

R^4 in type II superstrings

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Challenges Beyond the Standard Model
BW2007, Serbia

Outline

- 1 Introduction
 - Motivation
 - Overview
 - Scattering Amplitudes

- 2 R^4 , purified
 - Introduction
 - The ingredients
 - The four-point Lagrangian

- 3 Conclusions

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Motivation

String/M-theory can be approximated by LEEA

String/M-theory predicts higher-derivative corrections

- String-theory ($10D$) implies l_{String} corrections
- M-theory ($11D$) receives l_{Planck} corrections
- κ -symmetric branes receive l_{String}, l_{Planck} corrections

Motivation

Important consequences

- Qualitative modifications to vacua
- Implications for no-go theorems
- Testing dualities beyond leading-order
- Black-hole precision measurements

Overview

Various approaches

- Corrections not fully under control
- Much recent progress

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1 Introduction

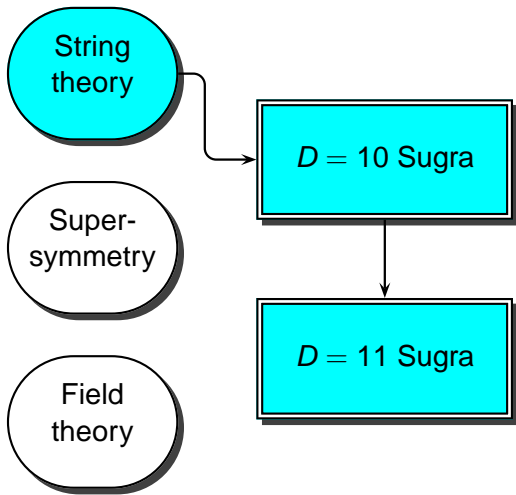
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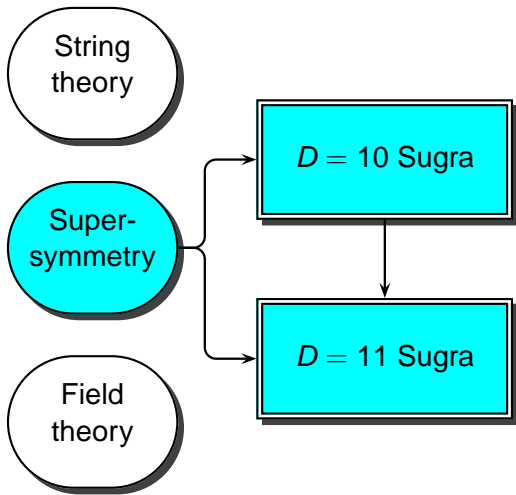
Overview



String perturbation

- Conformal invariance
- Scattering amplitudes

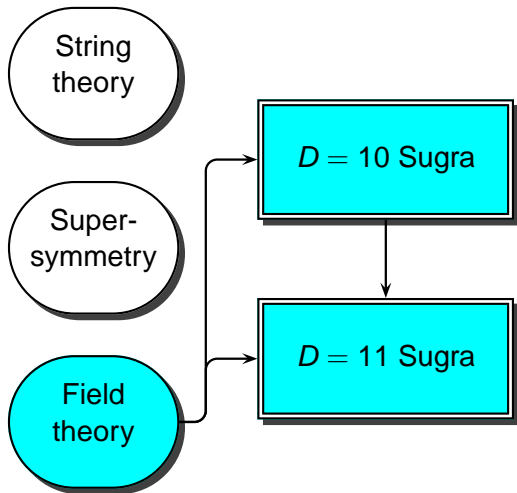
Overview



Supersymmetry

- Component approach
- Harmonic superspace
- Action principle
- Spinorial cohomology

Overview



Field theory

- Higher-loop counterterms
- New techniques

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Scattering Amplitudes

RNS/GS

- Time-honored technique
- Expansions in α' , g_s
- General N -point expressions
- Difficulty with RR fields

-  M.B. Green, J. Schwarz, PLB 109 (1982)
-  D. Gross, E. Witten, NPB 277 (1986)
-  N. Sakai, Y. Tani, NPB 287 (1987)
-  D. Gross, J. Sloan, NPB 291 (1987)
-  K. Peeters, P. Vanhove, A. Westerberg, CQG 18 (2001); 19 (2002)
-  K. Peeters, A. Westerberg, CQG 21 (2004)
-  D. Oprisa, S. Stieberger, hep-th/0509042

Scattering Amplitudes

RNS/GS

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hep-th/0509042

Scattering Amplitudes

Pure-spinor

- Bypasses difficulties with RNS/GS
- Easier to treat RR fields



N. Berkovits, JHEP 0004;
0409; 0601



N. Berkovits B.C. Vallilo,
JHEP 0007



N. Berkovits, CRP 6 (2005)



C.R. Mafra, JHEP 0601



G. Policastro, DT,
CQG 23 (2006)



N. Berkovits, C.R. Mafra,
JHEP 0611



C. Stahn, 0704.0015

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Pure-spinor superstring

BRST operator

$$Q = \oint dz \lambda^\alpha d_\alpha$$

where

$$\lambda \gamma^a \lambda = 0$$

Massless vertex operators

$$QU = 0, \quad QV = \alpha' \partial U$$

Tree Amplitudes

N -point amplitude

$$\mathcal{A} = \langle U_1(z_1) U_2(z_2) U_3(z_3) \int dz_4 V_4(z_4) \dots \int dz_N V_N(z_N) \rangle$$

Integrate out nonzero modes:

$$\mathcal{A} = \int dz_4 \dots \int dz_N \langle \lambda^\alpha \lambda^\beta \lambda^\gamma f_{\alpha\beta\gamma}(z_r, k_r, \theta) \rangle$$

Zero-mode integration

$$\mathcal{A} = T^{\alpha\beta\gamma} \int dz_4 \dots \int dz_N f_{\alpha\beta\gamma}(z_r, k_r, \theta)$$

where:

$$T^{\alpha\beta\gamma} (\gamma^i \theta)_\alpha (\gamma^j \theta)_\beta (\gamma^k \theta)_\gamma (\theta \gamma_{ijk} \theta) = 1$$

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Vertices

Integrated vertex

$$U = \lambda^\alpha A_\alpha(x, \theta)$$

10D SYM in Superspace

- Field-strength: $F = DA$
- Bianchi identities: $DF = 0$
- $F_{\alpha\beta} = 0 \implies$ **ordinary** SYM



B.E.W. Nilsson, Göteborg-ITP-81-6

The expansion of the vertices

Wess-Zumino gauge

$$\theta^\alpha A_\alpha = 0$$

so that:

$$\theta^\alpha D_\alpha = \theta^\alpha \frac{\partial}{\partial \theta^\alpha}$$

Integrate on-shell conditions

$$A_m = [\cosh\sqrt{\mathcal{O}}]_m^q a_q + [\mathcal{O}^{-\frac{1}{2}} \sinh\sqrt{\mathcal{O}}]_m^q (\theta\gamma_q \xi)$$

where:

$$[\mathcal{O}]_m^q := \frac{1}{2}(\theta\gamma_m^{qp}\theta)\partial_p; \quad a_q := A_q|, \quad \xi^\alpha := W^\alpha|$$

Solvability in linearized approximation is generic



DT, JHEP 0411

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DT, JHEP 0411

The Kawai-Lewellen-Tye Relations



$$\int d^2z = \int dz \otimes \int d\bar{z}$$

The Kawai-Lewellen-Tye Relations



$$\int d^2z = \int dz \otimes \int d\bar{z}$$

Four-point amplitude factorizes

$$\mathcal{A}_4^{\text{cl}} = -g^2 \sin(\pi\alpha' k_2 \cdot k_3) \mathcal{A}_4^{\text{op}}\left(\frac{\alpha's}{2}, \frac{\alpha't}{2}\right) \otimes \tilde{\mathcal{A}}_4^{\text{op}}\left(\frac{\alpha't}{2}, \frac{\alpha'u}{2}\right)$$

The Kawai-Lewellen-Tye Relations



$$\int d^2z = \int dz \otimes \int d\bar{z}$$

Four-point Lagrangian factorizes

$$\mathcal{L}_4^{\text{cl}} = \mathcal{L}_4^{\text{op}} \otimes \tilde{\mathcal{L}}_4^{\text{op}}$$

Pole Subtraction

Unitarity

Poles in N -pt amplitudes come from 1PRD's with N' vertices, where $N' < N$

Before taking the 1PRD's into account:

$$\mathcal{A} \sim f(s, t, u) \{ \dots \}$$

where:

$$f(s, t, u) = -\frac{8\pi}{\alpha'^3 stu} - 2\pi\zeta(3) + \mathcal{O}(\alpha'^2)$$

Taking 1PRD's into account:

$$f(s, t, u) \longrightarrow \hat{g}(s, t, u) := f(s, t, u) + \frac{8\pi}{\alpha'^3 stu}$$

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Four-point Lagrangian: schematically

Four-point Lagrangian

$$\mathcal{L}_{4pt} \propto (\alpha')^3 \widehat{\mathcal{G}} \left\{ \widehat{R}^4 + (\partial F)^2 \widehat{R}^2 + (\partial F)^4 \right\}$$

where:

$$\widehat{R}_{mn}{}^{pq} := R_{mn}{}^{pq} + 2\kappa e^{-\frac{\kappa D}{\sqrt{2}}} \nabla_{[m} H_{n]}{}^{pq} - \sqrt{2}\kappa \delta_{[m}{}^{[p} \nabla_{n]} \nabla^{q]} D$$



G. Policastro, DT,
CQG 23 (2006)

Four-point Lagrangian: the gore and the glory

- NS-NS

$$\mathcal{L}_{NS} = \frac{(\alpha')^3}{4!} \hat{g} t_8 t_8 \hat{R}^4$$

where:

$$\hat{R}_{mn}{}^{pq} := R_{mn}{}^{pq} + 2\kappa e^{-\frac{\kappa D}{\sqrt{2}}} \nabla_{[m} H_{n]}{}^{pq} - \sqrt{2}\kappa \delta_{[m}{}^{[p} \nabla_{n]} \nabla^{q]} D$$

Four-point Lagrangian: the gore and the glory

- RR-NS

$$\mathcal{L}_{(\partial F)^2 R^2} = -2^6 \kappa (\alpha')^3 \widehat{\mathcal{G}} \left(A_1 + \frac{1}{2} A_2 + \frac{1}{4} A_3 \right)$$

where:

$$A_1 := \widehat{R}^i{}_{n'}{}^j{}_{n'} \widehat{R}_{ipjp'} \langle \gamma^n \partial^p \mathcal{F} \gamma^{(n'} \partial^{p')} \mathcal{F}^{Tr} \rangle$$

$$A_2 := \widehat{R}_{mn}{}^i{}_{n'} \widehat{R}_{pqip'} \left(\langle \gamma^{mnp} \partial^q \mathcal{F} \gamma^{(n'} \partial^{p')} \mathcal{F}^{Tr} \rangle + F \leftrightarrow F^{Tr} \right)$$

$$A_3 := \widehat{R}_{mnm'}{}_{n'} \widehat{R}_{pqp'q'} \langle \gamma^{[mnp} \partial^q] \mathcal{F} \gamma^{m' n' p'} \partial^{q'} \mathcal{F}^{Tr} \rangle$$

Four-point Lagrangian: the gore and the glory

- RR-RR

$$\mathcal{L}_{(\partial F)^4} = \frac{32}{9}(\alpha')^3 \kappa^2 \widehat{\mathcal{G}} (B_1 - 5B_2 + B_3 + 4B_4 - B_5)$$

where:

$$B_1 := \langle \partial_m \partial_p F \gamma_q \partial^m \partial^p F^{\text{Tr}} \gamma_n F \gamma^q F^{\text{Tr}} \gamma^n \rangle$$

$$B_2 := \langle \partial_m \partial_p F \gamma_q F^{\text{Tr}} \gamma_n \partial^m \partial^p F \gamma^q F^{\text{Tr}} \gamma^n \rangle$$

$$B_3 := \langle \partial_m \partial_p F \gamma_q F^{\text{Tr}} \gamma_n F \gamma^q \partial^m \partial^p F^{\text{Tr}} \gamma^n \rangle$$

$$B_4 := \langle \partial_m \partial_p F \gamma_q F^{\text{Tr}} \gamma_n \rangle \langle \partial^m \partial^p F \gamma^q F^{\text{Tr}} \gamma^n \rangle$$

$$B_5 := \langle F \gamma_q F^{\text{Tr}} \gamma_n \rangle \langle \partial^m \partial^p F \gamma^q \partial_m \partial_p F^{\text{Tr}} \gamma^n \rangle$$

Future Directions

- Physical applications
- Higher points
- Factorization
- IIB

Thank You