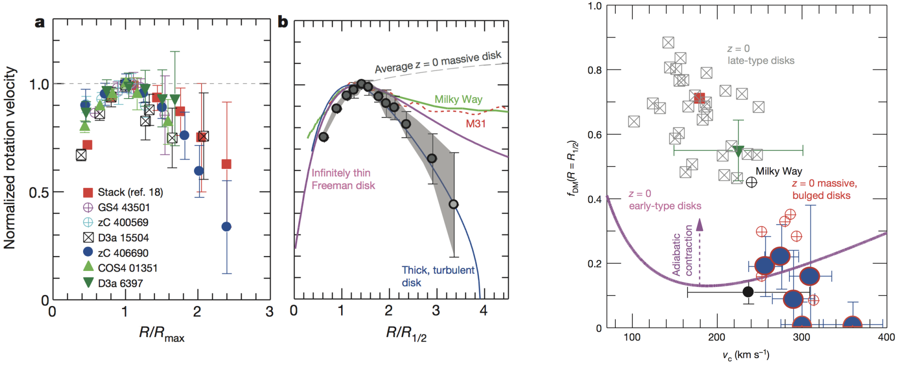
The collision velocity of the bullet cluster in conventional and modified dynamics

G. W. Angus[1★] and S. S. McGaugh[2★]

LETTER

doi:10.1038/nature21685

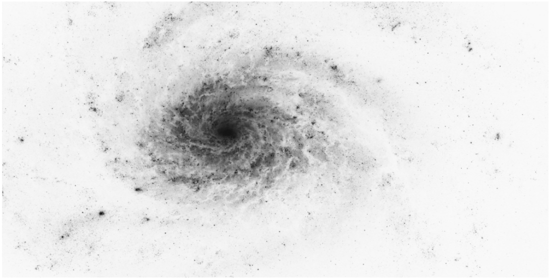
Strongly baryon–dominated disk galaxies at the peak of galaxy formation ten billion years ago

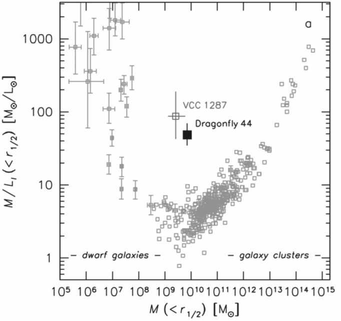


THE ASTROPHYSICAL JOURNAL LETTERS, 828:L6 (6pp), 2016 September 1 doi:10.3847/2041-8205/828/1/L6

A HIGH STELLAR VELOCITY DISPERSION AND ~100 GLOBULAR CLUSTERS FOR THE ULTRA-DIFFUSE GALAXY DRAGONFLY 44

PIETER VAN DOKKUM[1], ROBERTO ABRAHAM[2], JEAN BRODIE[3], CHARLIE CONROY[4], SHANY DANIELI[1], ALLISON MERRITT[1], LAMIYA MOWLA[1], AARON ROMANOWSKY[3,5], AND JIELAI ZHANG[2]





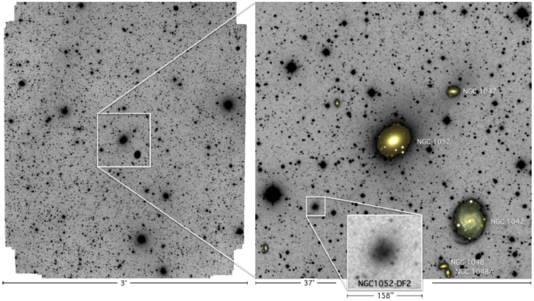
Dwarf galaxy NGC1052-DF2

LETTER

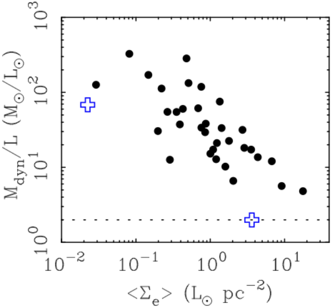
doi:10.1038/nature25767

A galaxy lacking dark matter

Pieter van Dokkum[1], Shany Danieli[1], Yotam Cohen[1], Allison Merritt[1,2], Aaron J. Romanowsky[3,4], Roberto Abraham[5], Jean Brodie[4], Charlie Conroy[6], Deborah Lokhorst[5], Lamiya Mowla[1], Ewan O'Sullivan[6] & Jielai Zhang[5]

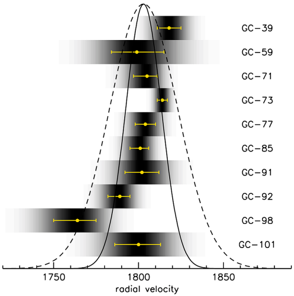


Extended Data Figure 1 | NGC1052–DF2 in the Dragonfly field. The full Dragonfly field, approximately 11 degree², centred on NGC 1052. The zoom-in shows the immediate surroundings of NGC 1052, with NGC1052–DF2 highlighted in the inset.



A galaxy lacking dark matter?

Furthermore, and paradoxically, the existence of NGC1052–DF2 may falsify alternatives to dark matter. In theories such as modified Newtonian dynamics (MOND)[25] and the recently proposed emergent gravity paradigm[26], a 'dark matter' signature should always be detected, as it is an unavoidable consequence of the presence of ordinary matter. In fact, it had been argued previously[27] that the apparent absence of galaxies such as NGC1052–DF2 constituted a falsification of the standard cosmological model and offered evidence for modified gravity. For a MOND acceleration scale of $a_0 = 3.7 \times 10^3\,\mathrm{km^2\,s^{-2}\,kpc^{-1}}$, the expected[28] velocity dispersion of NGC1052–DF2 is $\sigma_M \approx (0.05 G M_{\mathrm{stars}} a_0)^{1/4} \approx 20\,\mathrm{km\,s^{-1}}$, where G is the gravitational constant—a factor of two higher than the 90% upper limit on the observed dispersion.



A galaxy lacking dark matter?

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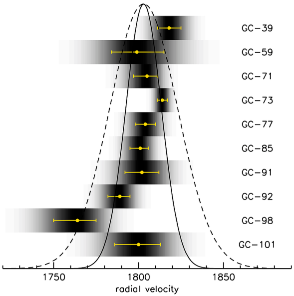

DRAFT VERSION APRIL 13, 2018
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CURRENT VELOCITY DATA ON DWARF GALAXY NGC1052-DF2 DO NOT CONSTRAIN IT TO LACK DARK MATTER

NICOLAS F. MARTIN[1,2], MICHELLE L. M. COLLINS[3], NICOLAS LONGEARD[1], ERIK TOLLERUD[4]

Draft version April 13, 2018