Tachyon inflation in a holographic braneworld

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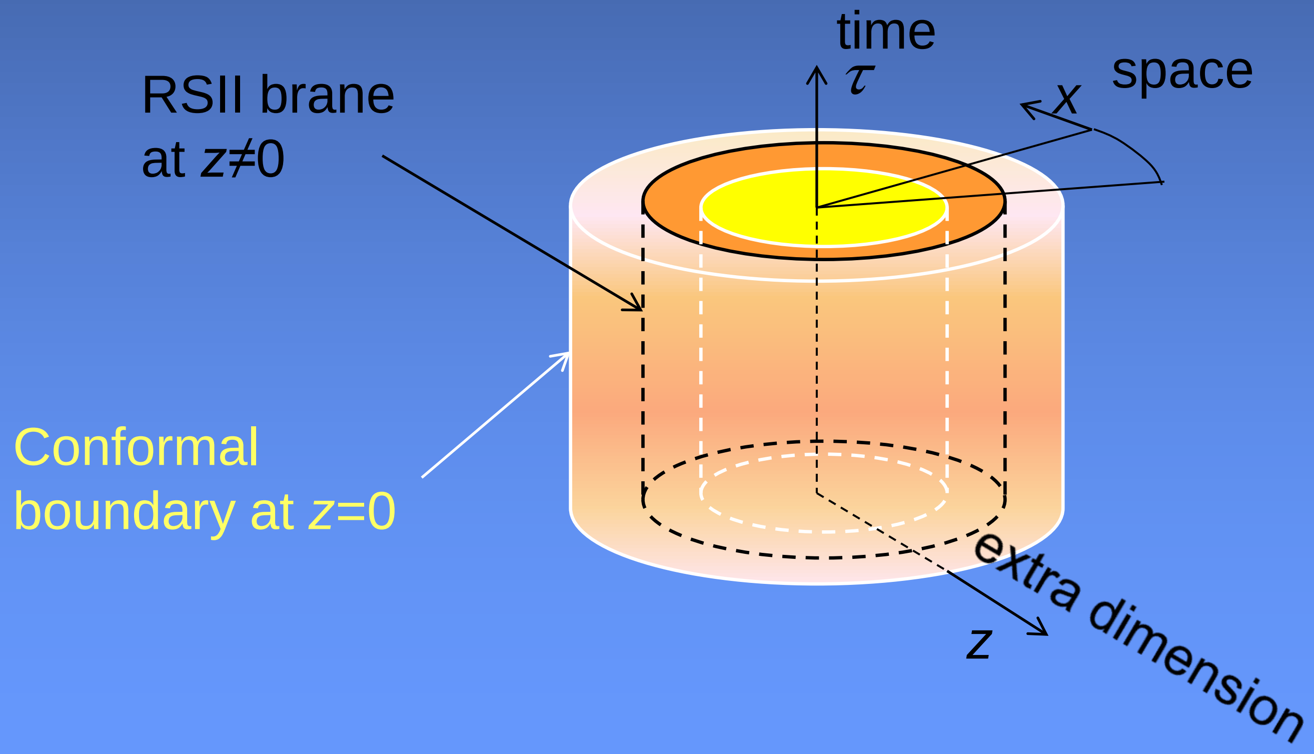
Basic idea

Holographic braneworld is a 3-brane located at the boundary of the asymptotic AdS_5. The cosmology is governed by matter on the brane in addition to the boundary CFT



Based on:

N.B. , S. Domazet and G. Djordjevic, Class. Quant. Grav. **34**, (2017) arXiv:1704.01072
N.B. , D. Dimitrijevic, G. Djordjevic and M. Milosevic, Int. J. Mod. Phys. A **32**, (2017) arXiv:1607.04524
N.B. et al., *Tachyon Inflation in a holographic braneworld*, in preparation

and related earlier works

P.S. Apostolopoulos, G. Siopsis and N. Tetradis, Phys. Rev. Lett. **102** (2009) arXiv:0809.3505
P. Brax and R. Peschanski, Acta Phys. Polon. **B 41** (2010) arXiv:1006.3054
N.B., Phys. Rev. D 93 (2016) arXiv:1511.07323

Outline

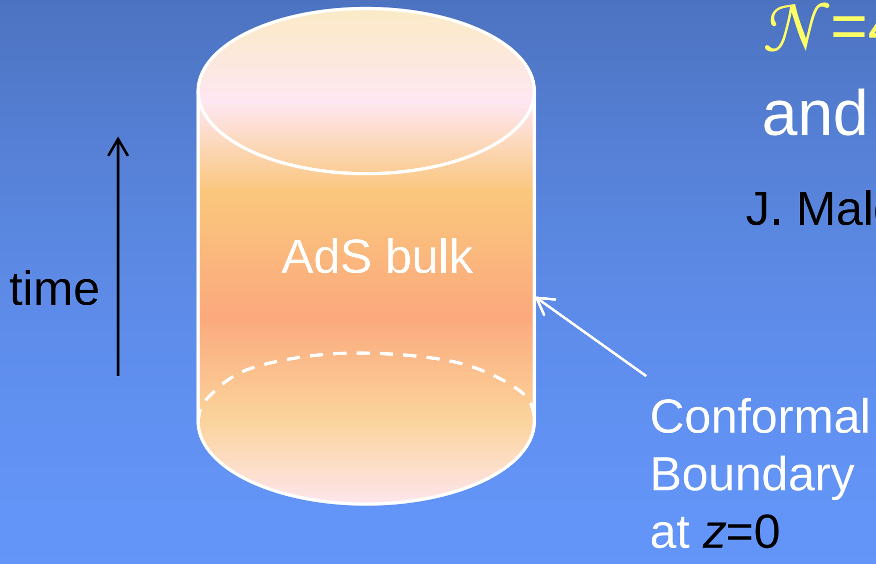
1. AdS/CFT
2. Holographic cosmology
3. Tachyon inflation
4. Conclusions and outlook

AdS/CFT

AdS/CFT correspondence is a holographic duality between gravity in $d+1$-dim space-time and quantum CFT on the d-dim boundary. Original formulation stems from string theory:

Equivalence of 3+1-dim
$\mathcal{N}=4$ Supersymmetric YM Theory
and string theory in $AdS_5 \times S_5$

J. Maldacena, Adv. Theor. Math. Phys. **2** (1998)



Consider a bulk action with only gravity in the bulk

$$S = \frac{1}{8\pi G_5} \int d^5x \sqrt{-G} \left(-\frac{R^{(5)}}{2} - \Lambda_5 \right)$$

Given induced metric $h_{\mu\nu}$ on the boundary the geometry is completely determined by the field equations obtained from the variation principle

$$\frac{\delta S_{(5)}}{\delta G_{\mu\nu}} = 0$$

Using the solution $G_{\mu\nu} = G_{\mu\nu}[h]$ we can define a functional

$$S[h] = S^{\substack{\text{on}\\\text{shell}}}\left[G_{\mu\nu}[h]\right]$$

where $S^{\substack{\text{on}\\\text{shell}}}\left[G_{\mu\nu}[h]\right]$ is the on shell bulk action

AdS/CFT conjecture: *S*[h] can be identified with the generating functional of a conformal field theory (CFT)on the boundary

$$S[h] \equiv \ln \int d\psi \exp\left\{-\int d^4x \sqrt{-h}\,\mathcal{L}^{\mathrm{CFT}}(\psi)\right\}$$

$\mathcal{L}^{\mathrm{CFT}}(\psi)$ – CFT Lagrangian

The induced metric $h_{\mu\nu}$ serves as the source for the **stress tensor** of the dual CFT so that its vacuum expectation value is obtained from the classical action

$$\left\langle T^{\mathrm{CFT}}_{\mu\nu} \right\rangle = \frac{1}{2\sqrt{-h}} \frac{\delta S}{\delta h^{\mu\nu}}$$

Holographic renormalization

The on-shell action is IR divergent and must be regularized and renormalized. The asymptotically AdS metric in the Fefferman-Graham form is

$$ds^2_{(5)} = G_{ab}dx^a dx^b = \frac{\ell^2}{z^2}(g_{\mu\nu}dx^\mu dx^\nu - dz^2)$$

where the length scale ℓ is the AdS curvature radius.
Near z=0 the metric can be expanded as

$$g_{\mu\nu} = g^{(0)}_{\mu\nu} + z^2 g^{(2)}_{\mu\nu} + z^4 g^{(4)}_{\mu\nu} + \cdots$$

Explicit expressions for $g^{(2n)}_{\mu\nu}$, in terms of arbitrary $g^{(0)}_{\mu\nu}$ can be found in

S. de Haro, S.N. Solodukhin, K. Skenderis, Comm. Math. Phys. **217** (2001)

We regularize the action by placing a brane (RSII brane) near the AdS boundary, i.e., at $z = \varepsilon\ell$, $\varepsilon \ll 1$, so that the induced metric on the brane is

$$h_{\mu\nu} = \frac{1}{\varepsilon^2}\left(g^{(0)}_{\mu\nu} + \varepsilon^2 \ell^2 g^{(2)}_{\mu\nu} + \cdots\right)$$

The bulk splits in two regions: $0 \le z \le \varepsilon\ell$, and $\varepsilon\ell \le z \le \infty$. We can either discard the region $0 \le z \le \varepsilon\ell$ (one-sided regularization) or invoke the Z_2 symmetry and identify two regions (two-sided regularization). For simplicity we shall use the one-sided regularization. The regularized bulk action is

$$S^{\mathrm{reg}}[h] = \frac{1}{8\pi G_5} \int_{z \ge \varepsilon\ell} d^5x \sqrt{-G}\left(-\frac{R^{(5)}}{2} - \Lambda_5\right) + S_{\mathrm{GH}}[h] + S_{\mathrm{br}}[h]$$

where $$S_{\mathrm{br}}[h] = -\sigma \int d^4x \sqrt{-h} + \int d^4x \sqrt{-h}\,\mathcal{L}_{\mathrm{matt}}$$

The renormalized boundary action is obtained by adding counter-terms and taking the limit ε→0

$$S^{\mathrm{reg}}[h] = \lim_{\varepsilon \to 0}\left(S^{\mathrm{ren}}[h] + S_1[h] + S_2[h] + S_3[h]\right)$$

The necessary counter-terms are

$$S_1[h] = -\frac{6}{16\pi G_5 \ell}\int d^4x\sqrt{-h},$$

$$S_2[h] = -\frac{\ell}{16\pi G_5}\int d^4x\sqrt{-h}\left(-\frac{R[h]}{2}\right),$$

$$S_3[h] = -\frac{\ell^3}{16\pi G_5}\int d^4x\sqrt{-h}\frac{\log\epsilon}{4}\left(R^{\mu\nu}[h]R_{\mu\nu}[h] - \frac{1}{3}R^2[h]\right)$$

S.W. Hawking, T. Hertog, and H.S. Reall, Phys. Rev. D **62** (2000)
de Haro et al. Commun. Math. Phys. **217** (2001)

Now we demand that the variation with respect to the induced metric $h^{\mu\nu}$ of the regularized on shell bulk action (RSII action) vanishes, i.e.,

$$\delta S^{\mathrm{reg}}[h] = 0$$

Which may be expressed as

matter on the brane

$$\delta\left[S^{\mathrm{ren}} - S_3 - \left(\sigma - \frac{3}{8\pi\ell G_5}\right)\int d^4x\sqrt{-h} + \int d^4x\sqrt{-h}\,\mathcal{L}_{\mathrm{matt}} + \frac{\ell}{16\pi G_5}\int d^4x\sqrt{-h}\,\frac{R}{2}\right] = 0.$$

cosmological constant

Einstein Hilbert term

AdS/CFT prescription

$$\delta(S^{\mathrm{ren}} - S_3) = \frac{1}{2}\int d^4x\sqrt{-h}\left\langle T^{\mathrm{CFT}}_{\mu\nu}\right\rangle \delta h^{\mu\nu}$$

The variation of the action yields Einstein's equations on the boundary

$$R_{\mu\nu} - \frac{1}{2} R g^{(0)}_{\mu\nu} = 8\pi G_{\mathrm{N}} \left(\left\langle T^{\mathrm{CFT}}_{\mu\nu} \right\rangle + T^{\mathrm{matt}}_{\mu\nu} \right)$$

where

$$\langle T^{\mathrm{CFT}}_{\mu\nu} \rangle = -\frac{\ell^3}{4\pi G_5} \left\{ g^{(4)}_{\mu\nu} - \frac{1}{8} \left[(\mathrm{Tr} g^{(2)})^2 - \mathrm{Tr}(g^{(2)})^2 \right] g^{(0)}_{\mu\nu} - \frac{1}{2} (g^{(2)})^2_{\mu\nu} + \frac{1}{4} \mathrm{Tr} g^{(2)} g^{(2)}_{\mu\nu} \right\}$$

de Haro et al, Comm. Math. Phys. **217** (2001)

This is an explicit realization of the AdS/CFT correspondence:
the vacuum expectation value of a boundary CFT operator is obtained in terms of geometrical quantities of the bulk.

Holographic cosmology

Starting from AdS-Schwarzschild static coordinates $(\tau, r, \chi, \vartheta, \varphi)$ and making the coordinate transformation $\tau = \tau(t,z),\ r = r(t,z)$ the line element will take a general form

$$ds_{(5)}^2 = \frac{\ell^2}{z^2}(g_{\mu\nu}dx^\mu dx^\nu - dz^2) = \frac{\ell^2}{z^2}\left[\mathcal{N}^2(t,z)dt^2 - \mathcal{A}^2(t,z)d\Omega_k^2 - dz^2\right]$$

$$d\Omega_\kappa^2 = d\chi^2 + \frac{\sin^2(\sqrt{\kappa}\chi)}{\kappa}(d\vartheta^2 + \sin^2\vartheta d\varphi^2)$$

Imposing the boundary conditions at z=0:

$$\mathcal{N}(t,0) = 1, \quad \mathcal{A}(t,0) = a(t)$$

we obtain the induced metric at the boundary in the FRW form

$$ds_{(0)}^2 = g_{\mu\nu}^{(0)}dx_\mu dx_\nu = dt^2 - a^2(t)d\Omega_k^2$$

Solving Einstein's equations in the bulk one finds

$$\mathcal{A}^2 = a^2\left[1-\left(H^2+\frac{\kappa}{a^2}\right)\frac{z^2}{4}\right]^2 + \frac{1}{4}\frac{\mu z^4}{a^4}, \qquad \mathcal{N} = \frac{\dot{\mathcal{A}}}{\dot{a}},$$

where $H \equiv \dot{a}/a$ is the Hubble rate at the boundary and μ is the dimensionless parameter related to the bulk BH mass

P.S. Apostolopoulos, G. Siopsis, and N. Tetradis, Phys. Rev. Lett. **102**, (2009)

P. Brax and R. Peschanski, Acta Phys. Polon. **B 41** (2010)

Comparing the exact solution with the expansion

$$g_{\mu\nu} = g_{\mu\nu}^{(0)} + z^2 g_{\mu\nu}^{(2)} + z^4 g_{\mu\nu}^{(4)} + \cdots$$

we can extract $g_{\mu\nu}^{(2)}$ and $g_{\mu\nu}^{(4)}$. Then, using the de Haro et al. expression for T^{CFT} we obtain

$$\left\langle T^{\rm CFT}_{\mu\nu}\right\rangle = t_{\mu\nu} + \frac{1}{4}\left\langle T^{{\rm CFT}\,\alpha}{}_{\alpha}\right\rangle g^{(0)}_{\mu\nu}$$

The second term is the conformal anomaly

$$\left\langle T^{{\rm CFT}\,\alpha}{}_{\alpha}\right\rangle = \frac{3\ell^3}{16\pi G_5}\frac{\ddot{a}}{a}\left(H^2 + \frac{\kappa}{a^2}\right)$$

The first term is a traceless tensor with non-zero components

$$t_{00} = -3t_i^i = \frac{3\ell^3}{64\pi G_5}\left[\left(H^2 + \frac{\kappa}{a^2}\right)^2 + \frac{4\mu}{a^4} - \frac{\ddot{a}}{a}\left(H^2 + \frac{\kappa}{a^2}\right)\right]$$

Hence, apart from the conformal anomaly, the CFT dual to the time dependent asymptotically AdS$_5$ metric is a conformal fluid with the equation of state $p_{\rm CFT} = \rho_{\rm CFT}/3$

where $\rho_{\rm CFT} = t_{00}$ $\qquad p_{\rm CFT} = -t_i^i$

From the boundary Einstein equations we obtain the holographic Friedmann equation (from now on we assume spatial flatness, i.e., we put $\kappa = 0$)

$$H^2 - \frac{\ell^2}{4} H^4 = \frac{8\pi G}{3}\rho + \frac{4\mu\ell^2}{a^4}$$

quadratic deviation

dark radiation

E. Kiritsis, JCAP **0510** (2005); Apostolopoulos et al, Phys. Rev. Lett. **102**, (2009)

The second Friedmann equation can be derived from the energy-momentum conservation

$$\dot{H}\left(1 - \frac{\ell^2}{2} H^2\right) = -4\pi G (p + \rho)$$

quadratic deviation

where $\rho = T_{00}^{\mathrm{matt}}, \quad p = -T^{\mathrm{matt}\,i}{}_{i}$

The holographic cosmology has interesting properties. Solving the first Friedmann equation as a quadratic equation for H^2 we find

$$H^2 = \frac{2}{\ell^2}\left(1 \pm \sqrt{1 - 8\pi\ell^2 G\rho / 3}\right),$$

Demanding that this equation reduces to the standard Friedmann equation in the low energy limit, i.e., in the limit when

$$\ell^2 G\rho \ll 1$$

it follows that we must discard the + sign solution. Then, it follows that the physical range of the Hubble rate is between 0 and $\sqrt{2}/\ell$ starting from its maximal value $H_{\max} = \sqrt{2}/\ell$ at an arbitrary initial time t_0. At that time, which may be chosen to be zero, the density and cosmological scale are both finite so the Big-Bang singularity is avoided!

Holographic cosmology appears also in other contexts. In particular it can be derived in

Modified Gauss-Bonnet gravity

$$S = \int d^4x \sqrt{-g}\left[\frac{1}{16\pi G}\left(-R + f\left(R,\mathcal{G}\right)\right) + \mathcal{L}^{\mathrm{matt}}\right]$$

where $\mathcal{G} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$ is the Gauss-Bonnet invariant

This model was shown to be ghost free.

see, e.g., I.L. Buchbinder, S.D. Odintsov, I.L. Shapiro, *Effective actions in quantum gravity* (IOP publishing, Bristol, 1992)

If in addition one requires that the second Friedmann equation is linear in $\dot{H}$, then *f* must be a function of only one variable *f* = *f*(*J*) where

$$J = \frac{1}{\sqrt{12}}\left(-R + \sqrt{R^2 - 6\mathcal{G}}\right)^{1/2}$$

C. Gao, Phys. Rev. D 86 (2012)

The second requirement cannot be fulfilled in a simple f(R) modified gravity including the Starobinski model

In a cosmological context with spatially flat metric one finds $J = \dot{a}/a \equiv H$, the function f becomes a function of H, and the first Friedmann equation takes the form

$$H^2 + \frac{1}{6} f - \frac{1}{6} H \frac{df}{dH} = \frac{8\pi G}{3} \rho$$

The left-hand side is a function of H only and takes the holographic form if

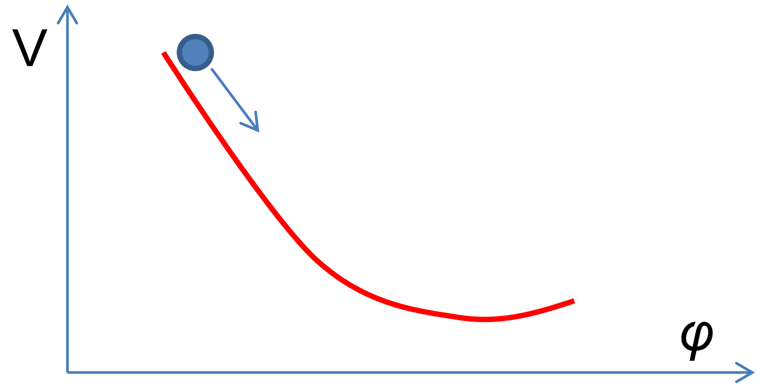
$$f(H) = \frac{1}{2} \ell^2 H^4$$

Hence, the holographic cosmology is reproduced in the modified Gauss-Bonnet gravity of the form

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{16\pi G} \left(-R - \frac{\ell^2}{288} \left(\sqrt{R^2 - 6\mathcal{G}} - R \right)^2 \right) + \mathcal{L}^{\mathrm{matt}} \right]$$

Inflation

One postulates a field, dubbed the *inflaton,* usually a self-interacting scalar that evolves towards the minimum of a slow roll potential. In conjunction with Friedman equation one solves the field equations from the beginning to the end of inflation. During inflation a slow roll regime is assumed, i.e., a very slow change of the Hubble rate so the Universe expands almost as a de Sitter spacetime with a large cosmological constant.



Quantum fluctuations of the inflaton field generate initial density perturbations of order $\delta\rho/\rho = 10^{-5}$ at the time of decoupling

$$t \approx 300\,000\,\text{years} \quad (z \approx 1000)$$

Tachyon inflation

One of the popular models of inflaton is the tachyon field θ of dimension of length with the Lagrangian

$$\mathcal{L} = -\ell^{-4} V(\theta / \ell) \sqrt{1 - g^{\mu\nu} \theta_{,\mu} \theta_{,\nu}}$$

Where ℓ is some length scale introduced to make the potential V dimensionless .

Our aim is to study tachyon inflation in the framework of holographic cosmology . The model is based on a holographic braneworld scenario with an effective tachyon field on a D3-brane located at the holographic boundary of ADS_5. In our model we identify ℓ as the curvature radius of AdS_5

The covariant Hamiltonian corresponding to $\mathcal{L}$ is

$$\mathcal{H} = \ell^{-4} V \sqrt{1 + \pi_\theta^2 / V^2}$$

where $$\pi_\theta = \ell^{-4} \sqrt{g_{\mu\nu} \pi_\theta^\mu \pi_\theta^\nu}$$

The conjugate momentum π_θ^μ is, as usual, related to $\theta_{,\mu}$ via

$$\pi_\theta^\mu = \frac{\partial \mathcal{L}}{\partial \theta_{,\mu}}$$

Now we assume that the holographic braneworld is a spatially flat FRW universe with line element

$$ds^2 = g^{\mu\nu} dx^\mu dx^\nu = dt^2 - a^2(t)\left[dr^2 + r^2 d\Omega^2 \right]$$

From the covariant Hamilton's equations

$$\theta_{,\mu} = \frac{\partial \mathcal{H}}{\partial \pi_\theta^\mu} \qquad \pi^\mu_{\theta\,;\mu} = -\frac{\partial \mathcal{H}}{\partial \theta}$$

we obtain two first order differential equations in comoving frame

$$\dot{\theta} = \frac{\pi_\theta}{\sqrt{V^2 + \pi_\theta^2}}, \qquad \dot{\pi}_\theta = -3H\pi_\theta - \frac{VV_{,\theta}}{\sqrt{V^2 + \pi_\theta^2}}$$

The Lagrangian and Hamiltonian are identified with the pressure and energy density: $p \equiv \mathcal{L}, \;\; \rho \equiv \mathcal{H}$, and we employ the holographic Friedmann equations

$$H^2 - \frac{\ell^2}{4} H^4 = \frac{8\pi G}{3}\rho \qquad \dot{H}\left(1 - \frac{\ell^2}{2} H^2\right) = 4\pi G(p + \rho)$$

We also demand that these equations reduce to the standard Friedmann equation in the low energy limit.

Inflation on the holographic brane

Tachyon inflation is based upon the slow evolution of the field θ with the slow-roll conditions

$$\dot{\theta}^2 \ll 1, \qquad |\ddot{\theta}| \ll 3H\dot{\theta}.$$

Then, during inflation we find

$$h^2 \equiv H^2\ell^2 \simeq 2(1-\sqrt{1-\kappa^2 V/3}),$$

where $\kappa^2 = 8\pi G/\ell^2$

and the evolution is constraint to the physical range of the Hubble rate

$$0 \le h^2 \le 2$$

In the following we will examine a simple exponential potential

$$V = e^{-\omega\theta/\ell}$$

where ω is a free dimensionless parameter. We will also consider the initial value h_i^2 as a free parameter ranging between 0 and 2.

The most important parameters that characterize inflation are the slow-roll inflation parameters ε_j defined recursively

$$\varepsilon_{j+1} \equiv \dot{\varepsilon}_j / (H\varepsilon_j)$$

starting from $\varepsilon_0 \equiv H_* / H$, where H_* is the Hubble rate at some chosen time. The next two are given by

$$\varepsilon_1 \equiv -\frac{\dot{H}}{H^2} = \frac{\omega^2(4-h^2)}{6h^2(2-h^2)}, \quad \varepsilon_2 \equiv \frac{\dot{\varepsilon}_1}{H\varepsilon_1} = 2\varepsilon_1\left(1-\frac{2h^2}{(2-h^2)(4-h^2)}\right)$$

During inflation $\varepsilon_{1,2} < 1$ and inflation ends once either ε_1 or ε_2 exceeds 1. Near the end of inflation $h^2 \simeq \kappa^2 V/3 \ll 1$ and $\varepsilon_2 \simeq 2\varepsilon_1$.

Another important quantity is the so called number of e-folds defined as

$$N \equiv \int_{t_\mathrm{i}}^{t_\mathrm{f}} H dt$$

where the subscripts **i** and **f** denote the beginning and the end of inflation. Typically $N \simeq 50-60$ is sufficient to solve the flatness and horizon problems

From the field equations we find an approximate equation

$$\dot{\theta} \simeq \frac{\omega}{3H}$$

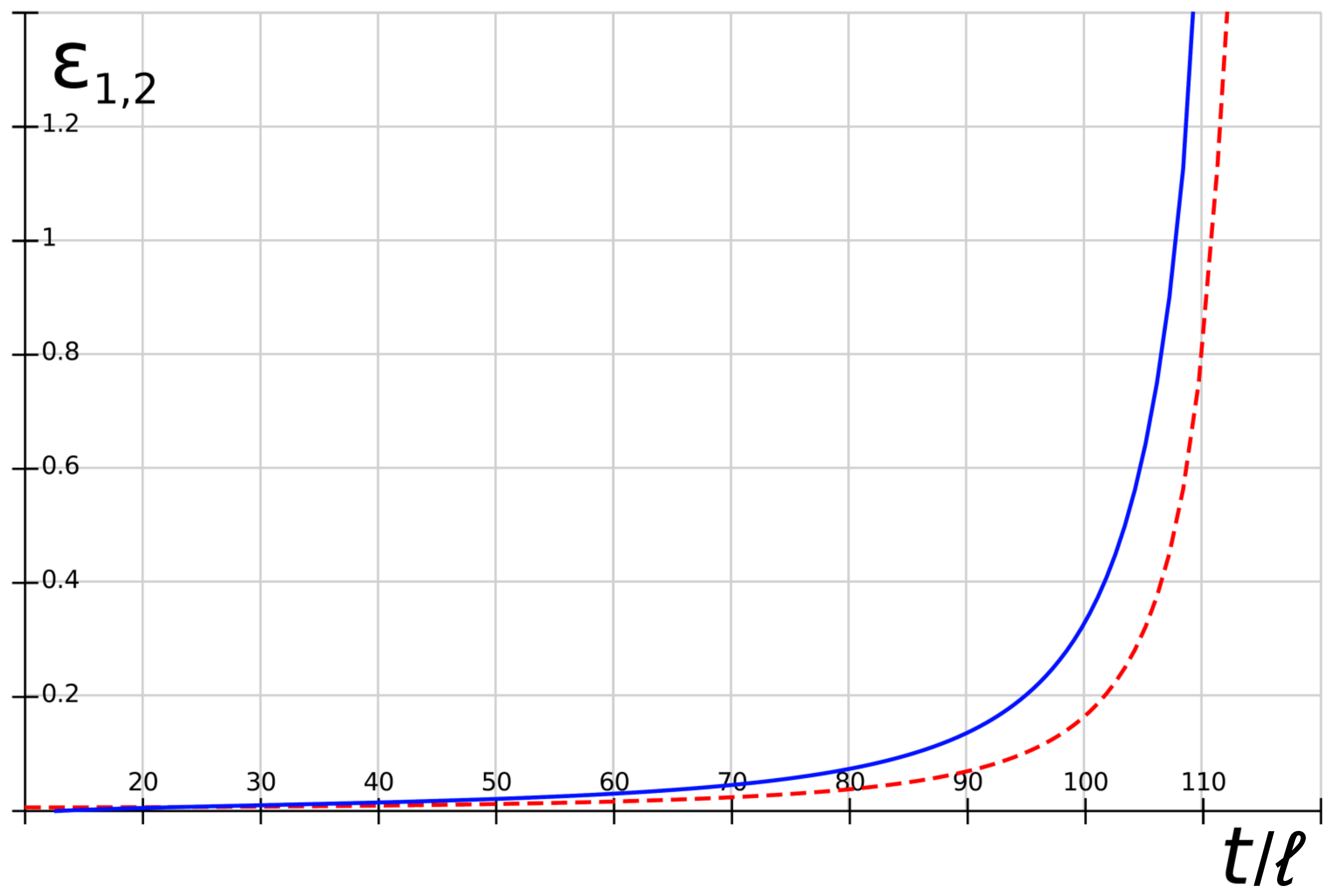
which can be easily integrated yielding the time as a function of H in the slow roll regime

$$t = \frac{3}{\omega^2}\left[2(\sqrt{2}-h) + \ln\frac{(\sqrt{2}-1)(2+h)}{(\sqrt{2}+1)(2-h)}\right], \qquad h \equiv H\ell$$

The number of e-folds can also be calculated explicitly yielding an expression that relates our free parameters h_i and ω to N

$$N = \frac{12}{\omega^2}\left[\sqrt{1-\frac{\omega^2}{3}} - 1 + \frac{h_i^2}{2} + \ln\left(2-\frac{h_i^2}{2}\right) - \ln\left(1+\sqrt{1-\frac{\omega^2}{3}}\right)\right]$$

Hence, for a fixed chosen N we have only one free parameter



Slow roll parameters ε_1 (dashed red line) and ε_2 (blue line) versus time for fixed N=60 and ω^2=0.027 corresponding to the initial h_i^2=0.6

The slow-roll parameters ε_j are related to the observational quantities such as the tensor-to-scalar ratio r and the scalar spectral index n_s defined by

$$r = \frac{\mathcal{P}_T}{\mathcal{P}_S}, \qquad n_s = \frac{d \ln \mathcal{P}_S}{d \ln q}$$

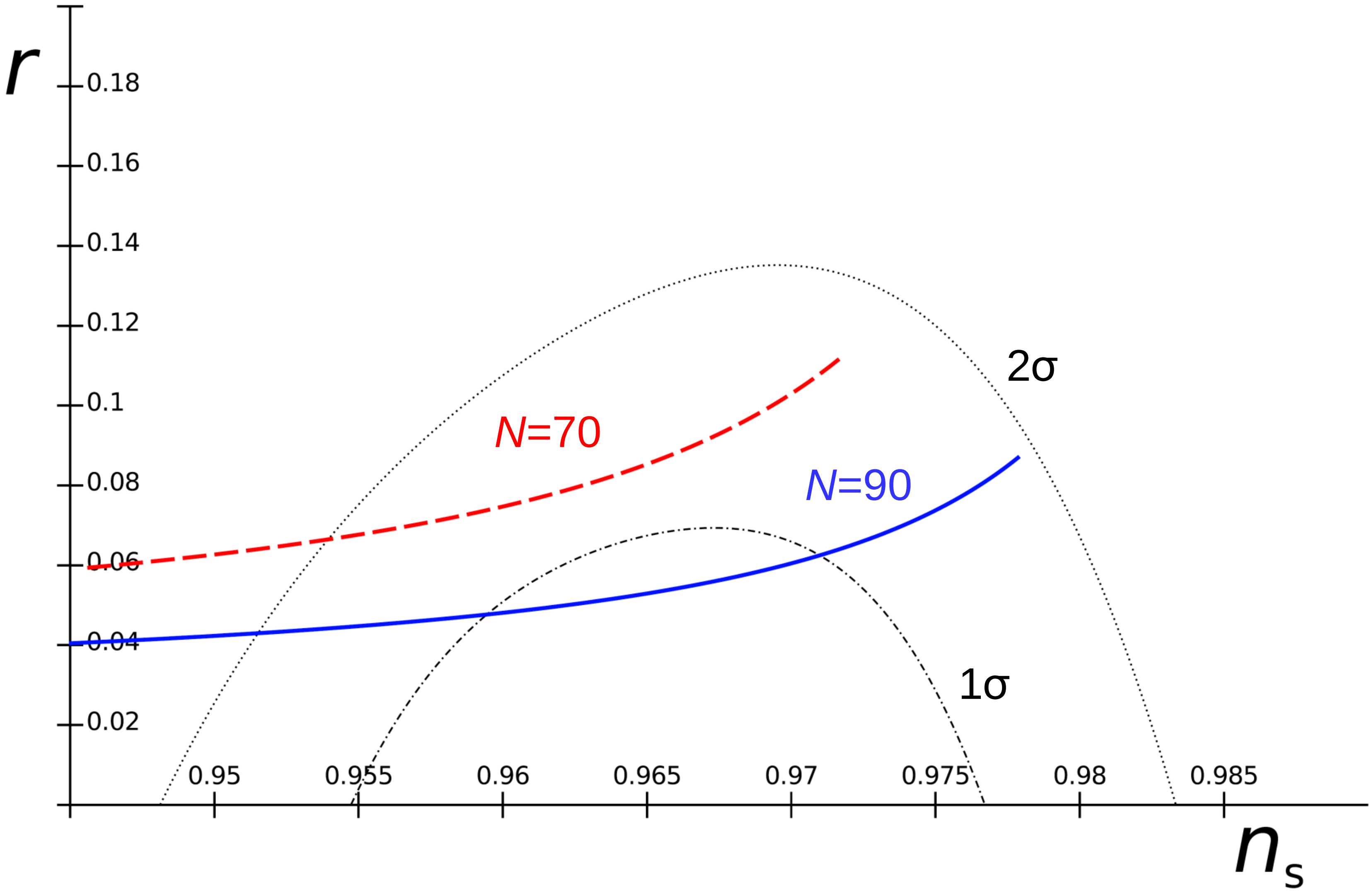
where $\mathcal{P}_S$ and $\mathcal{P}_T$ are the power spectra of scalar and tensor perturbations, respectively, evaluated at the horizon, i.e., for a wave-number satisfying $q=aH$. One finds at the lowest order in ε_1 and ε_2

$$r = 16(1-h^2)\varepsilon_1 \left(1 - C\varepsilon_2 - \frac{4-2h^2}{12-3h^2}\varepsilon_1 \right)$$

$$n_s - 1 = \left(-3 + \frac{2-3h^2}{2-h^2} \right)\varepsilon_1 - \varepsilon_2$$

Deviations from the standard tachyon inflation

where $C = -2 + \ln 2 + \gamma \simeq -0.72$



r versus n_s for fixed *N* and varying initial value h_i^2 ranging from 0 to 2 along the lines. The parameter *ω* is also varying in view of the functional dependence $N=N(h_i, \omega)$. The black lines denote the Planck constraints contours of the 1*σ* (dash-dotted) and 2*σ* (dotted) confidence level

Conclusions and outlook

- We have shown that the slow-roll equations of the tachyon inflation with exponentially attenuating potential on the holographic brane are quite distinct from those of the standard tachyon inflation with the same potential
- The n_s - r relation depends on the initial value of the Hubble rate and on the assumed value of the number of e-folds N and show a reasonable agreement with the Planck 2015 data for $N > 70$.
- The presented results obtained in the slow roll approximation are preliminary. What remains to be done is to solve the exact equations numerically for the same potential and for various other potentials that have been exploited in the literature.

Thank you

Basic idea

Braneworld cosmology is based on the scenario in which matter is confined on a brane in a higher dimensional bulk with only gravity allowed to propagate in the bulk. The brane can be placed, e.g., at the boundary of a 5-dim asymptotically Anti de Sitter space (AdS_5)

Why AdS?

Anti de Sitter space is dual to a conformal field theory at its boundary (AdS/CFT correspondence)
AdS is a maximally symmetric solution to Einstein's equations with negative cosmological constant. In 4+1 dimensions the symmetry group is $AdS_5 \equiv SO(4,2)$

The 3+1 boundary conformal field theory is invariant under conformal transformations: Poincare + dilatations + special conformal transformation = conformal group ≡ SO(4,2)

AdS/CFT conjecture: $S[\varphi, h]$ can be identified with the generating functional of a conformal field theory on the boundary

$$S[\varphi,h] \equiv \ln \int d\psi \exp\left\{-\int d^4x\sqrt{-h}\left[\mathcal{L}^{\mathrm{CFT}}(\psi(x)) - O(\psi(x))\varphi(x)\right]\right\}$$

where the boundary fields serve as sources for CFT operators

$\mathcal{L}^{\mathrm{CFT}}(\psi)$ – conformal field theory Lagrangian

$O(\psi)$ – operators of dimension Δ

In this way the CFT correlation functions can be calculated as functional derivatives of the on-shell bulk action, e.g.,

$$\frac{\delta^2 S}{\delta\varphi(x)\delta\varphi(y)} = \langle O(\psi(x))O(\psi(y))\rangle - \langle O(\psi(x))\rangle\langle O(\psi(y))\rangle$$

The induced metric $h_{\mu\nu}$ serves as the source for the stress tensor of the dual CFT so that its vacuum expectation value is obtained as

$$\frac{1}{2\sqrt{-h}}\frac{\delta^2 S}{\delta h^{\mu\nu}} = \langle T^{\mathrm{CFT}}_{\mu\nu}\rangle$$

The Planck mass scale is determined by the curvature of the five-dimensional space-time

$$\frac{1}{G_{\mathrm{N}}} = \frac{\gamma}{G_5}\int_0^\infty e^{-2y/\ell}\,dy = \frac{\gamma\ell}{2G_5} \qquad \gamma = \begin{cases} 1 & \text{one-sided} \\ 2 & \text{two-sided} \end{cases}$$

One usually imposes the RS fine tuning condition

$$\sigma = \sigma_0 \equiv \frac{3\gamma}{8\pi G_5 \ell} = \frac{3}{8\pi G_{\mathrm{N}} \ell^2}$$

which eliminates the 4-dim cosmological constant.

Bound on the AdS_5 curvature radius ℓ:

The classical 3+1 dim gravity is altered on the RSII brane
For $r \gg \ell$ the Newtonian potential of an isolated source on the brane is given by

$$\Phi(r) = \frac{G_N M}{r}\left(1 + \frac{2\ell^2}{3r^2}\right)$$

J. Garriga and T. Tanaka, Phys. Rev. Lett. 84, 2778 (2000)

Table top tests of Long et al find no deviation of Newton's potential and place the limit

$$\ell < 0.1\,\mathrm{mm} \quad \text{or} \quad \ell^{-1} > 10^{-12}\,\mathrm{GeV}$$

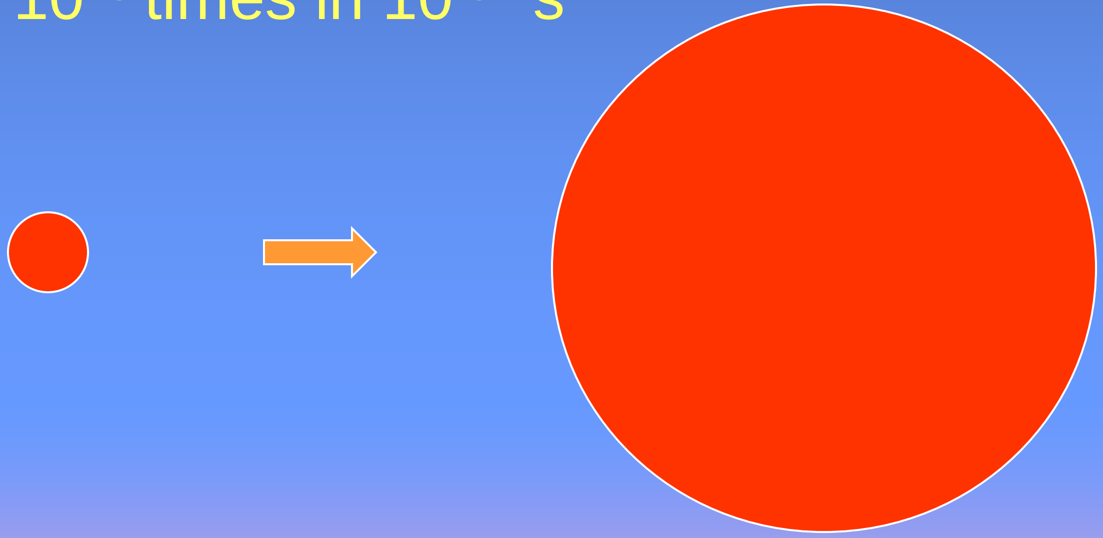
Holographic type cosmologies appear also in other contexts:

1. The saddle point of the spatially closed mini superspace partition function dominated by matter fields conformally coupled to gravity
   A. O. Barvinsky, C. Deffayet and A. Y. Kamenshchik, JCAP **0805**, (2008) arXiv:0801.2063
2. A modified Friedmann equation with a quartic term $\sim H^4$ derived from the generalized uncertainty principle and the first low of thermodynamics applied to the apparent horizon entropy.
   J. E. Lidsey, Phys. Rev. D **88** (2013) arXiv:0911.3286
3. The quartic term as a quantum correction to the Friedmann equation using thermodynamic arguments at the apparent horizon [37]
   S. Viaggiu, Mod. Phys. Lett. A, **31** (2016) arXiv:1511.06511
4. Modified Gauss-Bonnet gravity
   G. Cognola et al Phys. Rev. D 73 (2006), C. Gao, Phys. Rev. D 86 (2012);

Inflation

A short period of rapid expansion at the very early universe (starting at about 10^{-43} s after the Big Bang)

10^{25} times in 10^{-32} s



The main problems of the standard cosmology solved by inflation

- Horizon problem – homogeneity and isotropy of the CMB radiation
- Flatness problem – fine tuning of the initial conditions
- Large scale structure problem – the origin of initial density perturbations that serve as seeds of the observed structure today
- Monopole problem – absence of topological defects: monopoles, cosmic strings, domain walls

Tachyon inflation

One of the popular models of inflaton is the *tachyon*. Our aim is to study tachyon inflation in the framework of holographic cosmology . The model is based on a holographic braneworld scenario with an effective tachyon field on the D3-brane located at the holographic bound of ADS bulk. A tachyon Lagrangian of the form

$$\mathcal{L} = -\ell^{-4} V(\theta / \ell)\sqrt{1 - g^{\mu\nu}\theta_{,\mu}\theta_{,\nu}}$$

can be derived in the context of a dynamical brane moving in a 4+1 background with a general warp

$$ds^2_{(5)} = \frac{1}{\chi^2(z)}(g^{\mu\nu}dx^\mu dx^\nu - dz^2)$$

The field θ is identified with the 5-th coordinate z and the potential is related to the warp

$$V(\theta / \ell) = \ell^4 / \chi^4(\theta)$$

N.B., S. Domazet and G. Djordjevic, Class. Quant. Grav. **34**, (2017) arXiv:1704.01072.