So (generalizing) we say that $|p,p\rangle$ is a $*$-algebra projector. Multiply by $|p,p\rangle$ to assign momentum to closed perturbative string states.

Check! Take boundary state in closed string theory

$$e^{-\sum_n \beta_n^{\mu\dagger} \tilde{\beta}_n^{\nu\dagger} \eta_{\mu\nu}} |0_c\rangle \otimes e^{\sum_{1}^{\infty} {}_n \beta_n^{i\dagger} \tilde{\beta}_n^{j\dagger} \eta_{ij}} |0_c\rangle \sim e^{-\frac{1}{2} \sum_{k=1}^{\infty} a_k^{\mu\dagger} C_{kl} a_l^{\nu\dagger} \eta_{\mu\nu}} |0\rangle$$

i.e. boundary state in closed string theory corresponds to identity state in open string theory!

Quite non-trivial check!!

Add momentum. Use lump solutions, i.e.

$$|V_3\rangle \longrightarrow |V'_{3,\perp}\rangle \otimes |V_{3,\parallel}\rangle$$

Transverse directions, $i, j = 1, \dots, k$

Parallel directions, $\mu, \nu = 0, \dots, p+1$

$$a_0^{(r)j} = \frac{1}{2}\sqrt{b}\,\hat{p}^{(r)j} - i\frac{1}{\sqrt{b}}\hat{x}^{(r)j}$$

$$a_0^{(r)j\dagger} = \frac{1}{2}\sqrt{b}\,\hat{p}^{(r)j} + i\frac{1}{\sqrt{b}}\hat{x}^{(r)j}$$

$$a_0^{\dagger}|\Omega_b\rangle = 0$$

Then

$$|V'_{3,\perp}\rangle = K\, e^{-E'}|\Omega_b\rangle \qquad M, N = (0, m, n)$$

$$E' = \frac{1}{2}\sum_{r,s=1}^{3}\sum_{M,N\geq 0} a_M^{(r)j\dagger}\, V'^{rs}_{MN}\, a_N^{(s)l\dagger}\,\eta_{jl}$$

We can repeat previous construction

$$s \rightarrow s' \qquad \beta_N^i = \sum_{M=0}^{\infty} b'_{NM}\, s'^i_M, \quad \tilde{\beta}_N^i = -\sum_{M=0}^{\infty}\tilde{b}'_{NM}\, s'^i_M$$

$$[\beta^i_M, \beta^{j\dagger}_N] = \eta^{ij}\delta_{MN} \qquad [\tilde{\beta}^i_M, \tilde{\beta}^{j\dagger}_N] = \eta^{ij}\delta_{NM}$$

Same as before, but now

$$|p, q\rangle = \frac{1}{K}\sqrt{\frac{b}{2\pi}}\, e^{-\frac{b}{4}(p^2+q^2) + \sqrt{b}(q\beta_0^\dagger + p\tilde{\beta}_0^\dagger) - \frac{1}{2}(\beta_0^{\dagger 2} + \tilde{\beta}_0^{\dagger 2})}|0'_c\rangle$$

with property

$$|p_1, q_1\rangle * |p_2, q_2\rangle = \delta(q_1 - p_2)|p_1, q_2\rangle$$

Closed string states

$$\beta_{m_1}^{\mu_1 +} \cdots \beta_{m_r}^{\mu_r +} \tilde{\beta}_{n_1}^{\nu_1 +} \cdots \tilde{\beta}_{n_s}^{\nu_s +} |0_c\rangle \longleftrightarrow |\Lambda_{\vec{m},\vec{n}}^{\vec{\mu},\vec{\nu}}\rangle$$

$\vec{m} = (m_1, \ldots, m_r)$, etc. We have the algebra

$$|\Lambda_{\vec{m},\vec{n}}^{\vec{\mu},\vec{\nu}}\rangle * |\Lambda_{\vec{\ell},\vec{k}}^{\vec{\lambda},\vec{\kappa}}\rangle = \eta^{\vec{\nu}\vec{\lambda}} \delta_{\vec{n},\vec{\ell}} |\Lambda_{\vec{m},\vec{k}}^{\vec{\mu},\vec{\kappa}}\rangle .$$

where $\eta^{\vec{\nu}\vec{\lambda}} = \eta^{\nu_1\lambda_1}\eta^{\nu_2\lambda_2}\cdots$, $\delta_{\vec{n},\vec{\ell}} = \delta_{n_1,\ell_1}\delta_{n_2\ell_2}\cdots$

Closed string theory requires that

$$N_L = \sum_{i=1}^{r} m_i \,, \quad N_R = \sum_{i=1}^{s} n_i \,, \qquad N_L = N_R$$

(<u>level matching</u> condition)

Level matching is true for $\beta_{m_1}^{\mu_1 +} \cdots \beta_{m_r}^{\mu_r +} \tilde{\beta}_{n_1}^{\nu_1 +} \cdots \tilde{\beta}_{n_s}^{\nu_s +} |0_c\rangle$

if and only if $|\Lambda_{\vec{m},\vec{n}}^{\vec{\mu},\vec{\nu}}\rangle * |\Lambda_{\vec{m},\vec{n}}^{\vec{\mu},\vec{\nu}}\rangle = |\Lambda_{\vec{m},\vec{n}}^{\vec{\mu},\vec{\nu}}\rangle$, i.e.

$$\boxed{\text{off-shell CS states}} \Longleftrightarrow \boxed{\text{VSFT solutions}}$$

at $p=0$

Closed string states and Laguerre polynomials

$$\frac{1}{n!}\left(\beta^+\right)^n\left(\tilde{\beta}^+\right)^n|0_c\rangle = \langle \xi^L(a^+ + Sa)\rangle^n \langle \xi^R(a^+ + Sa)\rangle^n e^{-\frac{1}{2}a^+Sa^+}|0\rangle$$

$$= (-\kappa)^n \sum_{k=0}^{n} \frac{n!}{k!\,(n-k)!}\left(-\frac{\langle \xi C a^+\rangle\langle \xi a^+\rangle}{\langle \xi \frac{T}{1-T^2}\xi\rangle}\right)^k \frac{1}{k!}|\Xi\rangle$$

$$= (-\kappa)^n L_n(z)\,|\Xi\rangle$$

where

$$\kappa = \langle \xi \frac{T}{1-T^2}\xi\rangle \qquad z = \frac{1}{\kappa}\langle \xi C a^+\rangle\langle \xi a^+\rangle$$

$$\boxed{\begin{array}{l}\text{Closed string algebra}\\[1em] [\beta^\mu_m, \beta^{\nu +}_n] = \delta_{m,n}\,\eta^{\mu\nu}\\[1em] [\tilde{\beta}^\mu_m, \tilde{\beta}^{\nu +}_n] = \delta_{m,n}\,\eta^{\mu\nu}\\[1em] [\beta^\mu_m, \tilde{\beta}^\nu_n] = 0\end{array}}$$

Let's start from the sliver

$$|\Xi\rangle = \mathcal{N}\, e^{-\frac{1}{2} a^\dagger S a^\dagger} |0\rangle \qquad a^\dagger S a^\dagger = \sum_{n,m=1}^{\infty} a_n^\dagger S_{nm} a_m^\dagger$$

$$|\Xi\rangle * |\Xi\rangle = |\Xi\rangle$$

Define

$$s = w(a + S a^\dagger) \qquad s^\dagger = w(a^\dagger + S a) \qquad w = (1 - S^2)^{-\frac{1}{2}}$$

then

$$[s_n, s_m^\dagger] = \delta_{n,m} \qquad s_n |\Xi\rangle = 0 \qquad n \geq 1$$

This is a Bogoliubov transformation:

$$|0\rangle \rightsquigarrow |\Xi\rangle \qquad a, a^\dagger \rightsquigarrow s, s^\dagger$$

Now define ξ s.t.

$$\rho_R \xi = 0 \qquad \rho_L \xi = \xi \qquad C_{nm} = (-1)^n \delta_{nm}$$

$$\rho_R C\xi = C\xi \qquad \rho_L C\xi = 0$$

and call

$$\xi^L = w\xi \qquad \xi^R = w C \xi$$

There exist a complete basis of such ξ: ξ_n, $n = 1, \dots$

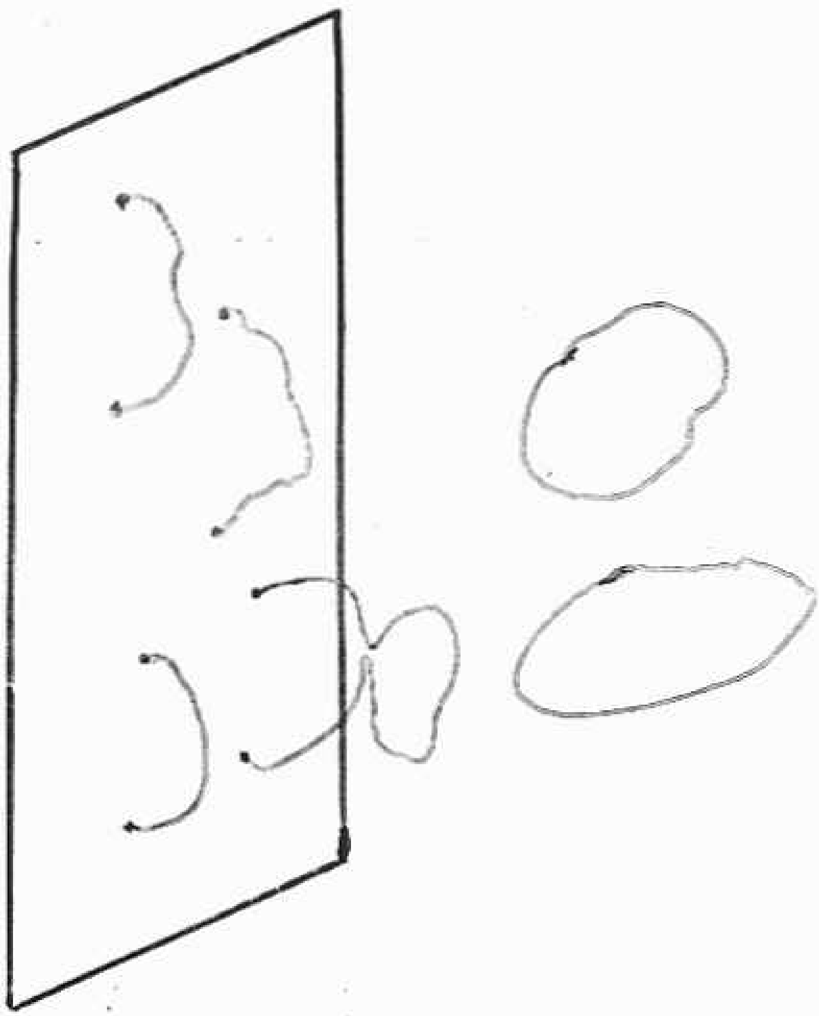
$$\beta_n^\mu = \sum_{l=1}^{\infty} b_{nl}\, s_l^\mu \qquad \tilde{\beta}_m^\mu = -\sum_{l=1}^{\infty} \tilde{b}_{ml}\, s_l^\mu$$

Towards open-closed string duality

Above we have seen a correspondence

open $\longleftrightarrow$ closed

This is the $\alpha' \to 0$ limit of a more complete correspondence taking care of α' corrections

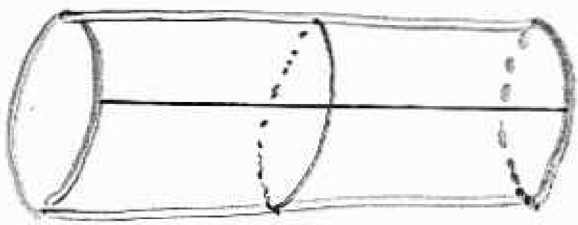


A. Sen's open-closed string duality:

closed string physics can be expressed in terms of open strings.

At $a = 2\pi$ $(\tilde{\lambda} = \frac{1}{2})$ the energy is stored in very massive closed string modes, which behave non-relativistically and are localized near the brane $\Longrightarrow$ tachyon matter

There seems to be an <u>open-closed string duality</u> at tree level.



traditional duality

"Tree level open strings know about closed strings"

OS

$$|\Xi\rangle = \mathcal{N}\, e^{-\frac{1}{2} a^+ S a^+} |0\rangle$$

$$a^+ S a^+ = \sum_{n=1}^{\infty} a_n^+ S_{nm} a_m^+$$

CS

$$|B\rangle \sim e^{-\sum_{n=1}^{\infty} a_n^+ \tilde{a}_n^+} |0\rangle$$

D-branes

$$\tilde{A}(p_1,p_2) = \int_0^1 d\rho\; \rho^{t/2-3}(1-\rho^2)^{s-2} = \frac{\Gamma(t/4-1)\,\Gamma(s-1)}{2\,\Gamma\left(\frac{t}{4}+s-2\right)}$$

Open string channel $\quad s = 1, 0, -1, \dots$

Closed string channel $\quad t = 4, 0, -4, \dots$

$$S(p_1,p_2) = \frac{1}{2\sinh\frac{a|E|}{2}} \sum_{k=0}^{\infty} f_k(s)\, \delta\left(\frac{t}{4}-1+k\right)$$

- S has poles in t, <u>not</u> in s (no open string poles)
- contributions to S come from $\rho \simeq 0$ (disk $\rightarrow$ sphere)

<u>Conclusion</u> $\quad S$ describes a sphere amplitude with two tachyon insertions and a tower of on-shell massive closed string states.

<u>BCFT description</u>

$$|W\rangle \equiv \frac{\delta(L_0+\bar{L}_0)}{2\sinh\frac{a|E|}{2}} (b_0+\bar{b}_0)\, |B^{p-1}\rangle_{|z|=1}$$

$$|B^{p-1}\rangle = \mathcal{N}\, \delta(x^\mu)\; e^{+\alpha_m^{\mu\dagger}\eta_{\mu\nu}\tilde{\alpha}_m^{\nu\dagger} - \alpha_m^{i\dagger}\delta_{ij}\tilde{\alpha}_m^{j\dagger}}\, |0\rangle$$

What is "tachyon matter"? (Gaiotto, Itzhaki, Rastelli)

The marginal deformation

$$\tilde{\lambda} \int_{\partial\Sigma} dt \cosh X^0(t)$$

seems to represent, for $\tilde{\lambda} = \frac{1}{2}$, the tachyon vacuum.

It is equivalent to an infinite array of D-branes at

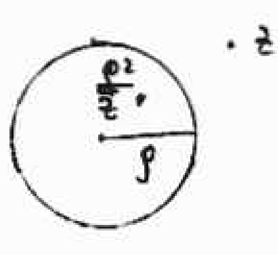
$$X^0 = i(2n+1)\pi a$$

Relation between Wick-rotated amplitudes $\tilde{A}(E)$ and inverse-Wick-rotated amplitudes $S(E)$

$$S(E) = \frac{1}{2\sinh\frac{aE}{2}} \, \mathrm{Disc}_E\left[\tilde{A}(iE)\right]$$

$$\mathrm{Disc}_E\, f(E) = \frac{1}{i}\left(f(E+i\epsilon) - f(E-i\epsilon)\right)$$

For instance, 2-point closed string tachyon on the disk $= \{z, |z| \geq \rho\}$



Tachyon driven cosmology

$$S = -\frac{1}{16\pi G}\int d^4x \left[-\sqrt{g}\, R + V(T)\sqrt{\det(g+\partial T\partial T)} + \Lambda\sqrt{g}\right]$$

with

$$V(T) = \frac{V_0}{\cosh\frac{T}{\sqrt{2}}}$$

Friedman eqs.

$$H^2 = \frac{8\pi G}{3}\left[\frac{V(T)}{\sqrt{1-\dot{T}^2}} + \Lambda\right]$$

$$\frac{\ddot{a}}{a} = \frac{8\pi G}{3}\left[\Lambda + \frac{V(T)}{\sqrt{1-\dot{T}^2}} - \frac{3}{2}\,\frac{\dot{T}^2 V(T)}{\sqrt{1-\dot{T}^2}}\right]$$

$$\ddot{T} = -(1-\dot{T}^2)\left[\frac{V'(T)}{V(T)} + 3\dot{T}\,\frac{\dot{a}}{a}\right]$$

<u>Not good inflation</u> $\eta \sim 1$

Perhaps good model for <u>dark matter</u>.

or <u>pre-inflationary cosmology</u>.

But OSFT potential not $V(T)$!

Effective Field Theory of Tachyonic Matter (Sen)

Proposal

$$S = -\int d^{p+1}x \, V(T) \sqrt{-\det A} \, , \qquad V(T) = e^{-\frac{T}{2}}$$

$$A_{\mu\nu} = \eta_{\mu\nu} + \partial_\mu T \, \partial_\nu T \qquad \mu, \nu = 0, \dots, p$$

For spatially homogeneous time-dependent field configurations

$$T_{00} = e^{-\frac{T}{2}} \left(1 - (\partial_0 T)^2\right)^{-1/2}$$

Since T_{00} is conserved, $\partial_0 T \to 1$ as $T \to \infty$.

Solution for large x^0:

$$T = x^0 + C e^{-x^0} + O(e^{-2x^0})$$

Pressure:

$$p = e^{-\frac{T}{2}} \sqrt{1 - (\partial_0 T)^2} \simeq -\sqrt{2C} \, e^{-x^0}$$

No plane wave solution. Candidate to represent tachyon condensation.

$$|B\rangle_{c=1} \sim \left[f(X^0(0)) + \alpha^0_{-1}\tilde{\alpha}^0_{-1}\, g(X^0(0)) \right] |0\rangle$$

where

$$f(x^0) = \frac{1}{1+e^{x^0}\sin\tilde{\lambda}\pi} + \frac{1}{1+e^{-x^0}\sin\tilde{\lambda}\pi} - 1$$

and

$$g(x^0) = \cos(2\pi\tilde{\lambda}) + 1 - f(x^0)$$

Interpretation

$$T_{00} = K\left(f(x^0) + g(x^0)\right) = K\left(\cos(2\pi\tilde{\lambda}) + 1\right)$$

$$T_{0i} = 0$$

$$T_{ij} = -2K\, f(x^0)\,\delta_{ij} \qquad\qquad K = \tfrac{1}{2}\tau_p$$

Comments.

- For $\tilde{\lambda} = \frac{1}{2}$ total energy vanishes, $f(x^0) = 0$ (equivalent to array of D-branes at $x^0 = i(2n+1)\pi$)

- $0 < \tilde{\lambda} < \frac{1}{2}$ system evolves.

$$f(x^0) \xrightarrow[x^0\to\infty]{} 0 \qquad\qquad g(x^0) \longrightarrow 1 + \cos(2\pi\tilde{\lambda})$$

So

$$T_{00} = \text{const} \qquad\qquad T_{ij} \to 0$$

Tachyon matter

Rolling tachyon &

open-closed string duality

A rolling tachyon is a classical solution of SFT which represents the evolution in time of the tachyon field $T(X^0)$. (Sen)

In classical field theory

$$T(X^0) = \lambda \cosh X^0 \qquad \begin{array}{l} T(0) = \lambda \\ T'(0) = 0 \end{array}$$

In CFT

$$-\frac{1}{2\pi}\int_\Sigma d^2z\, \partial_z X^0 \partial_{\bar z} X^0 + \tilde\lambda \int_{\partial\Sigma} dt \cosh X^0(t)$$

with $\tilde\lambda \simeq \lambda$.

Wick rotate $X_0 \to iX$ and study boundary state $|B\rangle$ perturbed by

$$\tilde\lambda \int dt \cos X(t)$$

After inverse-Wick rotating, the relevant part of $|B\rangle$ is

There is a map between a solution of cubic SFT and Berkovits SFT

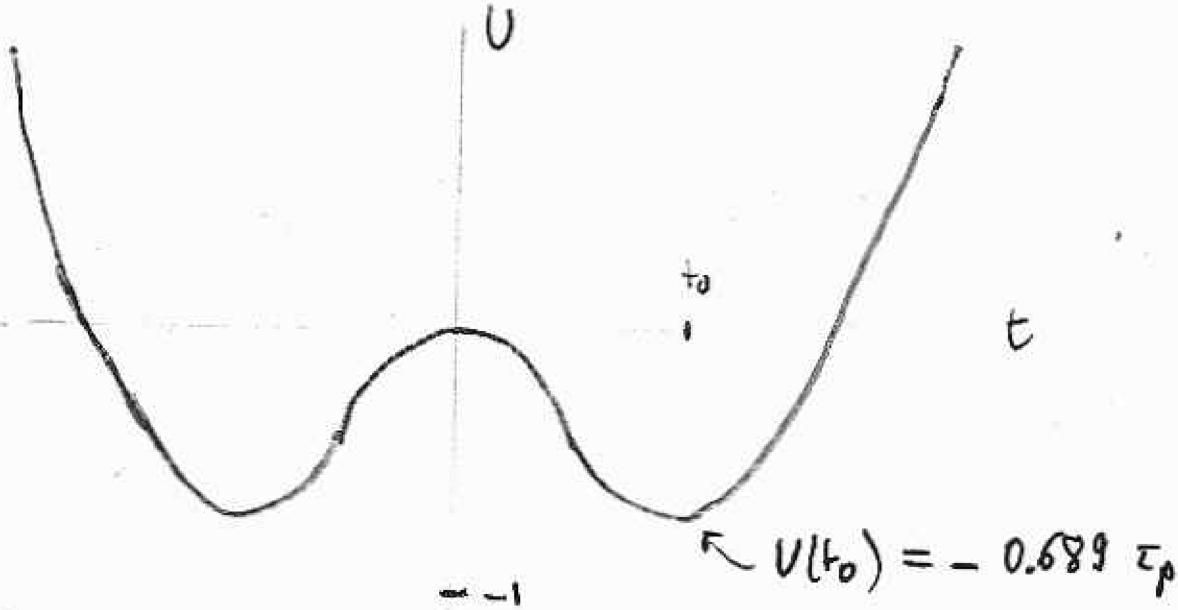
$$\Phi_1 = -c\,\xi\partial\xi\, e^{-2\varphi}(z)\,\Psi_1$$

Inserting Schnabl's solution in either theory we get a solution in GSO+. It seems to be ok.

No solution is known yet involving GSO-

At level $\left(\frac{1}{2}, 1\right)$ one finds the potential

$$V^{(1)}(t) = \tau_p \frac{\pi^2 \alpha'^3}{2\sqrt{2}} \left(\frac{3^4 \alpha'}{2^{10}} t^4 - \frac{1}{4\alpha'} t^2 \right)$$



$$\tau_p = \frac{2\sqrt{2}}{g_0^2 \pi^2 \alpha'^{\frac{p+1}{2}}}$$

<u>Another Superstring Field Theory</u> (Berkovits)

$$S = \frac{1}{2g_0^2} \int \left(e^{-\Phi} \left(Q e^{\Phi} \right) e^{-\Phi} \eta_0 e^{\Phi} - \int_0^1 dt\, e^{-t\Phi} \partial_t e^{t\Phi} \left[e^{-t\Phi} \eta_0 e^{t\Phi}, e^{-t\Phi} Q e^{t\Phi} \right] \right)$$

EOM:

$$\eta_0 \left(e^{-\Phi} Q e^{\Phi} \right) = 0$$

Picture changing operators X, Y

$$Y = c\,\partial\xi\, e^{-2\varphi} \qquad \text{pic. lowering}$$

Modified Witten's SSFT

$$\boxed{S = -\frac{1}{g_0^2}\int Y_{-2}\left(\frac{1}{2}\Psi * Q\Psi + \frac{1}{3}\Psi * \Psi * \Psi\right)}$$

EOM

$$Y_{-2}\left(Q\Psi + \Psi * \Psi\right) = 0 \qquad Y_{-2}(z,\bar z) = Y(z)\,Y(\bar z)$$

This is ok, but contains only GSO+ sector (no tachyon)

We must allow for GSO−. The action for a non-BPS D-brane is

$$\boxed{\begin{aligned} S^{non-BPS}_{NS} = -\frac{1}{g_0^2}\int Y_{-2}\Big(&\frac{1}{2}\Psi_+ * Q\Psi_+ + \frac{1}{3}\Psi_+ * \Psi_+ * \Psi_+ \\ &-\frac{1}{2}\Psi_- Q\Psi_- + \Psi_+ * \Psi_- * \Psi_-\Big)\end{aligned}}$$

where $\Psi_\pm \in$ GSO$\pm$

Imposing Feynman-Siegel gauge

$$b_0 \Psi_\pm = 0$$

one finds two tachyons (one disappears, the other $\sim c\, e^{-\varphi}$ is the good one)

SuperString Field Theory

2D Fields : $X^\mu, \psi^\mu, b, c, \beta, \gamma$

2 sectors : R , NS

bosonization: φ $\qquad \varphi(z)\varphi(w) = -\log(z-w)$ (picture)

EM tensor : $T_m(z) = -\frac{1}{2}\partial X^\mu \partial X_\mu(z) - \frac{1}{2}\psi^\mu\partial\psi_\mu(z)$

Supercurrent: $G_m(z) = i\psi^\mu \partial X_\mu(z)$

BRST charge:

$$Q = \frac{1}{2\pi i}\oint dz\, J_B(z)$$

$$J_B(z) = c\,T_m + \gamma\, G_m + c\partial c\, b - \frac{c}{2}\left(3\beta\partial\gamma + \partial\beta\gamma\right) - b\gamma^2$$

Introduce η, ξ s.t.

$$\beta = e^{-\varphi}\partial\xi \qquad \gamma = \eta\, e^{\phi}$$

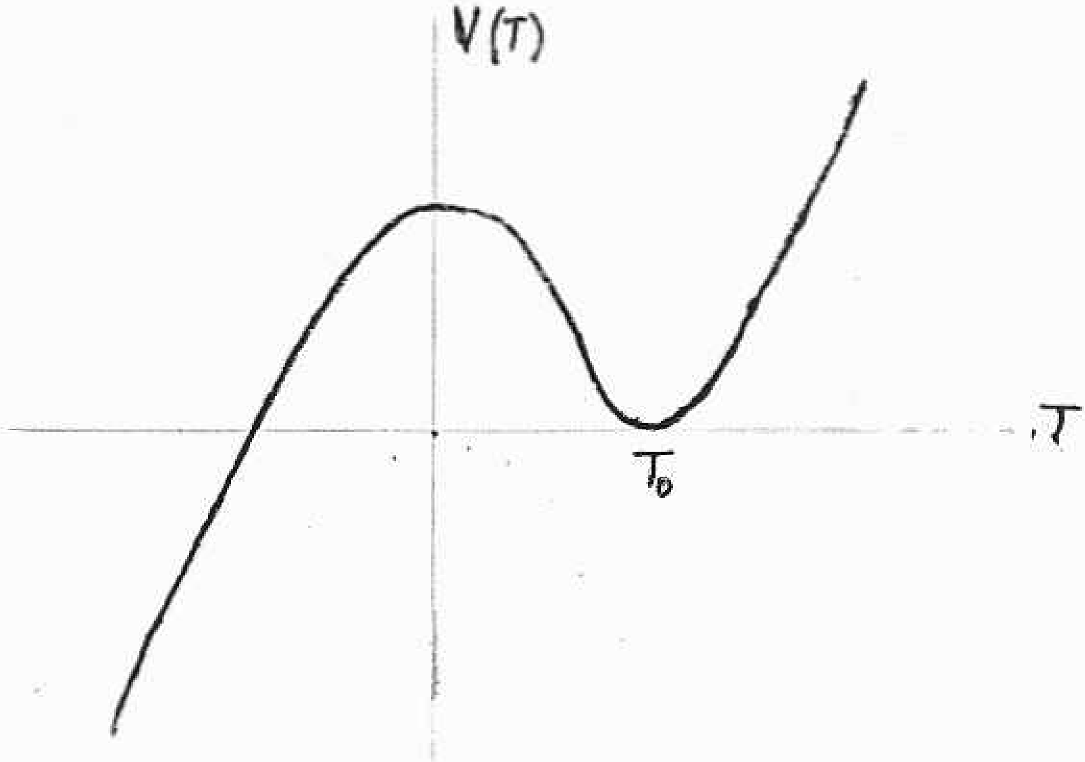
In amplitudes we must soak $\#_c = -3$, picture $= 2$ at infinity.

For instance, for tachyons

$$V^{(-1)} = e^{-\varphi}\, e^{ik.X}$$

$$V^{(0)} = -k\cdot\psi\, e^{ik.X}$$

Sen's conjectures (on D=26 OBS)



$$V(T) = M\left(1 + f(T)\right) \qquad M = T_{25}$$

1) $f(T_0) = -1$ ← demonstrated by M. Schnabl hep-th/0511286

2) There exist soliton lumps that correspond to lower dimensional branes

3) The vacuum at T_0 is the closed string vacuum ← proved by I. Ellwood and M. Schnabl hep-th/0606142

Sen's first conjecture

One can prove that

$$H_0 = \lim_{N\to\infty} \langle \psi_N , Q\psi_N \rangle = \frac{1}{2} + \frac{2}{\pi^2}$$

$$H_1 = \lim_{N\to\infty} \sum_{m=0}^{N} \langle \psi_N , Q\psi'_m \rangle = \frac{1}{2} + \frac{2}{\pi^2}$$

$$H_2 = \lim_{N\to\infty} \sum_{n=0}^{N} \sum_{m=0}^{N} \langle \psi'_n , Q\psi'_m \rangle = \frac{1}{2} - \frac{1}{\pi^2}$$

Therefore

$$\langle \psi , Q\psi \rangle = H_2 - 2H_1 + H_0 = -\frac{3}{\pi^2}$$

Since

$$\langle \psi , Q\psi \rangle = -\langle \psi , \psi * \psi \rangle$$

it follows

$$\boxed{\frac{1}{g_0^2}\left[\frac{1}{2}\langle \psi , Q_B\psi \rangle + \frac{1}{3}\langle \psi , \psi * \psi \rangle\right] = -\frac{1}{2\pi^2 g_0^2}}$$

Schnabl's solution

$$\boxed{\Psi = \lim_{N\to\infty}\left(\sum_{n=0}^{N} \psi'_n - \psi_N\right)}$$

where

$$\psi_n = \frac{-1}{\pi}\, c_1|0\rangle * (B_1^L - B_1^R)\,|n\rangle * c_1|0\rangle$$

and

$$\psi'_n = c_1|0\rangle * K_1^R|n\rangle * B_1^R c_1|0\rangle$$

$$\psi'_0 = K_1^R c_1|0\rangle + B_1^R c_0 c_1|0\rangle$$

One can prove that

$$Q\psi'_0 = 0$$

$$Q\psi'_{n+1} = -\sum_{m=0}^{n} \psi'_m * \psi'_{n-m}$$

Therefore

$$Q\psi_\lambda + \psi_\lambda * \psi_\lambda = 0 \qquad \forall\lambda$$

with

$$\psi_\lambda = \sum_{n=0}^{\infty} \lambda^{n+1}\psi'_n$$

Schnabl's solution corresponds to $\lambda = 1$

New representation of wedge states

$$|n\rangle = e^{\frac{2-n}{2}(\mathcal{L}_0+\mathcal{L}_0^\dagger)}|0\rangle$$

This is a squeezed state (matter part)

$$e^{\frac{2-n}{2}(\mathcal{L}_0+\mathcal{L}_0^\dagger)}|0\rangle = e^{\frac{2-n}{2}(a^\dagger A a^\dagger + a B a + a^\dagger C a}|0\rangle =$$

$$= e^{\eta}\, e^{a^\dagger \alpha a^\dagger}\, e^{a^\dagger \gamma a}\, e^{a \beta a}|0\rangle = e^{\eta}\, e^{a^\dagger \alpha a^\dagger}|0\rangle$$

where $\left(t = \frac{2-n}{2}\right)$

$$\alpha(t) = A\,\frac{\sinh\left(\sqrt{C^2-4A^2}\,t\right)}{\sqrt{C^2-4A^2}\cosh\left(\sqrt{C^2-4A^2}\,t\right) - C\sinh\left(\sqrt{C^2-4A^2}\,t\right)}$$

In particular

$$|n\rangle = \mathcal{N}_n\, e^{-\frac{1}{2}a^\dagger S_n a^\dagger}|0\rangle$$

The matrix $T_n = CS_n$ can be diagonalized and satisfies

$$T_{n+1} = X - \frac{1-T_n}{1-T_n X}$$

i.e.

$$T_n = \frac{T+(-T)^{n-1}}{1-(-T)^n}$$

where

$$T = \lim_{n\to\infty} T_n \qquad \text{is the sliver}$$

Some formulas

$$\mathcal{L}_n = \oint \frac{d\xi}{2\pi i} \left(1+\xi^2\right) \left(\arctan \xi\right)^{n+1} T_{\xi\xi}$$

$$\mathcal{L}_n^+ = \oint \frac{d\xi}{2\pi i} \left(1+\xi^2\right) \left(\operatorname{arccot} \xi\right)^{n+1} T_{\xi\xi}$$

$$\left[\mathcal{L}_n, \mathcal{L}_m\right] = (n-m)\,\mathcal{L}_{n+m}$$

Important example

$$\mathcal{L}_0 = L_0 + \sum_{k=1}^{\infty} \frac{2(-1)^{k+1}}{4k^2-1} L_{2k}$$

$$\left[\mathcal{L}_0, \mathcal{L}_0^+\right] = \mathcal{L}_0 + \mathcal{L}_0^+ \qquad \left[\mathcal{L}_0, K_1\right] = K_1$$

$$K_1 = \mathcal{L}_0 + \mathcal{L}_0^+$$

$$K_1^L = \frac{1}{2} K_1 + \frac{1}{\pi}\left(\mathcal{L}_0 + \mathcal{L}_0^+\right)$$

$$K_1^R = \frac{1}{2} K_1 - \frac{1}{\pi}\left(\mathcal{L}_0 + \mathcal{L}_0^+\right)$$

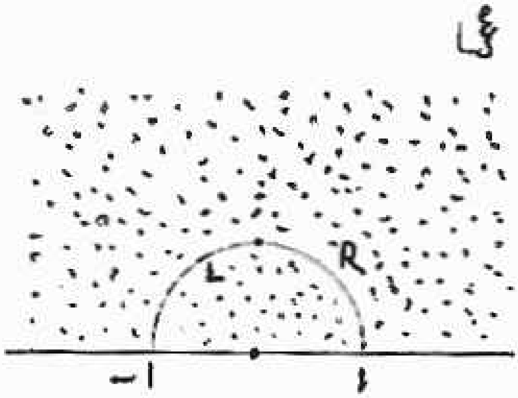
Other important objects

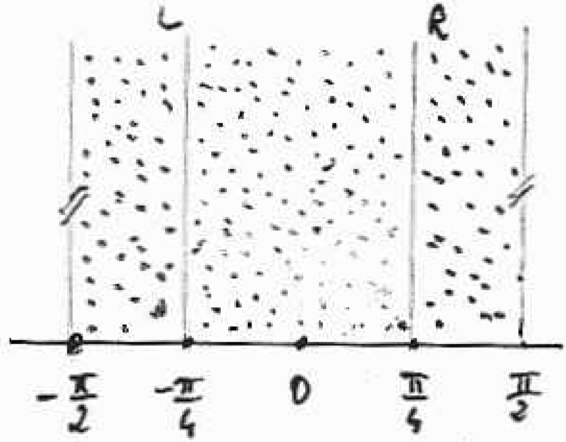
$$\mathcal{B}_0 = b_0 + \sum_{k=1}^{\infty} \frac{2(-1)^{k+1}}{4k^2-1} b_{2k}$$

$$B_1 = b_1 + b_{-1}$$

$$B_1^L = \frac{1}{2} B_1 + \frac{1}{\pi}\left(\mathcal{B}_0 + \mathcal{B}_0^+\right)$$

$$B_1^R = \frac{1}{2} B_1 - \frac{1}{\pi}\left(\mathcal{B}_0 + \mathcal{B}_0^+\right)$$



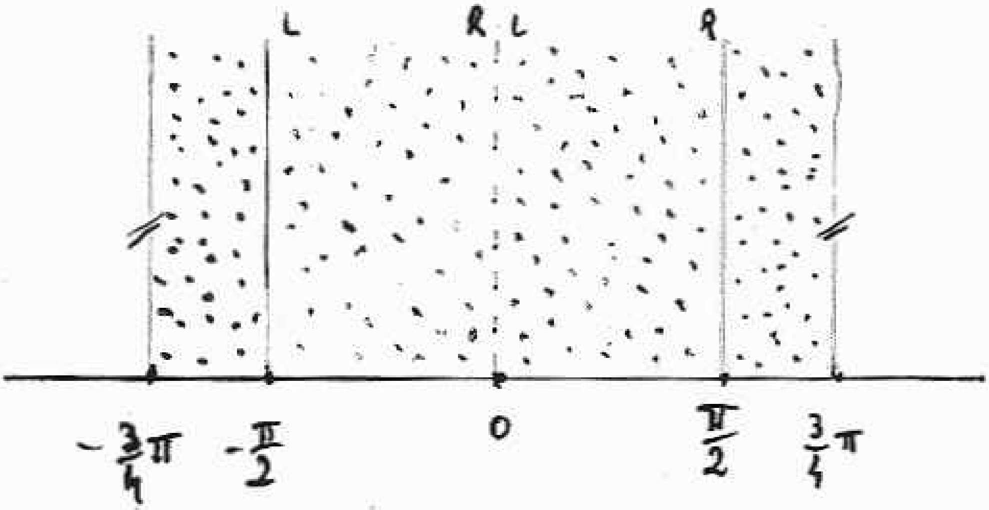


$$\boxed{z = \operatorname{arctg} \xi}$$

Wedge state $|2\rangle \equiv |0\rangle$ corresponds to map $\xi \to z$

Star product easy!

$$|2\rangle * |2\rangle = |3\rangle$$



Representation of wedge states $|n\rangle$

1) $\langle n|\phi\rangle \equiv \langle f_n \circ \phi(0)\rangle$ for any state $|\phi\rangle = \phi(0)|0\rangle$

$$f_n(z) = \frac{n}{2}\,\mathrm{tg}\left(\frac{2}{n}\,\mathrm{tg}^{-1}(z)\right)$$

2) $$|n\rangle = \exp\left(-\frac{n^2-4}{3n^2}L_{-2} + \frac{n^4-16}{30n^4}L_{-4} - \frac{(n^2-4)(176+128n^2+11n^4)}{1890n^6}L_{-6} + \dots\right)|0\rangle$$

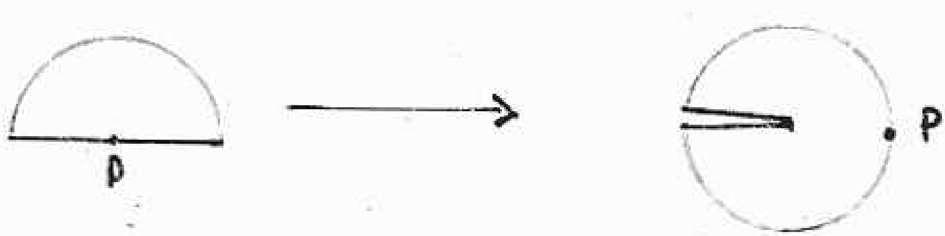
Star product of wedge states

$$|n\rangle * |m\rangle = |n+m-1\rangle$$

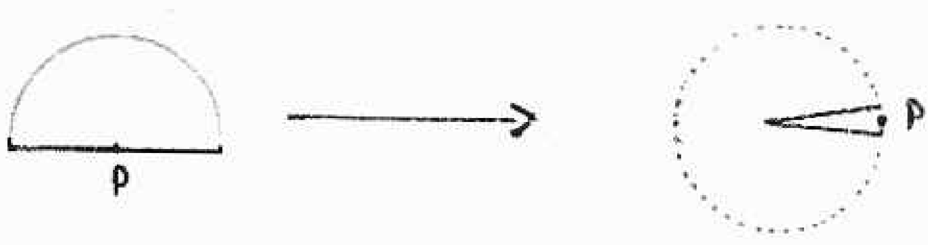
Two states satisfy $\psi * \psi = \psi$

$n = 1$ <u>identity state</u> $|I\rangle \equiv |1\rangle$

$n = \infty$ <u>sliver state</u> $|\Xi\rangle \equiv |\infty\rangle$



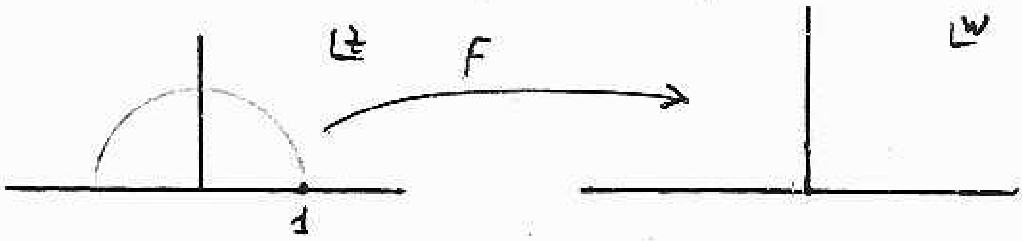
$|I\rangle$



$|\infty\rangle$

Surface states

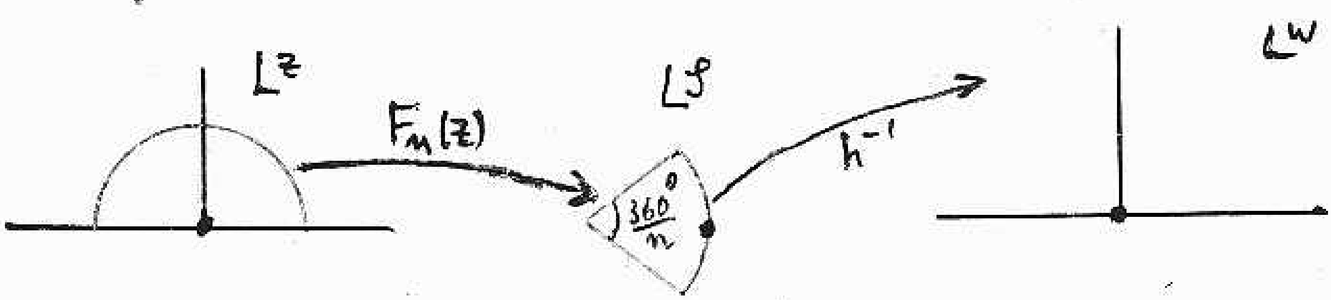
defined via conformal map $F(z)$ of the upper half disk to the upper half plane



$\langle f|$ is defined via

$$\langle f|\phi\rangle = \langle f\circ\phi(0)\rangle \qquad |\phi\rangle = \phi(0)|0\rangle$$

Wedge states



$$F_n(z) = \left(\frac{1+iz}{1-iz}\right)^{2/n} \qquad h^{-1}(\xi) = -i\,\frac{\xi-1}{\xi+1}$$

$$f_n = h^{-1}\circ F_n(z) = \mathrm{tg}\left(\frac{2}{n}\,\mathrm{arctg}(z)\right)$$

Then

$$|n\rangle * |m\rangle = |n+m-1\rangle$$

and

$$|n=1\rangle = |I\rangle \qquad |n=\infty\rangle = \text{sliver}$$