Emergent Gravity from Noncommutative Gauge Theory

Harold Steinacker

Department of Physics, University of Vienna

BW 2007

Introduction

Classical space-time meaningless at Planck scale due to gravity $\leftrightarrow$ Quantum Mechanics

$\Rightarrow$ “Quantized” (noncommutative?) spaces:,

$[x_i, x_j] = i\theta_{ij}$

Space-time uncertainty relations $\Delta x_i \Delta x_j \geq \theta_{ij}$

Realized in string theory (D-branes with $B$-field)

Physics on quantized space:

Noncommutative Quantum Field Theory well developed; some problems
Introduction

Classical space-time meaningless at Planck scale due to gravity ↔ Quantum Mechanics

⇒ “Quantized” (noncommutative?) spaces:, e.g. \([x_i, x_j] = i\theta_{ij}\)

space-time uncertainty relations \(\Delta x_i \Delta x_j \geq \theta_{ij}\)
realized in string theory (D-branes with \(B\)-field)

Physics on quantized space:
Noncommutative Quantum Field Theory
well developed; some problems

Relation with gravity ??
should be simple & naturally related no NC
Introduction

- Classical space-time meaningless at Planck scale due to gravity ↔ Quantum Mechanics
- ⇒ “Quantized” (noncommutative?) spaces: e.g. \([x_i, x_j] = i\theta_{ij}\)
  - space-time uncertainty relations \(\Delta x_i \Delta x_j \geq \theta_{ij}\)
  - realized in string theory (D-branes with \(B\)-field)
- Physics on quantized space: Noncommutative Quantum Field Theory well developed; some problems
- Relation with gravity ?? should be simple & naturally related no NC
Main Message:

- NC gauge theory (as Matrix Model) does contain gravity surprisingly, intrinsically NC mechanism
  gravity tied with NC
  cf. stringy Matrix Models (IKKT)

- Not precisely general relativity
  appears to agree with GR at low energies (?)
  - gravitational waves
  - Newtonian limit
  - linearized metric: $R_{ab} \sim 0$
Main Message:

- NC gauge theory (as Matrix Model) does contain gravity surprising, intrinsically NC mechanism gravity tied with NC
  cf. stringy Matrix Models (IKKT)

- Not precisely general relativity appears to agree with GR at low energies (?)
  - gravitational waves
  - Newtonian limit
  - linearized metric: $R_{ab} \sim 0$
Main result:

The model:

\[
S_{YM} = - \text{Tr}[X^a, X^b][X^{a'}, X^{b'}] \eta_{aa'} \eta_{bb'}
\]

where \( X^a \in L(\mathcal{H}) \) ... matrices (operators), \( a = 0, 1, 2, 3 \)

low-energy effective action:

\[
S_{YM} = \int d^4 y \rho(y) tr \left( 4 \eta(y) - G^{cc'} G^{dd'} F_{cd} F_{c'd'} \right) + 2 \int \eta(y) tr F \wedge F
\]

where

\[
G^{ab}(y) = - \theta^{ac}(y) \theta^{bd}(y) g_{cd} \text{ effective dynamical metric}
\]

\[
F_{ab} \text{ ... } su(n) \text{ field strength}
\]

contains dynamical gravity, close to general relativity
Main result:

The model:

\[ S_{YM} = - \text{Tr} [X^a, X^b] [X^{a'}, X^{b'}] \eta_{aa'} \eta_{bb'} \]

where \( X^a \in L(\mathcal{H}) \) ... matrices (operators), \( a = 0, 1, 2, 3 \)

low-energy effective action:

\[
S_{YM} = \int d^4 y \rho(y) \text{tr} \left( 4\eta(y) - G^{cc'} G^{dd'} F_{cd} F_{c'd'} \right) + 2 \int \eta(y) \text{tr} F \wedge F
\]

where

\[
G^{ab}(y) = - \theta^{ac}(y) \theta^{bd}(y) g_{cd} \quad \text{effective dynamical metric}
\]

\[
F_{ab} \quad \text{su}(n) \quad \text{field strength}
\]

contains dynamical gravity, close to general relativity
Outline

- NC gauge theory as Matrix Model
  → dynamical quantum spaces
- Effective metric, geometry
- Low-energy effective action and emergent gravity
- Some checks:
  - Gravitational waves, linearized metric
  - Newtonian Limit
- Remarks on quantization, UV/IR
- Conclusion
Consider Matrix Model:

\[ S_{YM} = - \text{Tr} \left( ([X^a, X^b] - i\theta^{ab}) ([X'^a, X'^b] - i\theta^{a'b'}) \right) \eta_{aa'}\eta_{bb'} \]

\( \theta^{ab} \) ... antisymmetric tensor, nondegenerate \( a = 0, 1, 2, 3 \)
dynamical objects:

\[ X^a = \overline{Y}^a + A^a \in L(H) \]

... hermitian matrices / operators ("covariant coordinates")

\[ [\overline{Y}^a, \overline{Y}^b] = i\theta^{ab} \]

"quantum plane"

(cf. Q.M. phase space, Heisenberg-algebra)

"conventional" point of view:

- describes \( U(1) \) Yang-Mills on quantum plane \( \mathbb{R}^4_\theta \)
- \( \rightarrow \) usual \( U(1) \) Yang-Mills on \( \mathbb{R}^4 \) for \( \theta \rightarrow 0 \)
Consider Matrix Model:

\[ S_{YM} = - \text{Tr} \left( ([X^a, X^b] - i\bar{\theta}^{ab})([X'^a, X'^b] - i\bar{\theta}^{a'b'}) \right) \eta_{aa'}\eta_{bb'} \]

\( \bar{\theta}^{ab} \) ... antisymmetric tensor, nondegenerate \( a = 0, 1, 2, 3 \)

dynamical objects:

\[ X^a = \overline{Y}^a + A^a \in L(H) \]

... hermitian matrices / operators ("covariant coordinates")

\[ [\overline{Y}^a, \overline{Y}^b] = i\bar{\theta}^{ab} \]

"quantum plane"

(cf. Q.M. phase space, Heisenberg-algebra)

"conventional" point of view:

- describes \( U(1) \) Yang-Mills on quantum plane \( \mathbb{R}^4_\theta \)
- \( \rightarrow \) usual \( U(1) \) Yang-Mills on \( \mathbb{R}^4 \) for \( \theta \rightarrow 0 \)
why? (“standard” analysis)

let \( [\overline{Y}^a, \overline{Y}^b] = i \Theta^{ab} \) ... quantum plane \( \mathbb{R}^4_\theta \),
then

\[
[\overline{Y}^a, f(\overline{Y})] \sim i \Theta^{ab} \partial_b f(\overline{Y}) \quad \text{for “smooth function” } f(\overline{Y}) \approx f(\overline{y}), \, \Theta \approx 0
\]

let \( X^a = \overline{Y}^a + \Theta^{ab} A_b \) then

\[
[X^a, X^b] - i \Theta^{ab} = \Theta^{aa'} \Theta^{bb'} (\partial_{a'} A_{b'} - \partial_{b'} A_{a'} + [A_{a'}, A_{b'}])
\]

so

\[
S_{YM} \sim \int F_{ab} F_{a'b'} \overline{g}^{aa'} \overline{g}^{bb'}, \quad \overline{g}^{ab} = -\Theta^{aa'} \Theta^{bb'} \eta_{a'b'}
\]

gauge fields ... fluctuations of covariant coordinates
why? ("standard" analysis)

let \( [\overline{Y}^a, \overline{Y}^b] = i\theta^{ab} \) ... quantum plane \( \mathbb{R}^4_{\theta} \),
then

\[ [\overline{Y}^a, f(\overline{Y})] \sim i\theta^{ab} \partial_b f(\overline{Y}) \quad \text{for "smooth function" } f(\overline{Y}) \approx f(y), \theta \approx 0 \]

let \( X^a = \overline{Y}^a + \theta^{ab} A_b \) then

\[
[X^a, X^b] - i\theta^{ab} = \theta^{aa'}\theta^{bb'} \left( \partial_{a'} A_{b'} - \partial_{b'} A_{a'} + [A_{a'}, A_{b'}] \right) = \theta^{aa'}\theta^{bb'} F_{a'b'}
\]

so

\[
S_{YM} \sim \int F_{ab} F_{a'b'} \overline{g}^{aa'} \overline{g}^{bb'}, \quad \overline{g}^{ab} = -\theta^{aa'}\theta^{bb'} \eta_{a'b'}
\]

gauge fields ... fluctuations of covariant coordinates
however:

- ∃ versions for compact ("fuzzy") spaces $S^2_N \times S^2_N, \mathbb{C}P^2$
  H. Grosse, H.S; W. Behr, F. Meyer, H.S

- space itself obtained as "vacuum" of similar matrix model
  $\Rightarrow$ space is dynamical;

- fluctuations of covariant coords $X^a \leftrightarrow$ gravity ?!

- string-theoretical matrix models (IKKT, BFSS)
  are supposed to contain gravity

- NC $u(1)$ gauge theory $\leftrightarrow$ gravity proposed in
  Yang [hep-th/0612231] (string theory)
However:

- There exist versions for compact ("fuzzy") spaces $S^2_N \times S^2_N$, $\mathbb{C}P^2$

  H. Grosse, H.S; W. Behr, F. Meyer, H.S

  - Space itself obtained as "vacuum" of similar matrix model
  - $\Rightarrow$ space is dynamical;
  - Fluctuations of covariant coords $X^a \leftrightarrow$ gravity ?!

- String-theoretical matrix models (IKKT, BFSS) are supposed to contain gravity

- Noncommutative $u(1)$ gauge theory $\leftrightarrow$ gravity proposed in
  Yang [hep-th/0612231] (string theory)
Geometry from NC $u(n)$ gauge theory:

- $u(n)$, naive:

$$X^a = \overline{Y}^a \otimes 1_n + \overline{\theta}^{ab} (A^0_b \otimes 1_n + A^\alpha_b \otimes \lambda_\alpha) \in \mathcal{A} \otimes u(n),$$

$$\mathcal{A} \ldots \text{functions on } \mathbb{R}^4_{\theta}$$

- ... obtain $u(n)$ Yang-Mills

- here: separate $u(1)$ and $su(n)$ components

$$X^a = (\overline{Y}^a + \overline{\theta}^{ab} A^0_b) \otimes 1_n + (\overline{\theta}^{ab} A^\alpha_b \otimes \lambda_\alpha)$$

$$=:\quad Y^a \otimes 1_n \quad + \quad \theta^{ab} A^\alpha_b \otimes \lambda_\alpha$$
Geometry from NC $u(n)$ gauge theory:

- $u(n)$, naive:

\[
X^a = \bar{Y}^a \otimes 1_n + \bar{\theta}^{ab} (A_0^b \otimes 1_n + A_\alpha^b \otimes \lambda_\alpha) \in \mathcal{A} \otimes u(n),
\]

$\mathcal{A}$ ... functions on $\mathbb{R}^4$

... obtain $u(n)$ Yang-Mills

- here: separate $u(1)$ and $su(n)$ components

\[
X^a = (\bar{Y}^a + \bar{\theta}^{ab} A_0^b) \otimes 1_n + (\bar{\theta}^{ab} A_\alpha^b \otimes \lambda_\alpha)
\]

\[
=: Y^a \otimes 1_n + \theta^{ab} A_\alpha^b \otimes \lambda_\alpha
\]

will see:

$u(1)$ component $Y^a$ ... dynamical geometry, gravity

$su(n)$ components $A_\alpha^a$ ... $su(n)$ gauge field coupled to gravity
Geometry from NC $u(n)$ gauge theory:

- $u(n)$, naive:

$$X^a = \bar{Y}^a \otimes 1_n + \bar{\theta}^{ab} (A^0_b \otimes 1_n + A^\alpha_b \otimes \lambda_\alpha) \in \mathcal{A} \otimes u(n),$$

$\mathcal{A}$ ... functions on $\mathbb{R}^4$

... obtain $u(n)$ Yang-Mills

- here: separate $u(1)$ and $su(n)$ components

$$X^a = (\bar{Y}^a + \bar{\theta}^{ab} A^0_b) \otimes 1_n + (\bar{\theta}^{ab} A^\alpha_b \otimes \lambda_\alpha)$$

$$=: Y^a \otimes 1_n + \theta^{ab} A^\alpha_b \otimes \lambda_\alpha$$

will see:

- $u(1)$ component $Y^a$ ... dynamical geometry, gravity

- $su(n)$ components $A^\alpha_a$ ... $su(n)$ gauge field coupled to gravity
\( u(1) \) components \( Y^a \) \( \leftrightarrow \) general Poisson structure:

\[
[Y^a, Y^b] = i\theta^{ab}(y)
\]

then

\[
[Y^a, \Phi(y)] = i\theta^{ab}(y)\partial_b \Phi(y) + O(\theta^2)
\]

consider additional scalar \( \Phi \) in adjoint

\[
S[\Phi] = -\text{Tr} \eta_{aa'}[X^a, \Phi][X^{a'}, \Phi]
= \text{Tr} G^{ab}(y) (\partial_a + [A_a, .])(\partial_b + [A_b, .])\Phi
\]

where

\[
G^{ab}(y) = -\theta^{ac}(y)\theta^{bd}(y)\eta_{cd}
\]

- \( \Phi \) couples to effective metric \( G^{ab} \) determined by \( \theta^{ab}(y) \)
- \( \theta^{ac}(y) \) ... vielbein
\( u(1) \) components \( Y^a \leftrightarrow \) general Poisson structure:

\[ [Y^a, Y^b] = i\theta^{ab}(y) \]

then

\[ [Y^a, \Phi(y)] = i\theta^{ab}(y)\partial_b \Phi(y) + O(\theta^2) \]

consider additional scalar \( \Phi \) in adjoint

\[
S[\Phi] = -\text{Tr} \eta_{aa'}[X^a, \Phi][X^{a'}, \Phi] \\
= \text{Tr} G^{ab}(y) (\partial_a + [A_a, .])\Phi(\partial_b + [A_b, .])\Phi
\]

where

\[
G^{ab}(y) = -\theta^{ac}(y)\theta^{bd}(y)\eta_{cd}
\]

- \( \Phi \) couples to effective metric \( G^{ab} \) determined by \( \theta^{ab}(y) \)
- \( \theta^{ac}(y) \) ... vielbein
nonabelian gauge fields (heuristic)

set $X^a = Y^a + \theta^{ab}(y)A_b(y)$ obtain

$$[X^a, X^b] = i\theta^{ab}(y) + i\theta^{ac}\theta^{bd}(\partial_c A_d - \partial_d A_c + [A_c, A_d] + O(\theta^{-1}\partial\theta))$$

$$= i\theta^{ab}(y) + i\theta^{ac}(y)\theta^{bd}(y)F_{cd} + O(\theta^{-1}\partial\theta))$$

hence

$$S_{YM} = -\text{Tr}[X^a, X^b][X^{a'}, X^{b'}]\eta_{aa'}\eta_{bb'}$$

$$\approx \text{Tr} \left( G^{ab}(y)\eta_{ab} - G^{cc'}(y) G^{dd'}(y) (F_{cd} F_{c'd'} + O(\theta^{-1}\partial\theta)) \right)$$

using $\text{Tr}(\theta^{ab}(y)F^{ab}) \approx 0$

similar to $\mathfrak{su}(n)$ YM coupled to metric $G^{ab}(y)$
nonabelian gauge fields (correct)

Seiberg-Witten map:

\[ X^a = Y^a + \theta^{ab} A_b - \frac{1}{2} (A_c [Y^c, \theta^{ad} A_d] + A_c F^{ca}) + O(\theta^3) \]

- expresses \( su(n) \) d.o.f. in terms of commutative \( su(n) \) gauge fields \( A_a \)
- relates NC g.t. \( i[\Lambda, X^a] \) in terms of standard \( su(n) \) g.t. of \( A_a \)

Volume element:

\[
(2\pi)^2 \ Tr f(y) = \int d^4 y \, \rho(y) \, f(y), \\
\rho(y) = \sqrt{\det(\theta^{-1}_{ab})} = \left(\det(\eta_{ab}) \det(G_{ab})\right)^{1/4}
\]

(cp. Bohr-Sommerfeld quantization)
Seiberg-Witten map:

\[ X^a = Y^a + \theta^{ab} A_b - \frac{1}{2}(A_c[Y^c, \theta^{ad} A_d] + A_c F^{ca}) + O(\theta^3) \]

- expresses \( su(n) \) d.o.f. in terms of commutative \( su(n) \) gauge fields \( A_a \)
- relates NC g.t. \( i[\Lambda, X^a] \) in terms of standard \( su(n) \) g.t. of \( A_a \)

Volume element:

\[(2\pi)^2 \text{Tr} f(y) = \int d^4 y \rho(y) f(y), \]
\[\rho(y) = \sqrt{\text{det}(\theta^{-1})} = (\text{det}(\eta_{ab}) \text{det}(G_{ab}))^{1/4}\]

(cp. Bohr-Sommerfeld quantization)
effective action to leading order:

\[
S_{YM} = \int d^4 y \, \rho(y) \text{tr} \left( 4 \eta(y) - G^{cc'} G^{dd'} F_{cd} F_{c'd'} \right) + 2 \int \eta(y) \text{tr} F \wedge F
\]

where

\[
\eta(y) = G^{ab}(y) \eta_{ab}
\]

indeed \( su(n) \) YM coupled to metric \( G^{ab}(y) \)
effective action to leading order:

\[ S_{YM} = \int d^4 y \rho(y) \text{tr} \left( 4\eta(y) - G^{cc'} G^{dd'} F_{cd} F_{c'd'} \right) + 2 \int \eta(y) \text{tr} F \wedge F \]

where

\[ \eta(y) = G^{ab}(y) \eta_{ab} \]

- indeed \( su(n) \) YM coupled to metric \( G^{ab}(y) \)
- additional term \( \int \eta(y) \text{tr} F \wedge F \), topological for \( \theta^{ab} = \text{const} \)
Effective action to leading order:

$$S_{YM} = \int d^4y \rho(y) \text{tr} \left( 4\eta(y) - G^{cc'} G^{dd'} F_{cd} F_{c'd'} \right) + 2 \int \eta(y) \text{tr} F \wedge F$$

where

$$\eta(y) = G^{ab}(y) \eta_{ab}$$

- indeed $su(n)$ YM coupled to metric $G^{ab}(y)$
- additional term $\int \eta(y) \text{tr} F \wedge F$, topological for $\theta^{ab} = \text{const}$
- “scalar action” $\int d^4y \rho(y) \eta(y)$ will imply vacuum equations $R_{ab} \sim 0$
effective action to leading order:

$$S_{YM} = \int d^4 y \rho(y) tr \left( 4 \eta(y) - G_{cc'} G_{dd'}^d F_{cd} F_{c'd'} \right) + 2 \int \eta(y) tr F \wedge F$$

where

$$\eta(y) = G^{ab}(y) \eta_{ab}$$

- indeed $su(n)$ YM coupled to metric $G^{ab}(y)$
- additional term $\int \eta(y) tr F \wedge F$, topological for $\theta^{ab} = const$
- “scalar action” $\int d^4 y \rho(y) \eta(y)$ will imply vacuum equations $R_{ab} \sim 0$
- $u(1)$ d.o.f. in dynamical metric $G^{ab}(y) = -\theta^{ac}(y) \theta^{bd}(y) \eta_{cd}$
  $\Rightarrow$ dynamical gravity
effective action to leading order:

\[ S_{YM} = \int d^4y \, \rho(y) \text{tr} \left( 4\eta(y) - G^{cc'} G^{dd'} F_{cd} F_{c'd'} \right) + 2 \int \eta(y) \, \text{tr} \, F \wedge F \]

where

\[ \eta(y) = G^{ab}(y) \eta_{ab} \]

- indeed \( su(n) \) YM coupled to metric \( G^{ab}(y) \)
- additional term \( \int \eta(y) \, \text{tr} \, F \wedge F \), topological for \( \theta^{ab} = \text{const} \)
- “scalar action” \( \int d^4y \, \rho(y) \eta(y) \) will imply vacuum equations \( R_{ab} \sim 0 \)
- \( u(1) \) d.o.f. in dynamical metric \( G^{ab}(y) = -\theta^{ac}(y) \theta^{bd}(y) \eta_{cd} \)

⇒ dynamical gravity
Remaining discussion:

- **linearized gravity:**
  - gravitational waves, Newtonian limit

- **quantization:**
  - induced Einstein-Hilbert action and UV/IR mixing
Remaining discussion:

- **linearized gravity:**
  gravitational waves, Newtonian limit

- **quantization:**
  induced Einstein-Hilbert action and UV/IR mixing
linearized NC gravity:

flat space: Moyal-Weyl \[ \bar{\theta}^{ab} = \text{const} \]
\[ \Rightarrow G^{ab} = -\theta^{ac} \theta^{bd} \eta_{cd} =: \bar{\eta}^{ab} \] ... flat Minkowski metric

small fluctuations:
\[ Y^a = \bar{Y}^a + \bar{\theta}^{ab} A_0^b \] (u(1) component)

\[ \theta^{ab}(y) = -i[Y^a, Y^b] = \bar{\theta}^{ab} + \theta^{ac}\theta^{bd} F^0_{cd}(y) \]

\[ F^0_{cd}(y) \] ... u(1) field strength
linearized NC gravity:

flat space: Moyal-Weyl \( \bar{\theta}^{ab} = \text{const} \)

\[ G^{ab} = -\bar{\theta}^{ac} \bar{\theta}^{bd} \eta_{cd} =: \bar{\eta}^{ab} \quad \text{... flat Minkowski metric} \]

small fluctuations:

\[ Y^a = \overline{Y}^a + \bar{\theta}^{ab} A^0_b \quad (u(1) \text{ component}) \]

\[ \theta^{ab}(y) = -i [Y^a, Y^b] = \bar{\theta}^{ab} + \bar{\theta}^{ac} \bar{\theta}^{bd} F^0_{cd}(y) \]

\[ F^0_{cd}(y) \quad \text{... } u(1) \text{ field strength} \]

\[ G^{ab}(y) = - (\bar{\theta}^{ac} + \bar{\theta}^{ae} \bar{\theta}^{ch} F^0_{eh})(\bar{\theta}^{bd} + \bar{\theta}^{bf} \bar{\theta}^{dg} F^0_{fg})\eta_{cd} \approx \bar{\eta}^{ab} - h^{ab} \]

where

\[ h_{ab} = \bar{\eta}_{bb'} \bar{\theta}^{b'c} F^0_{ca} + \bar{\eta}_{aa'} \bar{\theta}^{a'c} F^0_{cb} \]

... linearized metric fluctuation (cf. Rivelles [hep-th/0212262])
linearized NC gravity:

flat space: Moyal-Weyl \( \bar{\theta}^{ab} = \text{const} \)

\[
\Rightarrow G^{ab} = -\bar{\theta}^{ac} \bar{\theta}^{bd} \eta_{cd} =: \bar{\eta}^{ab} \quad \text{... flat Minkowski metric}
\]

small fluctuations: \( Y^a = \bar{Y}^a + \bar{\theta}^{ab} A^0_b \) (\( u(1) \) component)

\[
\theta^{ab}(y) = -i[Y^a, Y^b] = \bar{\theta}^{ab} + \bar{\theta}^{ac} \bar{\theta}^{bd} F^0_{cd}(y)
\]

\( F^0_{cd}(y) \) ... \( u(1) \) field strength

\[
G^{ab}(y) = -(\bar{\theta}^{ac} + \bar{\theta}^{ae} \bar{\theta}^{ch} F^0_{eh})(\bar{\theta}^{bd} + \bar{\theta}^{bf} \bar{\theta}^{dg} F^0_{fg})\eta_{cd}
\]

\[
\approx \bar{\eta}^{ab} - h^{ab}
\]

where

\[
h_{ab} = \bar{\eta}_{bb'} \bar{\theta}^{b'c} F^0_{ca} + \bar{\eta}_{aa'} \bar{\theta}^{a'c} F^0_{cb}
\]

... linearized metric fluctuation (cf. Rivelles [hep-th/0212262])
e.o.m for gravitational d.o.f.:

\[
[\gamma^a, \theta^{ab}(y)] = 0 \iff G^{ac} \partial_c \theta^{-1}_{ab}(y) = 0
\]

implies vacuum equations of motion (linearized)

\[
R_{ab} = 0 + O(\theta^2)
\]

moreover \( R_{abcd} = O(\theta) \neq 0 \) ... nonvanishing curvature

\( \Rightarrow \) on-shell d.o.f. of gravitational waves on Minkowski space

**note**

- \( G^{ab} = -\theta^{ac}(y) \theta^{bd}(y)\eta_{cd} \) ... restricted class of metrics
- same on-shell d.o.f. as general relativity (for vacuum)
e.o.m for gravitational d.o.f.:

\[ [Y^a, \theta^{ab}(y)] = 0 \iff G^{ac} \partial_c \theta^{-1}_{ab}(y) = 0 \]

implies vacuum equations of motion (linearized)

\[ R_{ab} = 0 + O(\theta^2) \]

moreover \[ R_{abcd} = O(\bar{\theta}) \neq 0 \] ... nonvanishing curvature

\[ \Rightarrow \text{on-shell d.o.f. of gravitational waves on Minkowski space} \]

**note**

- \[ G^{ab} = -\theta^{ac}(y) \theta^{bd}(y) \eta_{cd} \] ... restricted class of metrics
- same **on-shell** d.o.f. as general relativity (for vacuum)
Newtonian limit

Question: sufficient d. o. f. in $G^{ab}$ for geometries with matter?

Answer: o.k. at least for Newtonian limit

$$ds^2 = -c^2 dt^2 \left(1 + \frac{2U}{c^2}\right) + d\vec{x}^2 \left(1 + O\left(\frac{1}{c^2}\right)\right)$$

where $\Delta(3) U(y) = 4\pi G \rho(y)$ and $\rho$ ...static mass density

can show: $\exists$ sufficient d.o.f. in $G^{ab}$ for arbitrary $\rho(y)$

moreover, vacuum e.o.m. imply

$$ds^2 = -c^2 dt^2 \left(1 + \frac{2U}{c^2}\right) + d\vec{x}^2 \left(1 - \frac{2U}{c^2}\right)$$

as in G.R.
Question: what about the Einstein-Hilbert action?

Answer:

- **tree level**: e.o.m. for gravity follow from $u(1)$ sector:
  \[ G^{ac} \partial_c \theta^{-1}_{ab}(y) = 0 \]
  implies $R_{ab} \sim 0$,
  at least for linearized gravity.

- **one-loop**: gauge or matter (scalar) fields couple to $G_{ab}$
  \[ \Rightarrow \text{(Sakharov) induced Einstein-Hilbert action:} \]
  \[
  S_{1-loop} \sim \int d^4 y \sqrt{G} \left( c_1 \Lambda_{UV}^4 + c_2 \Lambda_{UV}^2 R[G] + O(\log(\Lambda_{UV})) \right)
  \]
  however, modifications due to different role of scaling factor
  $\det(G)$ in density.
Relation with UV/IR mixing

Recall **UV/IR mixing:**

- Quantization of NC field theory $\rightarrow$ new divergences in IR, similar to UV divergences; non-renormalizable?
- for NC gauge theories: restricted to trace-$\mathfrak{u}(1)$ sector

  - *here:* trace-$\mathfrak{u}(1)$ sector understood as geometric d. o. f., $\mathfrak{su}(n)$ YM coupled to $G_{ab}$
  - $\Rightarrow$ expect new divergences in IR due to induced gravity (E-H action)
Recall **UV/IR mixing**:

- Quantization of NC field theory $\rightarrow$ new divergences in IR, similar to UV divergences; non-renormalizable?
- for NC gauge theories: restricted to trace-$u(1)$ sector
- here: trace-$u(1)$ sector understood as geometric d. o. f., $su(n)$ YM coupled to $G_{ab}$
  $\Rightarrow$ expect new divergences in IR due to induced gravity (E-H action)

natural “explanation” for UV/IR mixing

conjecture: resolved by interpretation as induced gravity
Relation with UV/IR mixing

Recall **UV/IR mixing:**

- Quantization of NC field theory $\rightarrow$ new divergences in IR, similar to UV divergences; non-renormalizable?
- for NC gauge theories: restricted to trace-$u(1)$ sector
- **here:** trace-$u(1)$ sector understood as geometric d. o. f., $su(n)$ YM coupled to $G_{ab}$
- $\Rightarrow$ expect new divergences in IR due to induced gravity (E-H action)

- natural “explanation” for UV/IR mixing
- **conjecture:** resolved by interpretation as induced gravity
Summary and outlook

- matrix-model $\text{Tr}[X^a, X^b][X^{a'}, X^{b'}] \eta_{aa'} \eta_{bb'}$ describes $SU(n)$ gauge theory coupled to gravity
- simple, intrinsically NC mechanism to generate gravity
  NC spaces $\leftrightarrow$ gravity
- solves problem how to define NC $\mathfrak{su}(n)$ gauge theory
Summary and outlook

- matrix-model \( \text{Tr}[X^a, X^b][X^{a'}, X^{b'}] \eta_{aa'} \eta_{bb'} \) describes \( SU(n) \) gauge theory coupled to gravity

- simple, intrinsically NC mechanism to generate gravity

- NC spaces \( \leftrightarrow \) gravity

- solves problem how to define NC \( su(n) \) gauge theory

- not same as G.R., but close to G.R. for small curvature
  - vacuum equation \( R_{ab} \sim 0 \) at least in linearized case
  - Newtonian limit, some post-newtonian corrections o.k.
Summary and outlook

- matrix-model $\text{Tr}[X^a, X^b][X^{a'}, X^{b'}] \eta_{aa'} \eta_{bb'}$ describes $SU(n)$ gauge theory coupled to gravity
- simple, intrinsically NC mechanism to generate gravity
  NC spaces $\leftrightarrow$ gravity
- solves problem how to define NC $su(n)$ gauge theory
- *not* same as G.R., but close to G.R. for small curvature
  - vacuum equation $R_{ab} \sim 0$ at least in linearized case
  - Newtonian limit, some post-newtonian corrections o.k.
- same mechanism in string-theoretical matrix models (IKKT)
Summary and outlook

- matrix-model $Tr[X^a, X^b][X^{a'}, X^{b'}] \eta_{aa'} \eta_{bb'}$ describes $SU(n)$ gauge theory coupled to gravity
- simple, intrinsically NC mechanism to generate gravity
  NC spaces $\leftrightarrow$ gravity
- solves problem how to define NC $su(n)$ gauge theory
- not same as G.R., but close to G.R. for small curvature
  - vacuum equation $R_{ab} \sim 0$ at least in linearized case
  - Newtonian limit, some post-newtonian corrections o.k.
- same mechanism in string-theoretical matrix models (IKKT)
  - suggest natural explanation (resolution?) for UV/IR mixing
  - promising for quantizing gravity
Summary and outlook

- matrix-model $\text{Tr}[X^a, X^b][X^{a'}, X^{b'}] \eta_{aa'} \eta_{bb'}$ describes $SU(n)$ gauge theory coupled to gravity
- simple, intrinsically NC mechanism to generate gravity
  - NC spaces $\leftrightarrow$ gravity
- solves problem how to define NC $\mathfrak{su}(n)$ gauge theory
- not same as G.R., but close to G.R. for small curvature
  - vacuum equation $R_{ab} \sim 0$ at least in linearized case
  - Newtonian limit, some post-newtonian corrections o.k.
- same mechanism in string-theoretical matrix models (IKKT)
- suggest natural explanation (resolution?) for UV/IR mixing
- promising for quantizing gravity