Dark Matter, Dark Energy, and Unification Models

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After a short introduction to theoretical cosmology we review the latest observations and theoretical models. Special attention is paid to the problems of dark matter and dark energy. According to the current knowledge, dark matter (DM) in small part consists of baryonic matter, which does not shine. For the most part DM consists of as yet unknown particles, which interact with the rest of the visible matter only through the gravitational force. Dark energy (DE) is a substance with negative pressure needed to explain the accelerated expansion of the Universe. DE is usually ascribed to vacuum energy, i.e., to the cosmological constant, but it could also be a homogeneously distributed fluid with negative pressure. The fluid description has been exploited to unify DE and DM in a single entity – quartessence. A class of unification models dubbed k-essence are based on pure kinetic scalar field theory. To this class belongs the scalar Born-Infeld theory which describes the Chaplygin gas – the prototype model of DM/DE unification.

Contents

1 Introduction

Modern cosmology rests on very precise observations as well as on the theoretical pillars provided by the General Theory of Relativity. However, several cosmological puzzles are still unsolved. The most important ones are:

Initial singularity Cosmological observations such as microwave background radiation demonstrate without a doubt that the Universe was formed after the so-called **Big Bang**, which happened about 14 billion years ago, formally at one point of space with the infinite energy density [1]. What caused the Big Bang? What happened before the Big Bang, if talking about the time before the beginning of the Universe makes sense at all?

Inflation Observational cosmology requires a very fast expansion, dubbed inflation, of the early Universe immediately after the Big Bang. Theoretically, the inflation happens as a result of the dynamics of a postulated scalar field, the so-called *inflaton field* [1, 2] with self-interaction besides the gravitational interaction. What is the origin of the inflaton field and inflation?

Dark matter According to many astrophysical and cosmological observations the amount of matter in the Universe is larger than the amount of the visible matter by two orders of magnitude [3, 4]. Out of the invisible matter only 20% is ordinary matter (so-called baryonic matter); the rest is an invisible substance of unknown origin. The most popular candidates for dark matter are hypothetic particles, which are predicted by the so-called *supersymmetric theories*, but which are not seen in experiments at least for the time being.

Dark energy Negative pressure substance necessary to explain the accelerating expansion of the Universe ascribed to vacuum energy, i.e. the *cosmological constant* [5]. The Standard Cosmological Model and the Standard Model of particle physics fail to give a satisfactory explanation about the origin of dark energy. For example, the vacuum energy density given by the quantum field theory is in disagreement with the favored value in the Standard Cosmological Model by about 100 orders of magnitude . Another problem is the so-called *coincidence problem*: why is the vacuum energy density almost equal to the energy density of other matter components of the Universe today?

The absence of antimatter Modern quantum physics associates every existing particle constituting ordinary matter with a so-called *antiparticle* having the same mass but the quantum numbers of opposite sign. Antiparticles and particles in their collisions transform into radiation energy in the process which is called *annihilation*. In theory, antiparticles can form even bigger structures (for example, atomic nuclei analogous to the nuclei of ordinary matter) that are called antimatter. Antimatter is produced in laboratory but it has not been observed in nature although it is reasonable to assume that due to the symmetry principle the same amount of matter and antimatter were produced in the Big Bang. A satisfactory answer to this question is not possible in the framework of the generally accepted so-called Standard Model of particle physics

2 History of the Early Universe - Universe Creation

It is believed that the beginning of the Universe is described by the so-called quantum cosmology. In analogy with quantum mechanics the Universe is formed as a quantum fluctuation of the initial vacuum in which there was no space and time. The total energy of the Universe (matter energy + radiation energy + gravitation) is equal or very close to zero. According to the uncertainty relation

$$\triangle E \triangle t \geq \hbar, \tag{1}$$

where $\hbar$ is the Planck constant divided by 2π, borrowing of small amount of energy (close to zero) enables long (almost infinite) duration of the Universe. The quantum fluctuation is manifested as the Big Bang.

3 History of the Early Universe - Inflation

After the creation which is called the Big Bang, the Universe goes through the phase of **inflation**: very fast expansion up to about 10^{25} times in the duration of about 10^{-35} seconds. Inflation is postulated as a theoretical model, which solves several basic problems of standard cosmology in an elegant way:

1. The horizon problem. Observations of the background radiation as well as the large scale structure show that the Universe is homogeneous and isotropic. The problem arises because the information about background radiation arrive from distant regions of the Universe which were not in a causal relationship at the moment when radiation had been emitted. This is in contradiction with the observational fact that the measured temperature of radiation is equal (up to the fluctuations of at most about 10^{-5}) everywhere and in all directions of observation.

2. The flatness problem. Observations of the average matter density, expansion rate and fluctuations of the background radiation show that the Universe is flat or with a very small curvature today. In order to achieve this a "fine-tuning" of the initial conditions is needed, which is rather unnatural.

3. The initial density perturbations problem. The question is how the initial deviations from homogeneity of the density are formed having in mind that they should be about 10^{-5} in order to yield today's structures (stars, galaxies, clusters). The answer is given by inflation: perturbations of density are created as quantum fluctuations of the inflaton field.

4 Cosmological Observations and History of the Universe

Based on cosmological observations we would like to know the properties of the Universe such as: space geometry, age of the Universe, rate of the Universe expansion, density ratios among particular forms of matter such as luminous matter (stars and gases), ("baryonic") dark matter (neutron stars, planet-like objects, brown dwarfs, black holes), amount and nature of nonbaryonic dark matter, origin and nature of dark matter (the cosmological constant), structure formation. The main experiments i.e. observations, which yield answers to all these questions are: (cosmological) background microwave radiation, large scale structure, abundances of light elements, rotation curves of galaxies, gravitational lenses, distant supernovae.

From cosmological observations we can make a reconstruction of the brief history of the Universe After the Big Bang about 14 billion years ago, the era of **quantum gravity** starts and lasts for about 10^{-43}s. Then, the inflation period of about 10^{-35}s follows. After that, within a millionth fraction of a second, several phase transitions take place in which the symmetry of the fundamental interactions and the very structure of matter change. These are the transitions of the grand unification, electroweak transition and the quark-hadron transition. Through these transitions the fundamental unified interaction (described, perhaps, by some unified theory of everything, for example the **string theory**) breaks into the four known interactions: gravitation, electromagnetic, weak and strong. The period of the so-called **Big-Bang nucleosynthesis** follows, when the nuclei of the light elements are formed: deuterium, helium, lithium. The period lasts from about a second to several minutes. During the following 400,000 years nothing special happens; the Universe expands and cools down up until the moment of the so-called **recombination**, i.e., binding of electrons in atoms. At that moment the Universe becomes transparent because radiation does not interact with matter any more. At about the same time, matter which was until then dominated by radiation becomes dominant.Structure formation starts: stars, galaxies and clusters. The Universe continues to expand and cool all the way down to today's temperature of about 3 K.

5 Empirical Grounds for Observational Cosmology

Modern observational cosmology is supported by three main empirical pillars:

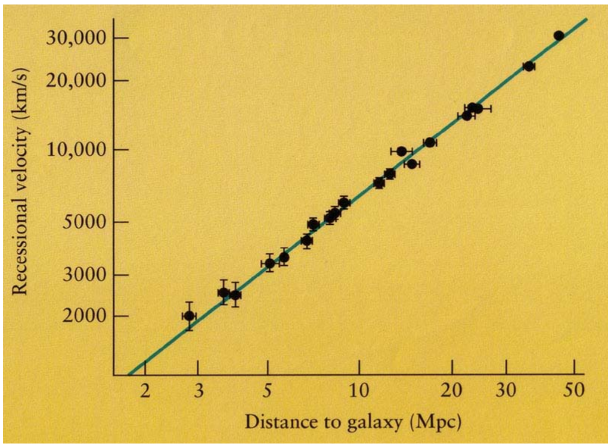

Figure 1: Hubble's diagram.

1. The expansion of the Universe - Hubble's law (Figure 1),
2. Cosmic microwave background radiation - very homogeneous in all directions,
3. Big Bang nucleosynthesis - proportions of light elements (H, D, He, Li).

5.1 Hubble's Law

By observing distant galaxies Hubble discovered that the Universe was expanding. The expansion rate is given by the so-called Hubble law

$$v = Hd, \tag{2}$$

where H is a constant and d is the distance to a galaxy. The distance d is determined by measuring the brightness, i.e. the luminosity of the object of the known brightness. The recession velocity v of the distant galaxies is determined by the so-called Doppler effect: the measured radiation spectrum of an observed object is compared to the standard spectrum of the object at rest. For objects, which are moving away, the wave theory predicts a red shift depending on the speed. The constant H, called the *Hubble constant*, is determined from the speed and the distance. It should be pointed out that the Hubble constant changes during the evolution of the Universe and in that sense it is actually not a constant but a function of time. The most recent measurements [19] of today's value H_0 of the Hubble constant are made by the Planck satellite mission [19] and give

$$H_0 = 67 \pm 1.2 \mathrm{(km/s)/Mpc}, \tag{3}$$

where Mpc (Mega parsec) is a unit of length which is used in astronomy for large distances, typically for a distance between two galaxies, and is equal to 3,262,000 light years. Hence, at a distance of 1 Mpc galaxies are moving away at speed of 67 km/s.

5.2 Cosmic Microwave Background (CMB) Radiation

The Universe is filled with radiation, which is, in a very good approximation, isotropic and homogeneous with a thermal spectrum at a temperature of about 2,728 K nowadays. In fact, what we actually observe is red-shifted radiation produced during the era of recombination, i.e., when the Universe became transparent. That took place about 400,000 years after the Big Bang or about 13.7 billion years ago. The latest precise observation by the COBE (Cosmic Background Explorer), WMAP (Wilkinson Microwave Anisotropy Probe), and ESA (European Space Agency) Planck satellite stations, and the BOOMERANG and MAXIMA balloons, however, show that there are deviations of isotropy at about the fourth decimal place. Figure 2 shows temperature fluctuations in the early Universe on the scale of the currently measured red-shift the spectrum.

The angular spectrum of deviations from isotropy gives us plenty of information on the basis of which the history of the Universe from the Big Bang to nowadays can be reconstructed. Figure 3 shows the angular spectrum of CMB. The deviation from homogeneity as a function of angle can be expanded in multipoles and the figure also shows the power of each multipole. The red line is a fit the standard cosmological model. The location and height of some peaks on the curve depend on the proportion of radiation, baryonic bright and dark matter, nonbaryonic dark matter as well as on the cosmological constant and the curvature of space.

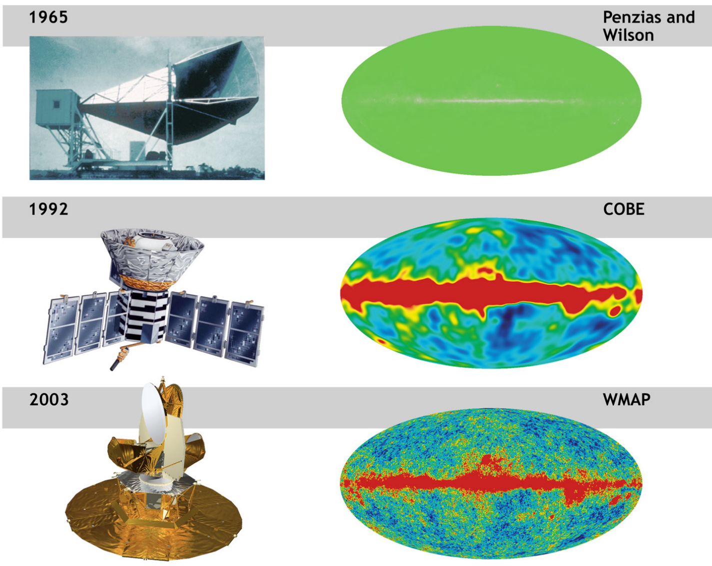

Figure 2: Measuring CMB; the temperature map of the sky. Top: homogeneous radiation of thermal spectrum at the temperature $T = 2.723K$. Middle: Deviation from isotropy of CMB in temperature area $\triangle T = 100\mu K$ (COBE). Bottom: Deviation in isotropy of CMB in temperature area $\triangle T = 200\mu K$ (WMAP [7]).

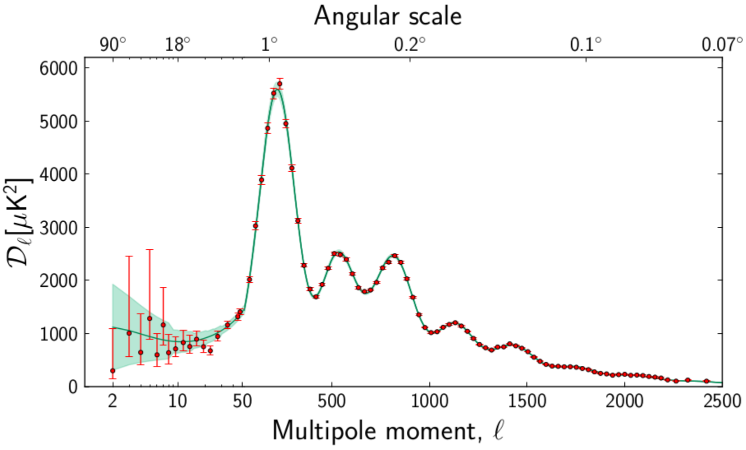

Figure 3: Angular (multipole) spectrum of the fluctuations of the microwave background radiation [19].

5.3 Big Bang Nucleosynthesis

Big Bang nucleosynthesis is the synthesis of light elements such as deuterium, helium isotopes and lithium, several minutes after the Big Bang. The production of these light elements lasted for about 20 minutes. During the time, some instable nuclei such as tritium and beryllium isotopes were created but very soon they decayed in a radioactive decay. For a short review on Big Bang Nucleosynthesis see [20].

By measuring the abundances of light elements in the shining matter in the Universe, a very important cosmological parameter can be calculated: the ratio of the number of photons to baryons. The calculation gives the mass concentrations of about 75% of hydrogen, 25% of helium-4, 0.01% of deuterium, very small traces of lithium and beryllium of order 10^{-10}, and no other heavy elements. The calculation agrees well with the measurements of abundances in the luminous matter provided the total density of baryon matter is about 3% to 4% (depending on the value of the Hubble constant) of the total density of matter in space. That very fact is one of the crucial evidences for the existence of nonbaryonic dark matter.

6 Theoretical Foundation of Modern Cosmology

The so called *Standard cosmology* provides a successful description of the evolution of the Universe from a fraction of a second after the creation until today. A short review on the standard model of cosmology is given in [21]. For our purpose, it is sufficient to state the basic underlying principles. Standard cosmology is based on the following three theoretical assumptions:

1. General Relativity. Gravity is described by Einstein's general theory of relativity governed by the *equivalence principle* and Einstein's equations of gravitational field. Einstein's equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + g_{\mu\nu} \Lambda = -8\pi G T_{\mu\nu}, \tag{4}$$

determine a mutual dependence of the space-time geometry described by the metric tensor $g_{\mu\nu}$ and the mass (i.e., energy) distribution described by the energy-momentum tensor $T_{\mu\nu}$. In this equation G is Newton's gravitational constant and the curvature tensor $R_{\mu\nu}$ and the curvature scalar R are functions of the metric tensor $g_{\mu\nu}$. The cosmological constant Λ, which Einstein introduced to describe the static Universe, is actually related to the vacuum energy density $\rho_{\rm vac} = \Lambda/(8\pi G)$ and in modern cosmological models is treated as a matter component which drives accelerated expansion. Hence, according to Einstein's equations the gravitational interaction is a consequence of space-time curvature and the properties of space and time are dictated by the amount and distribution of mass.

2. Perfect fluid. In a very good approximation the distribution of matter in space can be described by the so-called *perfect* (or *ideal*) The energy-momentum tensor then takes a simple form

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu - g_{\mu\nu} \, , \tag{5}$$

where u_μ is the four-velocity of the fluid, p is the pressure and ρ is the energy density. The energy momentum conservation is expressed by

$$T^{\mu\nu}{}_{;\nu} = 0 \tag{6}$$

Then, the longitudinal part of (6) $u_\mu T^{\mu\nu}{}_{;\nu} = 0$ yields the continuity equation

$$\dot{\rho} + 3\mathcal{H}(\rho + p) = 0, \tag{7}$$

and its transverse part the Euler equation

$$\dot{u}^\mu = \frac{1}{\rho + p} h^{\mu\nu} p_{,\nu} \, , \tag{8}$$

where we define

$$3\mathcal{H} = u^\nu{}_{;\nu} \, ; \qquad \dot{\rho} = u^\nu \rho_{,\nu} \, ; \qquad \dot{u}^\mu = u^\nu u^\mu{}_{;\nu} \, . \tag{9}$$

The tensor

$$h_{\mu\nu} = g_{\mu\nu} - u_\mu u_\nu \tag{10}$$

is a projector onto the three-space orthogonal to u^μ. The quantity $\mathcal{H}$ is the local Hubble parameter. Overdots indicate the proper time derivative.

3. Cosmological principle. The cosmological principle asserts that the Universe is homogeneous and isotropic on large scales (for example about 1Gpc). The most general metric satisfying the cosmological principle is the Friedmann-Robertson-Walker metric [21, 22]

$$ds^2 = dt^2 - a(t)^2 \left[\frac{dr^2}{1 - kr^2} - r^2(d\theta^2 + \sin\theta d\phi^2) \right] , \tag{11}$$

where the curvature constant k takes on the values 1, 0, or -1, for a closed, flat, or open universe, respectively. The time-dependent quantity $a(t)$ is the scale factor of the expansion conveniently normalized to unity at present time, i.e., $a(t_0) = 1$. In other words, a is the radius of the Universe measured in units of its current radius.

With these assumptions, the set of Einstein's equations reduces to the Friedmann-Robertson-Walker (FRW) equations

$$H(t)^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3} - \frac{k}{a^2} + \frac{\Lambda}{3}\,, \tag{12}$$

$$\frac{\ddot{a}}{a} = \frac{\Lambda}{3} - 4\pi G(\rho + 3p), \tag{13}$$

On the basis of empirical and theoretical foundations the standard cosmological model has been generally accepted today. It is sometimes called the **concordance** model, and is often referred to as ΛCDM abbreviation, where Λ stands for the cosmological constant and CDM for the so-called *cold dark matter*.

7 Density of Matter in Space

Cosmological observations show that the space today is flat. According to the Standard Cosmological Model a flat space requires the matter density to have the so-called critical value $\rho_{cr} \approx 10^{-29} g/cm3$ today. However, from astronomical observations the ratio of the total matter density to the critical density must be $\rho/\rho_{cr} \approx 31\%$. Besides, the density of luminous matter (stars, galaxies, gases...) is considerably smaller, $\rho_{lum}/\rho_{cr} \leq 0.5\%$. From the proportions of light elements and the comparison with the Big Bang nucleosynthesis the baryonic matter density (protons, neutrons, nuclei) with respect to the critical value is $\rho_{Bar}/\rho_{cr} \leq 5\%$. Consequently, based on today's observations one can conclude that more than 99% of matter is not luminous. Out of that less than 5% is ordinary ("baryonic") matter. Thus, about 26% is dark nonbaryonic matter and about 69% is the so-called vacuum energy. The density ratios of various kinds of matter **today** in comparison with the total density are

$$\Omega_B = \frac{\rho_B}{\rho_{tot}} \approx 0.048, \quad \Omega_{DM} = \frac{\rho_{DM}}{\rho_{tot}} \approx 0.26, \quad \Omega_\Lambda = \frac{\rho_\Lambda}{\rho_{tot}} \approx 0.69. \tag{14}$$

These ratios change in time but for the **flat Universe** we always have

$$\rho_{tot} = \rho_{crit}, \qquad \Omega_B + \Omega_{DM} + \Omega_\Lambda = 1. \tag{15}$$

8 Dark Matter

Besides the cosmological arguments (Big Bang nucleosynthesis and the CMB spectrum) there exist astrophysical observations from which we conclude that a larger part of the gravitating matter is dark. For example, the rotation curves of the galaxies (Figure 4) show that in almost all galaxies the amount of invisible matter is considerably greater than the amount of the visible matter [4].

In other words, the star dynamics in galaxies is inconsistent with Newton's Law of Gravity if one assumes that a galaxy contains only luminous matter such as stars and gases. Two solutions are possible: A modification of Newton's law or a hypothesis that besides luminous there is also invisible matter which interacts only gravitationally. One of the latest

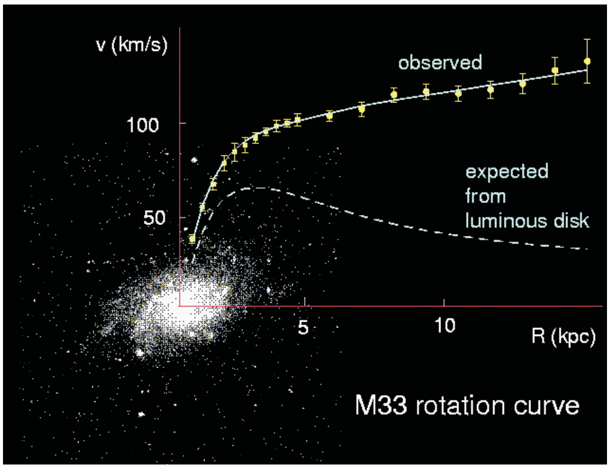

Figure 4: Rotation curve of the galaxy M33

observations [8] of the 1E0657-558 cluster, also called the Bullet cluster, shows that a larger part of dark matter in the cluster is separated from the bulk of ordinary matter in the form of gas(Figure 5). In these observations dark matter is detected by gravitational lensing.

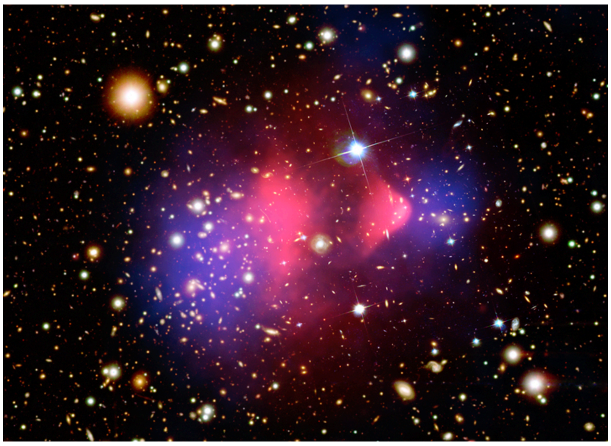

Figure 5: The 1E0657-558 cluster (The Bullet cluster) [18].

An obvious interpretation is that the Bullet cluster was created in a collision of two smaller clusters so that the matter which does not interact except gravitationally (dark matter and galaxies) is separated from the clouds of ionized particles in the intergalactic space which are slowed down and partly stopped because of interactions. The explanation of this effect by a modified theory of gravity is practically excluded.

8.1 Dark Matter Constituents - Possible Candidates

What is dark matter made of? A smaller part of about 20% is ordinary, baryonic DM, most likely in the form of relatively standard astrophysical objects, which do not radiate or radiate so weakly that they are not visible to astronomic observations. These are, e.g., cold hydrogen clouds and compact astrophysical objects such as brown dwarfs, planets, the so-called MACHOs (massive compact halo objects), and even black holes.

However, the main part of dark matter consists of yet undetected hypothetical nonbaryonic particles. Possible candidates are: sterile neutrino, axion and supersymmetric particles such as gravitino, neutralino, axino. Particle physicists mostly believe in supersymmetry (SUSY) as a unifying theory and expect a stable SUSY particle to be detected in future experiments at CERN. By combining the Standard Cosmological Model with SUSY it is possible to calculate today's concentration of these particles in space and if the calculation is (is not) consistent with the empirical value of about 24% of the total amount of matter in space, it would make a strong case for (against) the Standard Cosmological Model as well as the SUSY theories.

The different DM scenarios are conveniently classified as *hot, warm,* and *cold* DM [9], depending on the thermal velocities of DM particles in the early Universe.

Hot DM refers to low-mass neutral particles that are still relativistic when galaxy-size masses ($\sim 10^{12} M_\odot$) are first encompassed within the horizon. Hence, fluctuations on galaxy scales are wiped out by the "free streaming" of the dark matter. Standard examples of hot DM are neutrinos and majorons. They are still in thermal equilibrium after the QCD deconfinement transition, which took place at $T_{\rm QCD} \simeq 150$ MeV. Hot DM particles have a cosmological number density comparable with that of microwave background photons, which implies an upper bound to their mass of a few tens of eV.

Warm DM particles are just becoming nonrelativistic when galaxy-size masses enter the horizon. Warm DM particles interact much more weakly than neutrinos. They decouple (i.e., their mean free path first exceeds the horizon size) at $T \gg T_{\rm QCD}$. As a consequence, their number is expected to be roughly an order of magnitude lower and their mass an order of magnitude larger, than hot DM particles. Examples of warm DM are $\sim$ keV sterile neutrinos, axinos [10], or gravitinos in soft supersymmetry breaking scenarios [11, 12].

Cold DM particles are already nonrelativistic when even globular cluster masses ($\sim 10^6 M_\odot$) enter the horizon. Hence, their free streaming is of no cosmological importance. In other words, all cosmologically relevant fluctuations survive in a universe dominated by cold DM. The two main particle candidates for cold dark matter are the lowest supersymmetric *weakly interacting massive particles* (WIMPs) and the *axion*.

Hence, the above mentioned particles are candidates for cold DM as opposed to hot DM whose constituents are particles of very small mass such as neutrino. Today a strong belief prevails that most if not all of dark matter in space is cold so that the standard cosmological model is described by cold dark matter abbreviated as as CDM.

The nature of DM is still an open question. In spite of the large-scale successes of CDM there is still some unresolved issues such as overproduction of small scale structure and halos with a central cusp [13]. These problems are somewhat alleviated by warm DM and in particular by sterile neutrino warm DM [14, 15]. However, a recent analysis [17] shows that a realistic warm DM scenario with $m_{\rm DM} \simeq 4$ keV in agreement with recent constraints from Lyman-α forest [16] is not able to alleviate the small scale crisis of cold DM structure formation.

9 Dark Energy

Observations of distant supernovae of the type Ia and comparison of the luminosity distance with the distance determined from the recession velocity redshift (Hubble's law) demonstrates the accelerated expansion of the Universe. In other words, the standard model yields acceleration of the expansion if the cosmological constant differs from zero or if in addition to baryonic and dark matter there exists a substance with negative pressure. The cosmological constant Λ appears in Einstein's field equations owing to the vacuum energy. The accelerated expansion is a consequence of the negative pressure of the vacuum energy! In the same way, a gas with negative pressure can lead to the accelerated expansion. A new term, **dark energy**, has been introduced as a common term for both the vacuum energy and a substance with negative pressure.

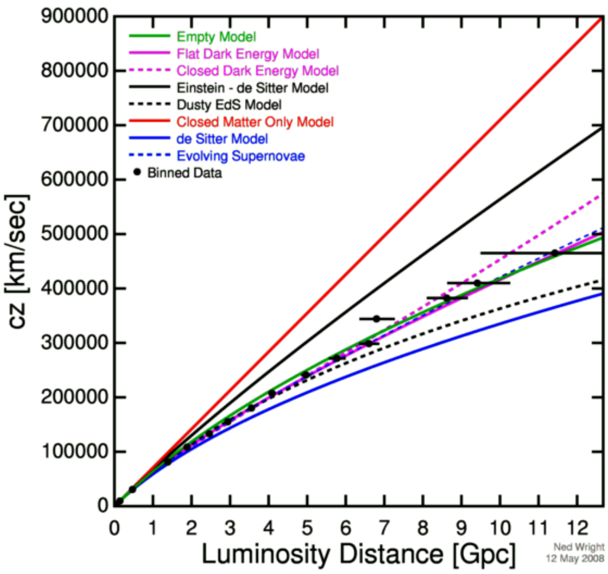

Figure 6: Hubble's diagram from the observation of SN type Ia. The curves represent fits for different cosmological models (for various values of Ω and Ω_Λ) [23].

The accelerated expansion and a comparison of the standard Big Bang model with observations (Figure 6) requires that the density of vacuum energy is about $\rho_\Lambda = 70\% \rho_{crit}$. In Figure 6 this corresponds to the purple solid line (for a flat Universe) or the dashed line (for the closed curved Universe).

Some of the most popular models of dark energy are

Cosmological Constant – vacuum energy density does not change in time.

Quintessence – a scalar field with a canonical kinetic term.

Phantom quintessence – a scalar field with a negative kinetic term.

k-essence – is a scalar field whose Lagrangian is a general function of kinetic energy. Energy density varies with time.

Quartessence – a model of unifying of dark energy and dark matter. Special subclass of k-essence. One of the popular models is the so-called *Chaplygin gas* [26, 27, 29],

Compared with dark matter, dark energy shows essentially different features. Dark matter has a positive pressure, creates non-homogeneous structures and goes together with baryonic matter in galaxies and galaxy clusters. Dark energy has a negative pressure, is distributed homogeneously through space and, as a rule, does not create structures. The exceptions are unified models of dark energy and dark matter where dark energy also takes part in creation of structures [67].

9.1 Vacuum Energy – Cosmological Constant Λ

The simplest model of dark energy – the cosmological constant – is very problematic from the point of view of particle physics and field theory. The calculation of the vacuum energy density in field theory of the Standard Model of particle physics gives the value about 10^{120} times higher than the value of the cosmological constant obtained from cosmological observations. One possible way out is the so called fine tuning: a rather unnatural assumption that all interactions of the standard model of particle physics somehow conspire to yield cancellation between various large contributions to the vacuum energy resulting in a small value of *Lambda* in agreement with observations. Another possible way out: particle physics is described by a new hypothetical theory (supersymmetry, superstrings ...) beyond the Standard model of particle physics where Λ is exactly zero due to some symmetry principle. In such a case, accelerated expansion must be explained by some other form of dark energy.

Another problem with Λ is the so called *coincidence problem*. The concordance (or ΛCDM) model begs the question of why this fine tuned value of Λ is such that DM and DE are comparable today, leaving one to rely on anthropic arguments. The coincidence problem is somewhat ameliorated in *quintessence* models which replace Λ by an evolving scalar field.

9.2 Quintessence

Quintessence is a canonical scalar field φ with a specific selfinteraction such that it effectively provides a slow roll inflation today [5]:

$$S = \int d^4x\, \mathcal{L}(X, \varphi) \tag{16}$$

$$\mathcal{L} = \frac{1}{2}X - V(\varphi) \tag{17}$$

where

$$X \equiv g^{\mu\nu}\varphi_{,\mu}\varphi_{,\nu}, \tag{18}$$

The energy momentum tensor

$$T_{\mu\nu} = \frac{2}{\sqrt{-\det g}} \frac{\delta S}{\delta g^{\mu\nu}} = \varphi_{,\mu}\varphi_{,\nu} - \mathcal{L} g_{\mu\nu} \tag{19}$$

for $X > 0$ that holds in a cosmological setting, describes a perfect fluid

$$T_{\mu\nu} = (\rho + p) u_\mu u_\nu - p g_{\mu\nu}, \tag{20}$$

where

$$p = \mathcal{L} = \frac{1}{2} X - V(\varphi), \tag{21}$$

$$\rho = X - \mathcal{L} = \frac{1}{2} X + V(\varphi). \tag{22}$$

The four-velocity is defined as

$$u_\mu = \eta \frac{\varphi_{,\mu}}{\sqrt{X}}, \tag{23}$$

where η is $+1$ or -1 according to whether $\varphi_{,0}$ is positive or negative, i.e., the sign of u_μ is chosen so u_0 is positive. A suitable choice of $V(\varphi)$ yields a desired cosmology, or vice versa: from a desired equation of state $p = p(\rho)$ one can derive the Lagrangian of the corresponding scalar field theory

9.3 Phantom quintessence

Phantom energy is a substance with negative pressure such that $|p|$ exceeds the energy density ρ so that the null energy condition (NEC) is violated, i.e., $p + \rho < 0$. Phantom quintessence is a scalar field with a negative kinetic term

$$p = \mathcal{L} = -\frac{1}{2} X - V(\varphi) \tag{24}$$

$$\rho = -\frac{1}{2} X + V(\varphi) \tag{25}$$

Obviously, for $X > 0$ we have $p + \rho \leq 0$ which demonstrates a violation of NEC! This model predicts a catastrophic end of the Universe, the so-called Big Rip - the total collapse of all bound systems.

9.4 k-essence

k-essence is a generalized quintessence which was first introduced as a model for inflation [24]. A minimally coupled k-essence model [24, 25], is described by

$$S = \int d^4x \sqrt{-g} \left[-\frac{R}{16\pi G} + \mathcal{L}(\varphi, X) \right], \tag{26}$$

where $\mathcal{L}$ is the most general Lagrangian, which depends on a single scalar field φ of dimension m^{-1}, and on the dimensionless quantity X defined in (18). For $X > 0$ the energy momentum tensor obtained from (26) takes the perfect fluid form,

$$T_{\mu\nu} = 2\mathcal{L}_X \, \varphi_{,\mu}\varphi_{,\nu} - \mathcal{L} g_{\mu\nu} = (\rho + p) u_\mu u_\nu - p \, g_{\mu\nu} \,, \tag{27}$$

with $\mathcal{L}_X$ denoting $\partial\mathcal{L}/\partial X$. The associated hydrodynamic quantities are

$$p = \mathcal{L}(\varphi, X), \tag{28}$$

$$\rho = 2X\mathcal{L}_X(\varphi, X) - \mathcal{L}(\varphi, X). \tag{29}$$

Examples

Kinetic k-essence The Lagrangian is a function of X only. In this case

$$p = \mathcal{L}(X) \tag{30}$$

and

$$\rho = 2X\mathcal{L}_X(X) - \mathcal{L}(X) \tag{31}$$

To this class belong the ghost condensate [31, 32]

$$\mathcal{L}(X) = A(1 - X)^2 + B \tag{32}$$

and the scalar Born-Infeld model [27, 34]

$$\mathcal{L}(X) = -A\sqrt{1 - X}. \tag{33}$$

10 DE/DM Unification – Quartessence

Pragmatically, the astrophysical and cosmological observational data can be accommodated by combining baryons with conventional CDM candidates and a simple cosmological constant Λ providing DE. This ΛCDM model, however, faces the mentioned fine tuning and coincidence problems associated to vacuum energy. The coincidence problem of the ΛCDM model is somewhat ameliorated in *quintessence* models which replace Λ by an evolving scalar field. However, like its predecessor, a quintessence-CDM model assumes that DM and DE are distinct entities. For a recent review of the most popular DM and DE models, see [30].

Another interpretation of the observational data is that DM/DE are different manifestations of a common structure. The general class of models, in which a unification of DM and DE is achieved through a single entity, is often referred to as *quartessence* [35, 36]. Among other scenarios of unification that have recently been suggested, interesting attempts are based on the *k-essence* [37, 32]. In particular a model called *Dusty Dark Energy* [33] achieves the DM-DE unification in the formalism of the $\lambda\phi$-fluid, resulting in the zero speed of sound and one scalar degree of freedom.

10.1 Chaplygin gas

The first definite model of DE/DM unification was proposed a few years ago [26, 27], based upon the Chaplygin gas, a perfect fluid obeying the equation of state

$$p = -\frac{A}{\rho}, \tag{34}$$

which has been extensively studied for its mathematical properties [34]. We will discuss this model in detail as it serves as a prototype for DE/DM unification.

The cosmological potential of equation (34) was first noted by Kamenshchik *et al* [26], who observed that integrating the energy conservation equation in a homogeneous model leads to

$$\rho(a) = \sqrt{A + \frac{B}{a^6}}\,, \tag{35}$$

where a is the scale factor normalized to unity today and B an integration constant. Thus, the Chaplygin gas interpolates between matter, $\rho \sim \sqrt{B}a^{-3}$, $p \sim 0$, at high redshift and a cosmological constant like $\rho \sim \sqrt{A} \sim -p$ as a tends to infinity. In this way one recovers a correct homogeneous cosmology that coincides with ΛCDM in the past and asymptotic future.

The Chaplygin gas model can be directly extended to include an inhomogeneous cosmology [27]. The essence of the idea is simply that in an *inhomogeneous* universe, highly overdense regions (galaxies, clusters) have $|w| = |p/\rho| \ll 1$ providing DM, whereas in underdense regions (voids) evolution drives ρ to its limiting value $\sqrt{A}$ giving DE.

Of particular interest is that the Chaplygin gas has an equivalent scalar field formulation [27, 34]. Considering the Lagrangian

$$\mathcal{L} = -\sqrt{A}\sqrt{1-X}\,, \tag{36}$$

where

$$X \equiv g^{\mu\nu}\varphi_{,\mu}\varphi_{,\nu}\,, \tag{37}$$

equation (34) is obtained by evaluating the stress-energy tensor $T_{\mu\nu}$, and introducing $u_\mu = \varphi_{,\mu}/\sqrt{X}$ for the four-velocity and $\rho = \sqrt{A}/\sqrt{1-X}$ for the energy density. One recognizes $\mathcal{L}$ as a Lagrangian of the Born-Infeld type, familiar in the D-brane constructions of string/M theory [38]. Geometrically, $\mathcal{L}$ describes space-time as the world-volume of a 3+1 brane in a 4+1 bulk via the embedding coordinate X^4 [39].

The Chaplygin gas model is equivalent to (scalar) Dirac-Born-Infeld description of a D-brane. A p-brane moving in a $p+2$-dimensional bulk is described by the Nambu-Goto action

$$S_{\rm DBI} = -\sqrt{A}\int d^{p+1}x\,\sqrt{(-1)^p \det(g^{\rm ind})} \tag{38}$$

where $g^{\rm ind}_{\mu\nu}$ is the induced metric ("pull back") of the bulk metric G_{ab}

$$g^{\rm ind}_{\mu\nu} = G_{ab}\frac{\partial X^a}{\partial x^\mu}\frac{\partial X^b}{\partial x^\nu} \tag{39}$$

We find a k-essence type of theory

$$S_{\rm DBI} = -\sqrt{A}\int dx^4\,\sqrt{1-X^2} \tag{40}$$

with

$$\rho = \frac{\sqrt{A}}{\sqrt{1-X^2}}; \qquad p = -\sqrt{A}\sqrt{1-X^2} \tag{41}$$

and hence

$$p = -\frac{A}{\rho} \tag{42}$$

10.2 Problems with Nonvanishing Sound Speed

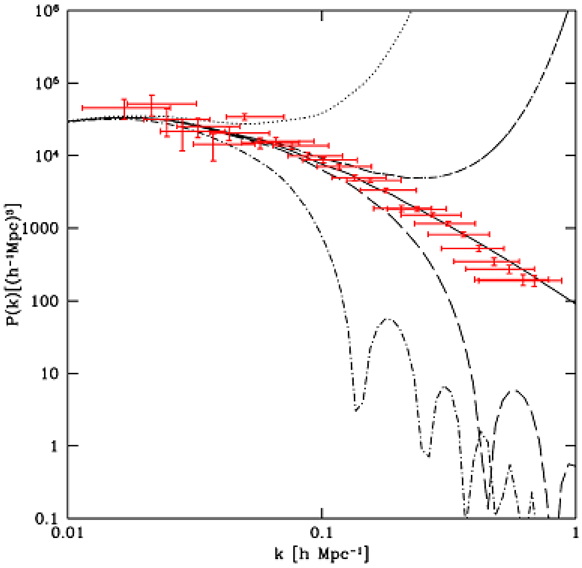

Figure 7: Power spectrum for $p = -A/\rho^\alpha$ for various α. From H.B. Sandvik et al [41]

To be able to claim that a field theoretical model actually achieves unification, one must be assured that initial perturbations can evolve into a deeply nonlinear regime to form a gravitational condensate of superparticles that can play the role of CDM. In [27, 28] this was inferred on the basis of the Zel'dovich approximation [40]. In fact, for this issue, the usual Zel'dovich approximation has the shortcoming that the effects of finite sound speed are neglected.

All models that unify DM and DE face the problem of nonvanishing sound speed and the well-known Jeans instability. A fluid with a nonzero sound speed has a characteristic scale below which the pressure effectively opposes gravity. Hence the perturbations of the scale smaller than the sonic horizon will be prevented from growing. Soon after the appearance of [26] and [27], it was pointed out that the perturbative Chaplygin gas (for early work see [29]) is incompatible with the observed mass power spectrum [41] and microwave background [42]. Essentially, these results follow from the adiabatic speed of sound

$$c_s^2 = \left.\frac{\partial p}{\partial \rho}\right|_s = \frac{A}{\rho^2} \, . \tag{43}$$

Although the adiabatic speed of sound is small until $a \sim 1$, the accumulated comoving acoustic horizon

$$d_s = \int dt \, \frac{c_s}{a} \, . \tag{44}$$

reaches Mpc scales by redshifts of about $z \sim 20$, thus frustrating the structure formation at galactic and subgalactic scales. This may be easily demonstrated in a simple spherical model.

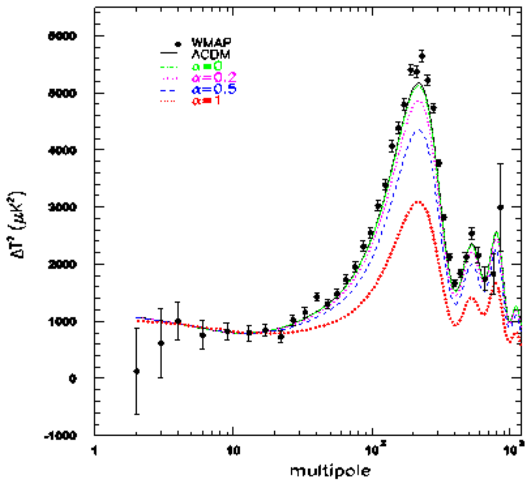

Figure 8: CMB spectrum for $p = -A/\rho^\alpha$ for various α. From L. Amendola et al [57]

10.2.1 Spherical Model

To study the evolution of perturbations of a model with nonvanishing pressure gradients and speed of sound we will use the spherical model. A variant of the spherical model has been applied to DE/DM mixtures in [43], however there the effects of pressure gradients were omitted. A variant of the spherical model for studying the evolution of density perturbations into the fully nonlinear regime has been developed using a Newtonian [58] and a general-relativistic formalism [67] applicable to any k-essence model. For a k-essence model or any one-component type of model, the Euler equation (8) combines with Einstein's equations to

$$3\dot{\mathcal{H}} + 3\mathcal{H}^2 + \sigma^2 + 4\pi G(\rho + 3p) = \left(\frac{h^{\mu\nu}p_{,\nu}}{p+\rho}\right)_{;\mu}. \tag{45}$$

where

$$\sigma^2 = \sigma_{\mu\nu}\sigma^{\mu\nu}. \tag{46}$$

with the shear tensor defined as

$$\sigma_{\mu\nu} = h^\alpha{}_\mu h^\beta{}_\nu u_{(\alpha;\beta)} - \mathcal{H}h_{\mu\nu}\,. \tag{47}$$

We thus obtain an evolution equation for the local Hubble expansion rate $\mathcal{H}$. Owing to the definition of the four-velocity (23) and the orthogonality $h^{\mu\nu}u_\nu = 0$ we may write

$$h^{\mu\nu}p_{,\nu} = c_s^2 h^{\mu\nu}\rho_{,\nu}. \tag{48}$$

Hence, if $c_s = 0$ or if the pressure gradient $p_{,\mu}$ is parallel to u_μ as for dust, the right-hand side of (45) vanishes in which case equations (7) and (45) comprise the original spherical model [44]. However, we are not interested in dust, since generally $c_s \neq 0$ and $h^{\mu\nu}p_{,\nu} \neq 0$, so we must generalize the spherical model to include the right-hand side of (45). The density contrast δ and the deviation $\delta\mathcal{H}$ of the Hubble parameter from the background value H are defined by

$$\rho = \bar{\rho}(1+\delta), \tag{49}$$

$$\mathcal{H} = H + \delta\mathcal{H}. \tag{50}$$

Then subtracting the background in equations (7) and (45), eliminating $\delta\mathcal{H}$ and $\delta\dot{\mathcal{H}}$, and neglecting shear in the Newtonian approximation $|p| << \rho$ we find [58]

$$\ddot{\delta} + 2\mathcal{H}\dot{\delta} - \frac{3}{2}\mathcal{H}^2\delta(1+\delta) - \frac{4}{3}\frac{\dot{\delta}^2}{1+\delta} - \frac{1+\delta}{a^2}\frac{\partial}{\partial x_i}\left(\frac{c_s^2}{1+\delta}\frac{\partial\delta}{\partial x_i}\right) = 0, \tag{51}$$

The root of the structure formation problem is the last term, as may be understood if we

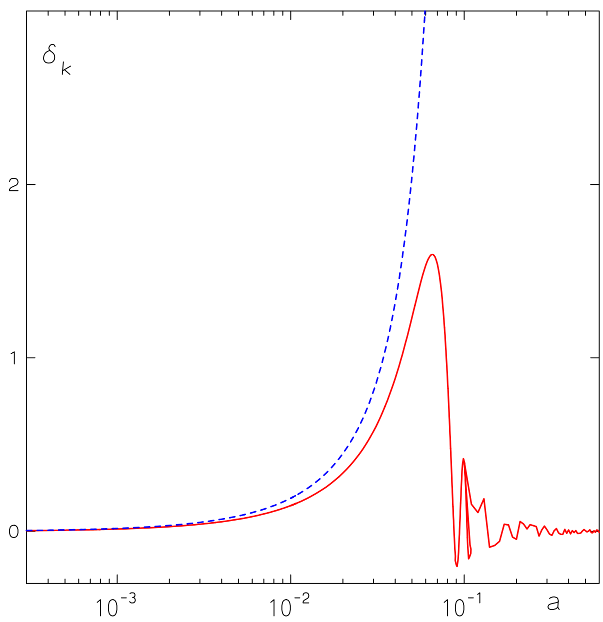

Figure 9: Evolution of the density contrast in the spherical model from $a_{\rm eq} = 3\times10^{-4}$ for the comoving wavelength $1/k = 0.34$ Mpc, $\delta_k(a_{\rm eq}) = 0.004$ (solid) $\delta_k(a_{\rm eq}) = 0.005$(dashed).

solve the equation at linear order which has been discussed in detail by Fabris *et al* [29]. Keeping only the terms linear in δ, $\dot{\delta}$, and $\ddot{\delta}$ one finds

$$\ddot{\delta} + 2H\dot{\delta} - \frac{3}{2}H^2\delta - \frac{c_s^2}{a^2}\Delta\delta = 0 \tag{52}$$

and using the expansion

$$\delta(a,\vec{x}) = \sum_k \delta_{\rm pert}(k,a)e^{i\vec{k}\vec{x}}, \tag{53}$$

one obtains an explicit solution for the perturbative density contrast which may be expressed as

$$\delta_{\rm pert}(k,a) \;\propto\; a^{-1/4} J_{5/14}(d_{\rm s}k)\,. \tag{54}$$

Here $J_\nu(z)$ is the Bessel function, k the comoving wave number, and $d_{\rm s}$ the comoving sonic horizon given by (44).

The perturbations whose comoving size $R \simeq 1/k$ is larger than d_s grow as $\delta = (\rho - \bar\rho)/\bar\rho \sim a$. Once the perturbations enter the acoustic horizon, i.e., as soon as $R < d_s$, they undergo damped oscillations. In the case of the Chaplygin gas we have $d_s \sim a^{7/2}/H_0$, where H_0 is the present day value of the Hubble parameter, reaching Mpc scales already at redshifts of order 10. However, to reiterate a point made in [27], small perturbations *alone* are not the issue, since large density contrasts are required on galactic and cluster scales. As soon as $\delta \simeq 1$ the linear perturbation theory cannot be trusted. An essentially nonperturbative approach is needed in order to investigate whether a significant fraction of initial density perturbations collapses in gravitationally bound structure - the condensate. If that happens the system evolves into a two-phase structure - a mixture of CDM in the form of condensate and DE in the form of uncondensed gas.

The case, where the Chaplygin gas is mixed with CDM, has been considered in a number of papers [45, 46, 47, 48, 49, 57, 59, 60]. Here, the Chaplygin gas simply plays the role of DE. In keeping with the quartessence philosophy, it would be preferred if CDM could be replaced by droplets of Chaplygin gas condensate, as in [61]. Homogeneous world models, containing a mixture of CDM and Chaplygin gas, have been successfully confronted with lensing statistics [45, 46] as well as with supernova and other tests [47, 48].

10.3 Generalized Chaplygin Gas

Another model, the so called "generalized Chaplygin gas" [50], was proposed in an attempt to solve the structure formation problem and has gained a wide popularity. The generalized Chaplygin gas is defined as [27, 50]

$$p = -\frac{A}{\rho^\alpha}, \qquad 0 \le \alpha \le 1 \tag{55}$$

As in the Chaplygin gas case, this equation of state has an equivalent field theory representation, the "generalized Born-Infeld theory" [50, 52]. However, the associated Lagrangian has no equivalent brane interpretation. The additional parameter does afford greater flexibility: e.g. for small α the sound horizon is $d_s \sim \sqrt{\alpha} a^2/H_0$, and thus by fine tuning $\alpha < 10^{-5}$, the data can be perturbatively accommodated [41]. Bean and Doré [49] and similarly Amendola *et al* [57] have examined a mixture of CDM and the generalized Chaplygin gas against supernova, large-scale structure, and CMB constraints. They have demonstrated that a thorough likelihood analysis favors the limit $\alpha \to 0$, i.e. the equivalent to the ΛCDM model. Both papers conclude that the standard Chaplygin gas is ruled out as a candidate for DE. However, analysis [60, 52] of the supernova data seems to indicate that the generalized Chaplygin gas with $\alpha \ge 1$ is favored over the $\alpha \to 0$ model and similar conclusions were drawn in [51]. But one should bear in mind that the generalized Chaplygin gas with $\alpha > 1$ has a superluminal sound speed that violates causality [53]. For a different view on this issue see [54, 55, 56].

10.4 Tachyon Condensate

The failure of the simple Chaplygin gas does not exhaust all the possibilities for quartessence. The Born-Infeld Lagrangian (36) is a special case of the string-theory inspired tachyon Lagrangian [68, 69] in which the constant $\sqrt{A}$ is replaced by a potential $V(\varphi)$

$$\mathcal{L} = -V(\varphi)\sqrt{1 - g^{\mu\nu}\,\varphi_{,\mu}\varphi_{,\nu}}\,. \tag{56}$$

In turn, tachyon models are a particular case of k-essence [24]. The possibility of obtaining both DM and DE from the tachyon with inverse square potential has been speculated in [70]. More recently, it was noted [71] that, in a Friedmann-Robertson-Walker (FRW) model, the tachyon model is described by the equation of state (34) in which the constant A is replaced by a function of the cosmological scale factor a, so the model was dubbed "variable Chaplygin gas". Related models have been examined in [72, 73], however, those either produce a larger d_s than the simple Chaplygin gas [72], or else need fine-tuning [73]. More precisely, the tachyon model [72] gives $d_s \sim a^2/H_0$, whereas the two-potential model [73] yields $d_s \sim \sqrt{1-h}a^2/H_0$, so it requires $1 - h < 10^{-5}$ like the generalized Chaplygin gas. Expanding in $1 - h$, the second potential reveals itself to be dominantly a cosmological constant.

A preliminary analysis of a unifying model based on the tachyon type Lagrangian (56) has been carried out in [67] for a potential of the form

$$V(\varphi) = V_n\varphi^{2n}, \tag{57}$$

where n is a positive integer. In the regime where structure function takes place, it has been shown that this model effectively behaves as the variable Chaplygin gas with $A(a) \sim a^{6n}$ with $n = 1(2)$ for a quadratic (quartic) potential. As a result, the much smaller acoustic horizon $d_s \sim a^{(7/2+3n)}/H_0$ enhances condensate formation by two orders of magnitude over the simple Chaplygin gas ($n = 0$). Hence this type of model may salvage the quartessence scenario.

10.5 Entropy Perturbations

One way to deal with the structure formation problem, is to assume entropy perturbations [62, 36, 63] such that the effective speed of sound vanishes. This type of quartessence is not different from [27], where the effects of nonvanishing c_s were tacitly neglected due to the Zeldovich approximation. In that picture we have $\delta p = c_s^2\delta\rho - \delta A/\rho = 0$ even if $c_s \neq 0$. But in a single field model it is precisely the adiabatic speed of sound that governs the evolution. Hence, entropy perturbations require the introduction of a second field on which A depends.

Suppose that the matter Lagrangian depends on two degrees of freedom, e.g., a Born-Infeld scalar field θ and one additional scalar field φ. In this case, instead of a simple barotropic form $p = p(\rho)$, the equation of state and may be written in the parametric form $p = p(\theta, \varphi)$, $\rho = \rho(\theta, \varphi)$. Besides, the calculation of the adiabatic speed of sound c_s involves the entropy density (entropy per particle) $s = s(\theta, \varphi)$

$$\delta p = \delta\rho\frac{\partial p}{\partial\rho} + \delta\varphi\frac{\partial p}{\partial\varphi} \tag{58}$$

$$\delta s = \delta\rho\frac{\partial s}{\partial \rho} + \delta\varphi\frac{\partial s}{\partial \varphi} \tag{59}$$

Then, the speed of sound is the sum of two nonadiabatic terms

$$c_{\rm s}^2 \equiv \left.\frac{\partial p}{\partial \rho}\right|_S = \frac{\partial p}{\partial \rho} - \frac{\partial s}{\partial p}\left(\frac{\partial s}{\partial \varphi}\right)^{-1}\frac{\partial p}{\partial \varphi} \tag{60}$$

Thus, even for a nonzero $\delta p/\delta\rho$ the speed of sound may vanish if the second term on the right-hand side cancels the first one. This cancellation will take place if in the course of an adiabatic expansion, the perturbation $\delta\varphi$ grows with a in the same way as $\delta\rho$. In this case, it is only a matter of adjusting the initial conditions of $\delta\varphi$ with $\delta\rho$ to get $c_s = 0$.

Aside from negating the simplicity of the one-field model, some attempts at realizing the nonadiabatic scenario [64, 65, 66] have convinced us that even if $\delta p = 0$ is arranged as an initial condition, it is all but impossible to maintain this condition in a realistic model for evolution.

10.6 Dusty Dark Energy

Another way to bypass the structure formation problem is to impose a constraint on pressure such that the pressure gradient is parallel to the fluid four-velocity. The model called *Dusty Dark Energy* [33] comprises two scalar fields λ and φ, λ being a Lagrange multiplier which enforces a constraint between φ and its kinetic energy term X. Starting from the action

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[K(\varphi, X) + \lambda\left(\frac{1}{2}X - V(\varphi)\right)\right], \tag{61}$$

where $K(\varphi, X)$ is an arbitrary function of X and φ. The field λ is a"Lagrange multiplier" and does not have a kinetic term, while X is a standard kinetic term for the field φ. The energy momentum tensor takes the usual perfect fluid form (20) with the four velocity given by (23).

The λ field equation

$$\frac{1}{\sqrt{-g}}\frac{\delta S}{\delta\lambda} = \frac{1}{2}X - V(\varphi) = 0 \tag{62}$$

imposes a constraint such that the pressure

$$p = K(\varphi, X) = K(\varphi, 2V(\varphi)) \tag{63}$$

becomes a function of φ only. Then, its gradient is proportional to $\varphi_{,\mu}$ and hence parallel to the 4-velocity. In this way the sound speed is always identically zero on all backgrounds. In particular, cosmological perturbations reproduce the standard behavior for hydrodynamics in the limit of vanishing sound speed. In a certain limit this model exactly reproduces the evolution history of ΛCDM, while deviations away from the standard expansion history produce a potentially measurable difference in the evolution of structure.

11 Conclusions and Outlook

We have discussed a just a few out of many attempts to unify DE/DM. The above mentioned examples illustrate more or less conventional trends in modern cosmology. Of course, there exist various alternative ideas such as modified theories of gravity. One of the popular ideas is the so-called **brane world cosmology** where our world is a four-dimensional membrane submerged in a five-dimensional space. Alternative theories explain more or less successfully a part of the phenomena related to dark energy and dark matter, but up until now there is no completely satisfactory theory which would solve all the puzzles. In any case, there remains a lot of work to be done for theoretical physicists.

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