ASYMPTOTIC SAFETY

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Introduction and motivation

Imagine a gauge theory without asymptotic freedom ($\beta_1 > 0$)

$$\mu \frac{dg}{d\mu} = \frac{\beta_1}{16\pi^2} g^3 \quad \to \quad g^2(\mu) = \frac{8\pi^2}{\beta_1 \log\left(\Lambda/\mu\right)}$$

$\Lambda =$ Landau pole $\qquad (g(\Lambda) = \infty)$

We got a Landau pole at 1-loop

What about higher loops?

$$\mu \frac{dg}{d\mu} = \frac{\beta_1}{16\pi^2} g^3 + \frac{\beta_2}{(16\pi^2)^2} g^5 + \dots$$

The Landau pole may be avoided if the gauge coupling (and eventually other couplings) flow to a finite value $g_* \neq 0$:

$$\frac{\beta_1}{16\pi^2} g_*^3 + \frac{\beta_2}{(16\pi^2)^2} g_*^5 + \dots = 0$$

$\rightarrow$ asymptotic safety

The problem is that unless β_1 parametrically small (see later), this important only if

$$\frac{g^2}{16\pi^2} \gtrsim \mathcal{O}(1)$$

destroying perturbativity.

Hard to work with non-perturbativity.

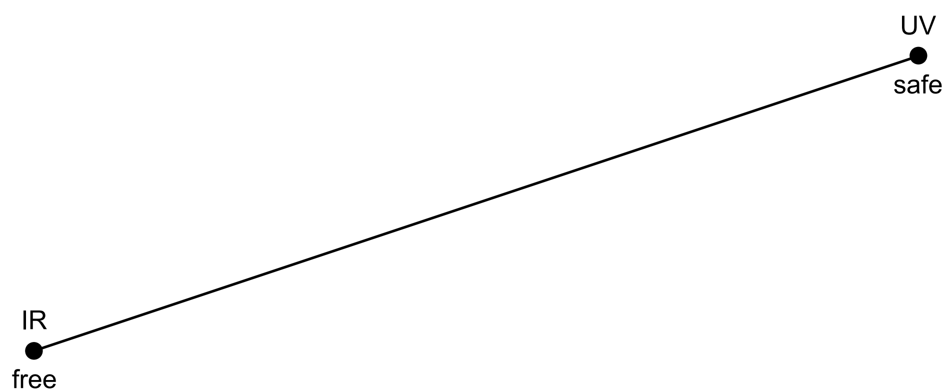
However if such a non-trivial UV fixed point exists, then the theory in the UV is asymptotically conformal (no running). We lost perturbativity but gained conformal symmetry

This we will use in connection with supersymmetry

Constraints on conformal field theories

Our set-up is a $d = 4$ supersymmetric theory

- free in the IR
- with hypothetical UV interacting fixed point= asymptotically safe theory



Trace anomaly for stress-energy tensor $T^{\mu\nu}$ in curved background :

$$T^{\mu}{}_{\mu} = -a \times E_4 + \ldots$$

Euler invariant

$$E_4 = R^{\alpha\beta\gamma\delta} R_{\alpha\beta\gamma\delta} - 4 R^{\alpha\beta} R_{\alpha\beta} + R^2$$

quadratic diffeomorphism invariant combination

$a \ldots$ central charge

The most important (non-perturbative) constraint on RG flows is the famous a-theorem (4d version of the 2d c-theorem):

$$\Delta a \equiv a_{UV} - a_{IR} > 0$$

Because of it

- RG flow is irreversible
- a provides a measure for # of d.o.f.

Calculation of the a-central charge

In a generic field theory a can be calculated perturbatively.

In most case this not useful because fixed point non-perturbative

Fortunately in supersymmetry central charges can be got exactly

(R_i, n_i) ... (R − charge, # d.o.f.) of chiral field i

$|G|$ ... dimension of gauge group G = # of gauge fields

$$a = \underbrace{2|G|}_{gaugino} + \underbrace{\sum_i n_i a_1(R_i)}_{chiral\ fields} \quad , \quad a_1(R) = 3(R-1)^3 - (R-1)$$

Total a equal to sum of single a_1 (one for each chiral multiplet)

This exact relation is due to the fact that

$T_{\mu\nu}$ and j_R^μ are different components of the same supermultiplet

$\rightarrow$ relations between $T^\mu{}_\mu$ and $\partial_\mu j_R^\mu$:

$$
\begin{aligned}
T^\mu{}_\mu &= -a\, E_4 + \ldots \\
\partial_\mu j_R^\mu &= \underbrace{\left[Tr\, U(1)_R\right]}_{\propto \sum_i n_i (R_i - 1)} R_{\alpha\beta\gamma\delta} \tilde{R}^{\alpha\beta\gamma\delta} + \underbrace{\left[Tr\, U(1)_R^3\right]}_{\propto \sum_i n_i (R_i - 1)^3} F_{R\mu\nu} \tilde{F}_R^{\mu\nu}
\end{aligned}
$$

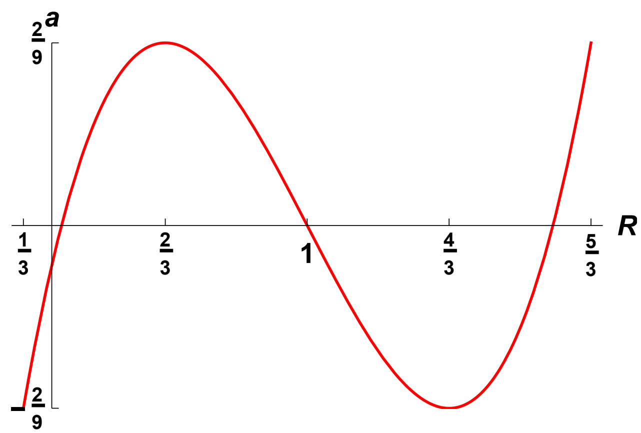
$U(1)_R$ symmetry unavoidable in supersymmetric fixed points (conformal theories): R charge part of the superconformal algebra

Since

$$R(\text{chiral superfield}) = \frac{2}{3} D(\text{chiral superfield})$$

for a free theory $(D(\phi_{free}) = 1)$

$$R(\phi_{free}) = 2/3$$



$a_1(R_{UV}) > a_1(R_{IR}) = a_1(2/3)$ not possible unless

at least one field in UV has $R > 5/3$ (necessary but not sufficient)

If we know the R-charges, we know the central charge a

How do we get the R-charges R_i?

In SCFT the β functions must vanish:

- NSVZ β function is proportional to

$$T(G) + \sum_i T(r_i)(R_i - 1) = 0$$

  $T \ldots$Dynkin index

- β function for superpotential coupling λ_a of

$$W = \lambda_a \prod_i \phi_i^{q_{ia}}$$

  is proportional to

$$\sum_i q_{ia} R_i - 2 = 0$$

Three possibilities:

1. # of constraints above bigger than number of chiral fields
   → no SCFT

2. # of constraints above equal to number of chiral fields
   → the solution to above equations unique and represents a possible candidate for CFT; to check consistency with $\Delta a > 0$

3. # of constraints above smaller than number of chiral fields
   → one uses the above equations to express some R-charges with the others; then applies the a-maximization to calculate the remaining R-charges:

a-maximization:

$$\frac{\partial a}{\partial R_i} = 0$$

This gives same number of equations than unknowns R_i.

Equations are quadratic so there can be several real solutions. One should choose the one with

$$\frac{\partial^2 a}{\partial R_i \partial R_j} \qquad \text{all negative eigenvalues}$$

Examples

We will see now

- 2 different ways of getting consistent UV fixed points
- an almost realistic UV safe GUT

SO(10) at large N_f

One type of representation only will not work. NSVZ:

$$T_G + n_1 T_1 (R_1 - 1) = 0 \quad \rightarrow \quad R_1 = 1 - \frac{T_G}{n_1 T_1} < 1$$

In the IR $R_1 = 2/3$ and $a_{IR} > a_{UV}$ (a-theorem violated)

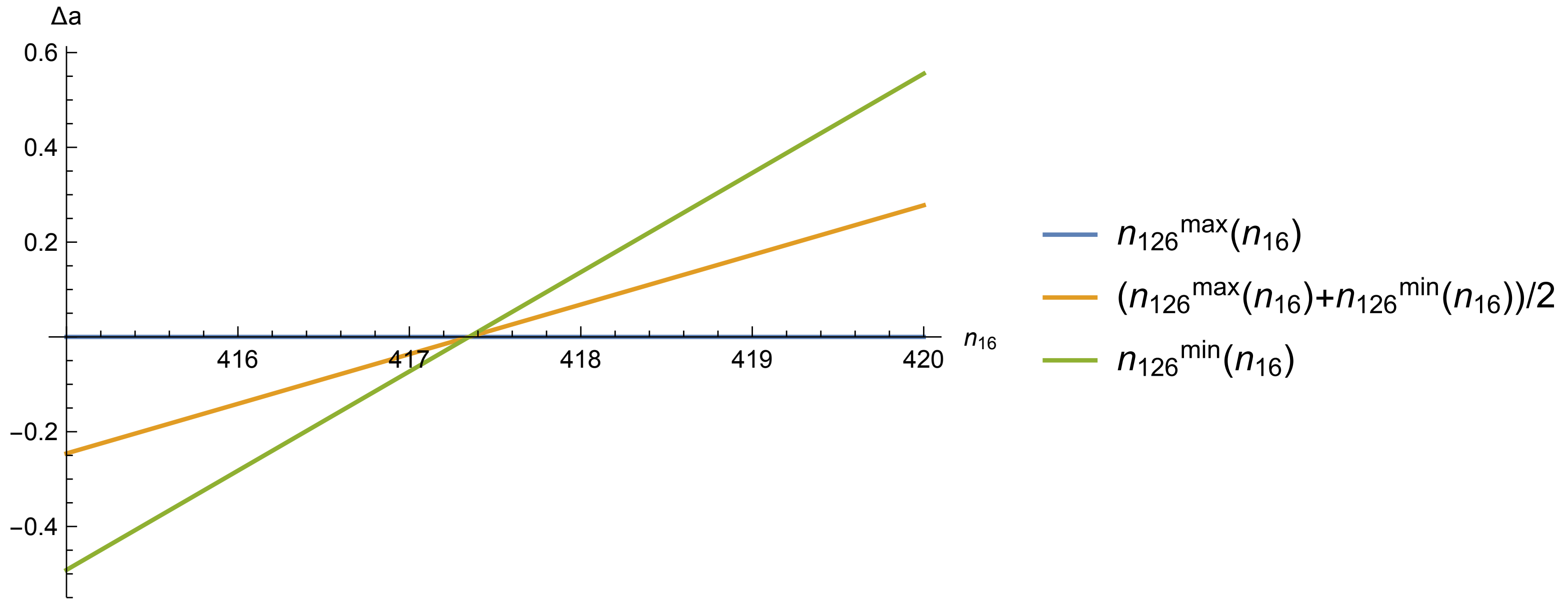
Imagine we have

n_{16} generations of representation 16

n_{126} generations of representation 126

$NSVZ = 0$ plus $a-$maximization to get R_{16} and R_{126}

The number of generations involved needs to be very large:
$n_{16} \geq 418$



Number of such solutions is ∞ (not bounded from above)

For $n_{16} \to \infty$ the solution exists providing

$$\frac{1}{7}\sqrt{\frac{3}{38}} < \frac{n_{126}}{n_{16}} < \frac{2(\sqrt{301}-11)}{315}$$

In numbers

$$0.0401394 < \frac{n_{126}}{n_{16}} < 0.0403133$$

This is a large $\underbrace{N_{f1}/N_c}_{n_{126}/10}$ and $\underbrace{N_{f2}/N_c}_{n_{16}/10}$ case with bounded $\underbrace{N_{f1}/N_{f2}}_{n_{126}/n_{16}}$

Non-zero superpotential

The first candidate for a supersymmetric UV fixed point found by Martin, Wells:

Take SU(N_c) with 2 adjoints X, Y plus $N_f \times (Q + \tilde{Q})$ and

$$W = y_1 \tilde{Q} X Q + y_2 Tr\, X^3$$

Automatically $R(Q) = R(\tilde{Q}) = R(X) = 2/3$ and

$$T_G + T_X\,(R(X) - 1) + T_Y\,(R(Y) - 1) + 2N_f T_Q\,(R(Q) - 1) = 0$$

$\rightarrow \quad \Delta a > 0 \quad$ if $\quad N_f > 4N_c$

The idea here is to make the superpotential terms determine R-charges of all fields except 1, the last one being determined by the vanishing NSVZ.

In Martin-Wells example, all fields (X, Q, $\tilde{Q}$) have $R = 2/3$ except one (Y) which has $R > 5/3$.

Possible to generalize. For example take $N_c/N_f = 0.46$, $N_c \to \infty$, and

$$W = y_1 \tilde{Q} X^4 Q + y_2 Tr\ X^6$$

leads to UV fixed point with all constraints satisfied.

A quasi-realistic UV safe theory

This is a phenomenological use of the above.

Minimal SO(10) grand-unified theory (GUT):

Matter fields:

$$3 \times 16 = 3 \times (\underbrace{Q + L + u^c + d^c + e^c}_{SM} + \nu^c)$$

Higgses:

$$10 = \underbrace{H_u + H_d}_{\text{MSSM Higgses}} + T + \bar{T}$$

210, 126, $\overline{126}$ needed to break SO(10)→SM and give correct masses to SM charged fermions and neutrinos (type I+II seesaw)

Huge SO(10) representations:

$$\beta_1 = +109$$

Naively 1-loop Landau pole

Right place to look for possible asymptotic safety

We can keep or not various trilinear terms in the superpotential

The only solution is the superpotential

$$W = y_1\, 210^3 + y_2\, 210\, 126\, \overline{126} + y_3\, 210\, 126\, 10 + y_4\, 210\, \overline{126}\, 10 + \sum_{a,b=2,3} 16_a\, 16_b\, \left(y_{5,ab}\, 10 + y_{6,ab}\, \overline{126} \right)$$

i.e. all the most general trilinear couplings except that 16_1 never appearing in W (first generation massless)

The constraints (all β-functions vanishing) fix

$$R(16_1) = \frac{113}{6}$$

and all other $R = 2/3$.

Duality

Seiberg type dualities connect theories with different gauge group and field content but same flavor structure. Example SQCD:

ELECTRIC	MAGNETIC
$SU(N_c)\ :\ g$	$SU(N_f - N_c)\ :\ \tilde{g}$
$N_f * \left(Q + \tilde{Q}\right)$	$N_f * (q + \tilde{q}) + N_f^2 * M$
$W = 0$	$W = \tilde{y}\, qMq$

- valid only in the IR (in the fixed point)
- valid only in the conformal window $3N_c/2 \leq N_f \leq 3N_c$
- quantum numbers of magnetic singlets $M \sim \tilde{Q}Q$
- at least one of the two theories must be strongly coupled
- duality of type strong↔weak

If a theory has a nontrivial UV fixed point and a nontrivial IR fixed point, and we know the duals of both of them, then reasonable that they are *dual in the whole flow*

The examples of UV fixed points considered so far do not have a nontrivial IR fixed points (no duals in the IR), but also in UV we do not know the dual

We can have the following simple example:

1. first at $t \to \infty$ have SQCD with SU(N_c) and $N_f + 1$ quarks
2. run down to the IR, usual duality with the magnetic theory
3. perturb the electric theory with a mass m for 1 quark pair; if this mass deep in the fixed point regime, duality still valid
4. this is the starting point (new UV) for the flow to the new IR
5. in the deep IR again (a new) duality

$t \equiv \log(\mu/m)$	IR $(t < 0)$	UV $(t > 0)$
magnetic theory	N_f flavours $\tilde{N}_c$ colours	$N_f + 1$ flavours $\tilde{N}_c + 1$ colours
electric theory	N_f flavours N_c colours	$N_f + 1$ flavours N_c colours

$\tilde{N}_c \equiv N_f - N_c$

Electric and magnetic theory are dual in the UV $(t = 0)$ and IR $(t = -\infty)$ so we believe they are dual to each other in the whole interval $-\infty < t \leq 0$

If we choose

$$N_f = 3\tilde{N}_c - 1$$

then the magnetic theory is weakly coupled

$$\beta_1 = 3\tilde{N}_c - N_f = 1$$

and we can calculate the flow perturbatively

$$\tilde{\alpha}_g \equiv \frac{\tilde{N}_c \tilde{g}^2}{(4\pi)^2} \quad , \quad \tilde{\alpha}_y \equiv \frac{\tilde{N}_c \tilde{y}^2}{(4\pi)^2}$$

Up to 2 loops

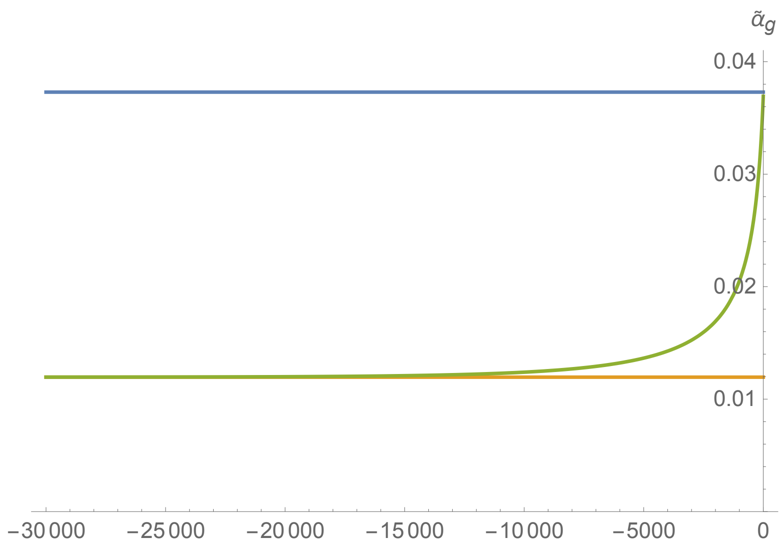
$$\frac{d}{dt}\begin{pmatrix}\tilde{\alpha}_g(t) \\ \tilde{\alpha}_y(t)\end{pmatrix} = \begin{pmatrix}\tilde{\alpha}_g(t) & \tilde{\alpha}_y(t)\end{pmatrix} M \begin{pmatrix}\tilde{\alpha}_g(t) - \tilde{\alpha}_g(-\infty) \\ \tilde{\alpha}_y(t) - \tilde{\alpha}_y(-\infty)\end{pmatrix}$$

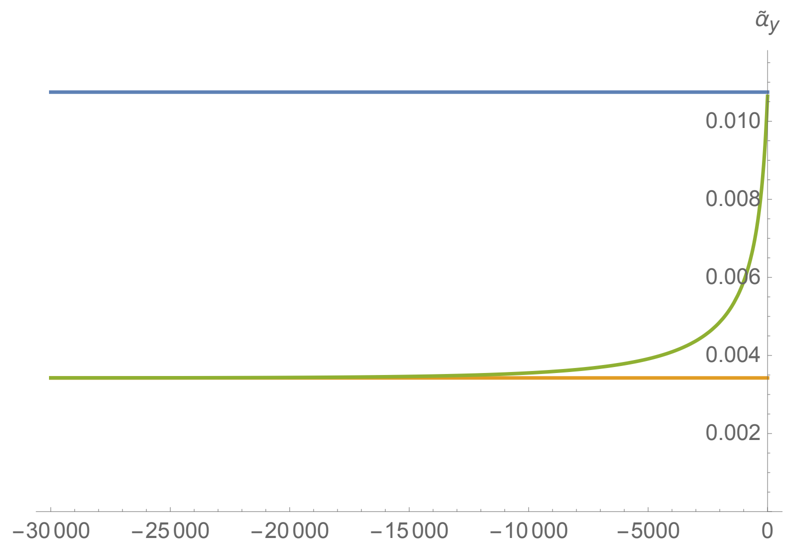
$M \ldots$ perturbatively calculable 2×2 matrix

$(\tilde{\alpha}_g, \tilde{\alpha}_y)(-\infty) \ldots$ perturbatively calculable fixed points values

This can be easily numerically solved

$\tilde{N}_c = 100 \quad (\to N_f = 299,\ N_c = 199)$





Magnetic theory solved

What about the electric theory?

$$\alpha_g \equiv \frac{N_c g^2}{(4\pi)^2} \quad , \quad f(x) \equiv \frac{1}{1-2x}$$

Formally we can write the RGE:

$$\frac{d\alpha_g(t)}{dt} = -\frac{6}{N_c}\alpha_g^2(t) f(\alpha_g(t)) \underbrace{\left(N_c + N_f \left(R_Q(t) - 1\right)\right)}_{\text{NSVZ } \beta \text{ function}}$$

Problem: theory non-perturbative so we do not know $R_Q(t)$ except in the fixed points (R-charges of Q in the conformal field theories)

$$R_Q(0) = 1 - \frac{N_c}{N_f + 1} \qquad \text{(UV)}$$

$$R_Q(-\infty) = 1 - \frac{N_c}{N_f} \qquad \text{(IR)}$$

How can duality help?

We need a physical quantity which have to be the same in the electric and magnetic theory.

What about the a central charge?

Two ways to define it outside the fixed points:

1) Komargodski, Schwimmer: through dilaton-dilaton ($\phi\phi$) scattering

$$a(\mu) = a_{UV} - \int_\mu^\infty d\mu \frac{\sigma_{\phi\phi\to\phi\phi}(\mu)}{\mu^3}$$

- since cross-section $\sigma > 0 \to a(\mu)$ decreasing from UV to IR (a-theorem)
- since cross-section σ is a physical quantity, so is $a(\mu)$

Non-perturbative electric $a_{el}(\mu)$ and perturbative magnetic $a_{mag}(\mu)$ should match along the whole flow

$$a_{el}(\mu) = a_{mag}(\mu)$$

Good to prove a-theorem but not easy to calculate

2) Kutasov: with Lagrange multipliers

$$a_{el} = 2(N_c^2 - 1) + 2N_f N_c a_1(R_Q) - \lambda_g(N_c + N_f(R_Q - 1))$$

Maximisation:

$$\frac{da_{el}}{dR_Q} = 2N_f N_c a_1'(R_Q) - \lambda_g N_f = 0$$

$$\rightarrow \lambda_g = 2N_c a_1'(R_Q)$$

$$\rightarrow a_{el} = 2(N_c^2 - 1) + 2N_f N_c \left(a_1(R_Q) - a_1'(R_Q)\left(R_Q - R_Q(-\infty)\right)\right)$$

Still we do not know what $R_Q(t)$ along the flow is

No proof that Komargodski-Schwimmer and Kutasov definitions coincide.

However possible to prove they do coincide for simple cases

We assume that they coincide in general

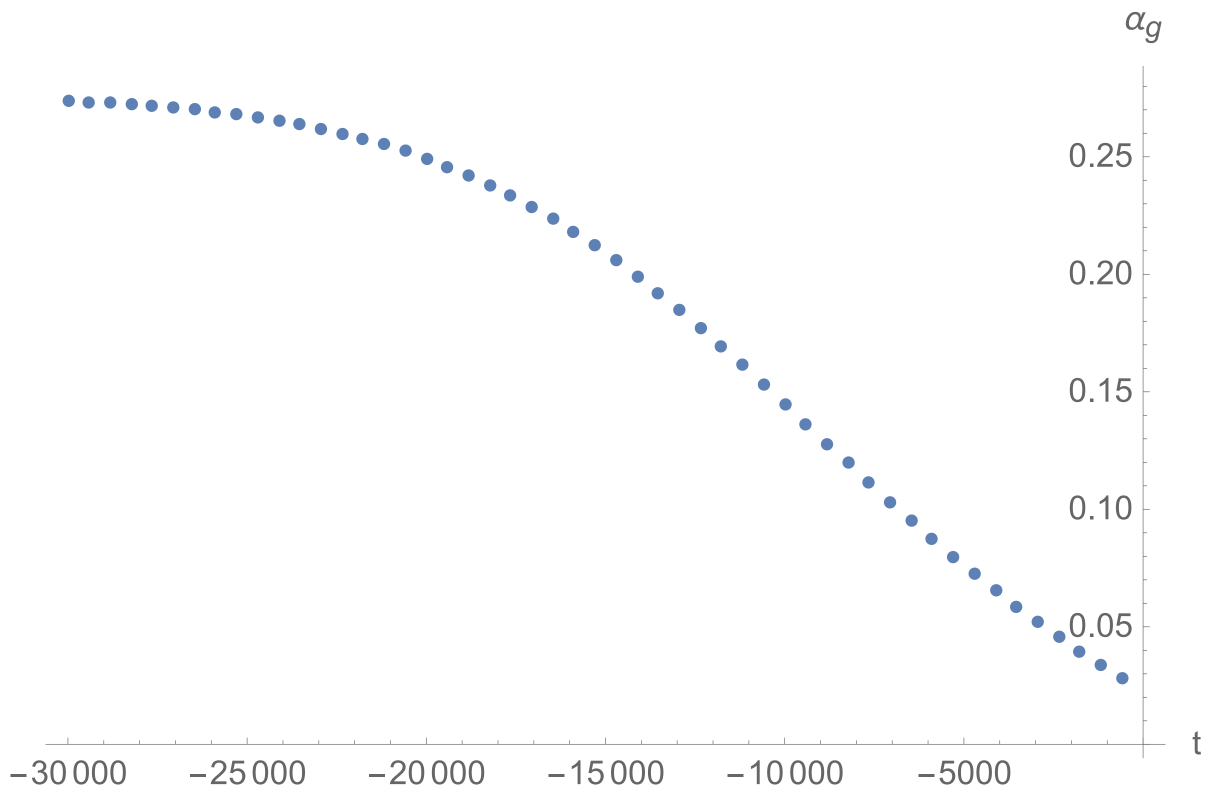
$$\rightarrow a_{el}(R_Q(t)) = a_{mag}(\tilde{\alpha}_g(t), \tilde{\alpha}_y(t))$$

r.h.s. perturbatively calculable $\rightarrow R_Q(t)$.

Then from formal RGE

$$\frac{d\alpha_g(t)}{dt} = -\frac{6}{N_c}\alpha_g^2(t) f(\alpha_g(t)) \left(N_c + N_f \left(R_Q(t) - 1\right)\right)$$

$\rightarrow \alpha_g(t)$ once we choose $\alpha_g(0) \leq 0.0216$



α_g
0.25
0.20
0.15
0.10
0.05
t
−30 000
−25 000
−20 000
−15 000
−10 000
−5000

Notice that

magnetic $\tilde{\alpha}_g$ decreasing towards IR (more perturbative)

but

electric α_g increasing towards IR (more non-perturbative)

strong-weak duality !

Conclusion

- instead of usual asymptotic freedom we considered theories with asymptotic safety
- Theory: two types of supersymmetric asymptotically safe theories presented
  1. large N_f
  2. Martin-Wells' type
- Phenomenology: in minimal renormalizable SO(10) GUT a quasi-realistic possibility for a UV safe theory found: one generation of matter fields decoupled from the superpotential
- explicit example of Seiberg duality in the whole flow