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Vacuum string field theory

L. Bonora, C. Maccaferri & PP, JHEP **0401** (2004) 038

L. Bonora, C. Maccaferri & PP, Phys. Rev. D **71** (2005) 026003

SFT – NP BI formulation of ST?

Standard formulations of string theory (RNS or GS) perturbative and onshell.

To directly address questions like vacua landscape we need formulation of string theory which is:

- offshell
- nonperturbative,
- background independent

Most straightforward - to apply basic logic of QFT.

In case of open bosonic ST

$\Longrightarrow$ Witten's cubic open SFT (OSFT):

- string field – functional on the space of string configurations

$$\Psi[x(\sigma); c(\sigma), b(\sigma)]$$

- action – Chern-Simons type (with Q BRST)

$$S[\Psi] = -\frac{1}{g^2}\int\left(\frac{1}{2}\Psi\star Q\Psi + \frac{1}{3}\Psi\star\Psi\star\Psi\right)$$

- $\Psi \star \Phi =$ $\int \Psi =$

- gauge symmetry

$$\delta\Psi = Q\Lambda + \Psi\star\Lambda - \Lambda\star\Psi$$

For practical purposes, better to represent Ψ as a gh#=1 state (or infinite family of fields):

$$|\Psi\rangle = \int d^{26}p\left[\phi(p)c_1|p\rangle + A_\mu(p)\alpha^\mu_{-1}c_1|p\rangle + \cdots\right]$$

Generalisation to algebra $\Psi \in \mathcal{A}$, star-product $* : \mathcal{A} \otimes \mathcal{A} \to \mathcal{A}$, inner product $\langle \cdot, \cdot \rangle$, and kinetic operator Q:

$$\begin{aligned}
Q^2 A &= 0 \\
Q(A * B) &= (QA) * B + (-1)^{\epsilon(A)} A * (QB) \\
\langle QA, B \rangle &= -(-1)^{\epsilon(A)} \langle A, (QB) \rangle \\
\langle A, B \rangle &= (-1)^{\epsilon(A)\epsilon(B)} \langle B, A \rangle \\
\langle A, B * C \rangle &= \langle A * B, C \rangle \\
A * (B * C) &= (A * B) * C
\end{aligned}$$

The Grassmanality ϵ and ghost number are additive with

$$\epsilon(*) = \mathrm{gh}(*) = 0 \,, \qquad \epsilon(Q) = \mathrm{gh}(Q) = 1 \,.$$

In OSFT string field has $\epsilon(\Psi) = \mathrm{gh}(\Psi) = 1$.

Above properties $\Longrightarrow$ gauge invariant action:

$$S[\Psi] = -\frac{1}{g^2} \left(\frac{1}{2} \langle \Psi, Q\Psi \rangle + \frac{1}{3} \langle \Psi, \Psi * \Psi \rangle \right)$$

Enables studying different backgrounds.

Oscillator approach – explicit manipulations with oscillator algebra (not fancy, but straightforward):

$$\langle A| = \mathrm{bpz}(|A\rangle)$$

$$|A * B\rangle = {}_1\langle A|\, {}_2\langle B|V_3\rangle_{123}$$

where 3-string vertex is a squeezed state:

$$|V_3\rangle = N \exp\left(-\frac{1}{2} a^\dagger V a^\dagger - c^\dagger \tilde{V} b^\dagger\right) |0_2\rangle |0_2\rangle |0_2\rangle$$

$V_{nm}^{(rs)}$, $\tilde{V}_{nm}^{(rs)}$ and N all c-number.

Feynman-Siegel gauge: $b_0|\Psi\rangle = 0$.

Remarkable results:

- perturbative expansion contains open and closed virtual strings
- gives a lot of evidence in support of Sen's conjectures:
  1. tachyon reflects instability of D25-brane – there exists locally stable minimum with $E = -T_{25}$
  2. D-branes are solitonic solutions of ST on D25-brane background
  3. no conventional open string excitations around "true" vacuum

Shows that SFT has potentials to be NP BI description of ST (open and closed).

However:

- results only numerical in level expansions
  - true vacuum has infinite expansion in modes
- (still) no solutions with higher energy then starting vacuum (e.g., for two D25 branes)
  - problems even for marginal case (singularities)

Problems of level expansion or something deeper?

Maybe more natural to consider the theory around true vacuum insted around one of unstable vacuua?

Vacuum SFT

Start from OSFT in which true vacuum is Ψ_0 and define $\tilde{\Psi}$ as

$$\Psi = \tilde{\Psi} + \Psi_0$$

OSFT action becomes (using EOM for Ψ_0):

$$S_o[\tilde{\Psi}] = -\frac{1}{g^2}\left(\frac{1}{2}\langle\tilde{\Psi}, \tilde{Q}\tilde{\Psi}\rangle + \frac{1}{3}\langle\tilde{\Psi}, \tilde{\Psi} * \tilde{\Psi}\rangle\right)$$

where the new kinetic term is given with

$$\tilde{Q}\tilde{\Psi} = Q\tilde{\Psi} + \Psi_0 * \tilde{\Psi} - \tilde{\Psi} * \Psi_0$$

Action around true vacuum has the same form, with the new kinetic operator (having all the properties needed for gauge invariance).

Remarks:

- one needs Ψ_0 (known only approx. in level expansion)
- even having Ψ_0 may not lead to the best form for the action (shifting background $\rightarrow$ nontrivial field redefinition)

Consider field redefinition $\hat{\Psi} = e^{O}\tilde{\Psi}$ where O is $\mathrm{gh}(O) = \epsilon(O) = 0$ operator satisfying

$$\begin{aligned} O(A * B) &= (OA) * B + A * (OB) \\ \langle OA, B\rangle &= -\langle A, OB\rangle \end{aligned}$$

Then the action for $\hat{\Psi}$ has the same form as for $\tilde{\Psi}$ but with

$$\hat{Q} = e^{O}\tilde{Q}e^{-O}$$

which can look very different from $\tilde{Q}$.

Option – to guess operator $\mathcal{Q}$ using Sen's conjectures. Then one requires:

- gauge invariance
- $\mathcal{Q}$ has vanishing cohomology (no open string modes around true vacuum)
- universality

RSZ conjecture: VSFT action with the same form as in OSFT:

$$S[\Psi] = -K\left(\frac{1}{2}\langle\Psi, \mathcal{Q}\Psi\rangle + \frac{1}{3}\langle\Psi, \Psi * \Psi\rangle\right)$$

where $\mathcal{Q}$ is purely ghost

$$\mathcal{Q} = c_0 + \sum_{n=0}^{\infty}(-1)^n\left(c_{2n} + c_{-2n}\right)$$

and is K some coupling constant (not necessarily equal to g).

Notice that $\mathcal{Q}$ can be written as

$$\mathcal{Q} = \frac{1}{2i}\left(c(i) - c(-i)\right)$$

i.e., a midpoint insertion (the most delicate point in 3-string product).

Properties of RSZ VSFT

EOM formally the same as in OSFT:

$$\mathcal{Q}\Psi + \Psi * \Psi = 0$$

If we consider factorized string fields

$$\Psi = \Psi_m \otimes \Psi_g$$

due to factorization property $* = *^m \otimes *^g$,
EOM separate into matter and ghost EOMs:

$$\begin{aligned} \Psi_m &= \Psi_m *^m \Psi_m \\ \mathcal{Q}\Psi_g &= -\Psi_g *^g \Psi_g \end{aligned} \tag{1}$$

Action for such solutions is:

$$S = -\frac{K}{6}\langle\Psi_m|\Psi_m\rangle\langle\Psi_g|\mathcal{Q}\Psi_g\rangle$$

Siegel gauge $b_0\Psi = 0$ gives:

$$\langle\Psi_g|\mathcal{Q}\Psi_g\rangle = \langle\Psi_g|c_0\Psi_g\rangle$$

Classical solutions – products of projectors.

D-branes in VSFT

Test for VSFT: Solutions having properties od D-branes?

Start from matter part of VSFT EOM

$$\Psi_m * \Psi_m = \Psi_m$$

and try to find translation invariant solution Ψ_{25} (possible D25 brane). As 3-string vertex $|V_3\rangle$ has a squeezed state form $\Longrightarrow$ try with squeezed state:

$$|\Psi(T)\rangle = \exp\left(-\frac{1}{2}a^\dagger \cdot CT \cdot a^\dagger\right)|0\rangle$$

T_{nm} is some matrix and $C_{nm} = (-1)^n \delta_{nm}$ is twist matrix ($n, m \geq 1$). This gives:

$$\Psi(T) * \Psi(T) \propto \Psi(T \star T)$$

$$T \star T = M_0 + (M_+, M_-)(1 - T\mathcal{M})^{-1}$$

where matrices $M_i = CV_i$ (V are Neumann coefficient matrices) mutually commute.

For $\Psi(T)$ to be a solution T should satisfy:

$$T = T \star T$$

Nontrivial because all matrices ∞-dim.

However, if we suppose that T commute with all M_i, then it is very easy to obtain the solution:

$$T = \frac{1}{2M_0}\left(1 + M_0 - \sqrt{(1-M_0)(1+3M_0)}\right)$$

For ghost part the game is the same because matrices $\tilde{M}$ satisfy the same algebraic relations as M.

Complete translation-invariant solution is

$$|\Psi_{25}\rangle = \mathcal{N}_c \exp\left(-\frac{1}{2}a^\dagger \cdot CT \cdot a^\dagger + c^\dagger \cdot C\tilde{T} \cdot b^\dagger\right) c_1|0\rangle$$

where the norm is:

$$\mathcal{N}_{25} = -[\det(1 - T\mathcal{M})]^{13}[\det(1 - \tilde{T}\tilde{\mathcal{M}})]^{-1}$$

and the energy density is:

$$\mathcal{E}_{25} = -\frac{S[\Psi_{25}]}{V_{26}} = \frac{K}{6}\mathcal{N}_{25}\frac{\det(1-\tilde{T}^2)}{\det(1-T^2)}$$

What about other Dp branes?

Note that the matter part of Ψ_{25} is a direct product of 26 terms (one for every target space dimension).

Consider solutions which are transl-inv only in $p+1$ dimensions (candidates for Dp-branes). Simmilar analysis gives for the matter part:

$$|\Psi_p\rangle = \mathcal{N}_p |\Psi(T)_{p+1}\rangle \otimes |\Psi(T')_{25-p}\rangle$$

where the first term is like matter part of Ψ_{25} (only $p+1$-dimensional), and the second term ($25-p$-dimensional) is algebraicaly the same with a difference that sums include mode $a_0^\dagger = (\hat{p} - i\hat{x})$.

Ratios of energy densities of Ψ_p give exactly standard ratios of tensions for Dp-branes.

States which has tensons of multiple-brane configurations – also easy to obtain.

Problems wih RSZ VSFT

- Singularities of sliver:
  - normalization factor and norm vanish
  - finite nonvanishing energy density requires $K \to \infty$ (infinite coupling constant - regularization needed)
  - EOM cannot be satisfied both when contracted with Fock and non-Fock states
- Spectrum of fluctuations:
  - masses OK, but polarisations not
  - relation between g_0 and T_{25} (obtained from 3-tachion amplitude) wrong (after regularisation)
- No coupling constant at all (classically)!!?
  - field redefinition $\hat{\Psi} = e^{O}\Psi$ with $O = \frac{1}{12}\ln\gamma(L_2 - L_{-2} - 4)$ just change coupling $K \to \hat{K} = K/\gamma$.

Can we avoid all this inside RSZ VSFT?

“Dressing” the sliver

To avoid first two problems – obviously new solutions needed. We'll show that this can settle also the third.

Try with squeezed state with modified (“dressed”) matrix T_ϵ:

$$T_\eta = T - \frac{\epsilon}{\kappa+1}\left(\vec{\xi}\otimes\vec{\xi} + C\vec{\xi}\otimes\vec{\xi}C\right)$$

where $\vec{\xi}$ and κ satisfy:

$$\vec{\xi}\frac{1}{1-T^2}\vec{\xi} = -1\,, \qquad \kappa = -\vec{\xi}\frac{T}{1-T^2}\vec{\xi}$$

Observe that $[T_\epsilon, M_i] \neq 0$ for $\epsilon \neq 0$.

These states are solutions of EOM if

$$\epsilon = 0, 1, \frac{\kappa+1}{\kappa} \qquad \text{and} \qquad \rho_1\vec{\xi} = 0$$

where $\epsilon = 0$ is HK sliver.

$*$-product for generic ϵ:

$$\Psi(T_{\epsilon_1}) * \Psi(T_{\epsilon_2}) = \Psi(T_{\epsilon_1 \star \epsilon_2})$$

where $\star$-product of ϵ's is:

$$\epsilon_1 \star \epsilon_2 = \frac{\epsilon_1 \epsilon_2}{(1-\epsilon_1)(1-\epsilon_2)\kappa + 1}$$

One can show that solutions with $\epsilon = 0$ and $\epsilon = \frac{\kappa+1}{\kappa}$ are "dual".

$\epsilon = 1$ solution more interesting, because the norm (which is proportional to energy density) due to dressing receives factor (when $\epsilon \approx 1$):

$$(\epsilon - 1)^{-D}$$

This factor, when properly scaled, can be used to make norm finite and nonvanishing, so also the energy density with finite coupling constant K.

Energy density is proportional to scaling factor, which then sort of dynamically generate scale in theory.

Additional results

- All states of open ST (with correct masses and polarizations) found in the spectrum of flucuations.
- The states with ratios of tensions equal to those of (multiple) Dp-branes can be constructed the same way as for $\epsilon = 0$.
- Freedom in scaling can be used to get:
  - correct relation between g_o and T_{25}
  - contracted EOM satisfied

These results suggest that there is “still life” in RSZ VSFT.

Open questions:

- is our regularization consistent?
- cohomological questions (some states should be gauged away)?

And some more general and heavier:

- closed string backgrounds (i.e., offshell closed string states)?
- extension to superstrings?