Tachyon Inflation in the RSII Framework

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Introduction

- We study (real) scalar field in cosmological context.

- General Lagrangian action:
$$S = \int d^4 x \sqrt{-g}\, \mathcal{L}(X(\partial\phi), \phi)$$

- Lagrangian (Lagrangian density) of the standard form:
$$\mathcal{L}(\phi, \partial\phi) = X(\partial\phi) - V(\phi)$$

- Non-standard Lagrangian:
$$\mathcal{L}_{tach}(T, X) = -V(T)\sqrt{1 + 2X(\partial T)} \qquad X = \frac{1}{2} g^{\mu\nu} \partial_\mu T \partial_\nu T$$

Introduction

- The action:

$$S = \int d^4 x \sqrt{-g}\, \mathcal{L}(X, T)$$

- In cosmology, scalar fields can be connected with a perfect fluid which describes (dominant) matter in the Universe.

- Components of the energy-momentum tensor:

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta S}{\delta g^{\mu\nu}}$$

$$T_{\mu\nu} = (P + \rho) u_\mu u_\nu - P g_{\mu\nu}$$

Introduction

$$T_{\mu\nu} = (P + \rho)u_\mu u_\nu - Pg_{\mu\nu}$$

- Pressure, matter density and velocity 4-vector, respectively:

$$P(X,T) \equiv \mathcal{L}(X,T)$$

$$\rho(X,T) \equiv 2X\frac{\partial \mathcal{L}}{\partial X} - \mathcal{L}(X,T)$$

$$u_\mu \equiv \frac{\partial_\mu T}{\sqrt{2X}}$$

Introduction

- Total action: term which describes gravity (Ricci scalar, Einstein-Hilbert action) plus term that describes cosmological fluid (scalar field Lagrangian):

$$S = \int d^4 x \sqrt{-g} \left(R + \mathcal{L}(X,T) \right)$$

- Einstein equations:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = T_{\mu\nu}$$

Tachyon field cosmology

- Tachyon lagrangian:

$$\mathcal{L}_{tach}(T,X) = -V(T)\sqrt{1 + g^{\mu\nu}\partial_\mu T \partial_\nu T}$$

- EoM:

$$\left(g^{\mu\nu} - \frac{\partial^\mu T \partial^\nu T}{1-(\partial T)^2} \right)\partial_\mu T \partial_\nu T = -\frac{1}{V(T)}\frac{dV}{dT}(1-(\partial T)^2)$$

Tachyon field cosmology

- Tachyon lagrangian:

$$\mathcal{L}_{tach}(T, X) = -V(T)\sqrt{1 + g^{\mu\nu}\partial_\mu T \partial_\nu T}$$

- Friedmann equations for spatially homogenous scalar field:

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3M_{Pl}^2}\frac{V}{(1 - \dot{T}^2)^{1/2}}$$

$$\frac{\ddot{T}}{1 - \dot{T}^2} + 3H\dot{T} + \frac{V'}{V} = 0$$

Tachyon field cosmology

- Rescaling:

$$x = \frac{T}{T_0} \qquad U(x) = \frac{1}{\lambda} V\left(\frac{T}{T_0}\right) = \frac{V(x)}{\lambda} \qquad \tau = \frac{t}{t_0}$$

- EoM:

$$\ddot{x} - 3HT_0 \dot{x}^3 - \frac{U'(x)}{U(x)} \dot{x}^2 + 3HT_o \dot{x} + \frac{U'(x)}{U(x)} = 0$$

- Hubble parameter rescaling:

$$\tilde{H} = T_0 \cdot H$$

Tachyon field cosmology

- Dimensionless equations:

$$\tilde{H}^2 = \frac{X_0^2}{3} \frac{U(x)}{\sqrt{1 - \dot{x}^2}}$$

$$\ddot{x} + X_0 \sqrt{3U(x)(1 - \dot{x}^2)^{3/2}} \, \dot{x} + \frac{(1 - \dot{x}^2)}{U(x)} \frac{dU(x)}{dx} = 0$$

$$X_0 = \frac{\lambda T_0^2}{M_{Pl}^2}, \qquad \lambda = \frac{M_s^4}{g_s (2\pi)^3}$$

The Inflation

- Slow-roll regime, slow-roll parameters:

$$\epsilon_{i+1} \equiv \frac{d \ln |\epsilon_i|}{dN}, \quad i \geq 0, \quad \epsilon_0 \equiv \frac{H_*}{H}$$

$$\epsilon_1 = -\frac{\dot{H}}{H^2}, \quad \epsilon_2 = \frac{1}{H}\frac{\ddot{H}}{\dot{H}} + 2\epsilon_1$$

$$\epsilon_1 = \frac{3}{2}\dot{x}^2, \quad \epsilon_2 = 2\frac{\ddot{x}}{\tilde{H}\dot{x}}$$

- Number of e-folds:

$$N(t) = \int_{t_i}^{t_e} H(t)dt$$

The Inflation

- Number of e-folds:

$$N(x) = X_0^2 \int_{x_i}^{x_e} \frac{U(x)^2}{|U'(x)|} dx, \qquad \text{where } \varepsilon_1(x_e) = 1$$

- The scalar spectral index:

$$n_s = 1 - 2\epsilon_1(x_i) - \epsilon_2(x_i)$$

- The tensor-to-scalar ratio:

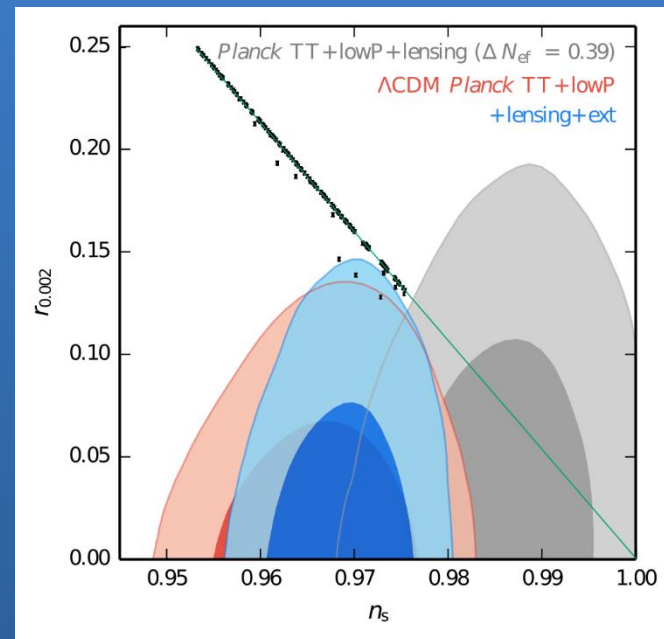$$r = 16\epsilon_1(x_i)$$

The Inflation

- Numerical results:

$$60 \leq N \leq 120, \ 1 \leq X_0 \leq 12$$

$$U(x) = \frac{1}{\tilde{x}^4}$$

Tachyon inflation in an AdS braneworld

- Randall–Sundrum models (1999) imagine that the real world is a higher-dimensional universe described by warped geometry. More concretely, our universe is a five-dimensional anti-de Sitter space and the elementary particles except for the graviton are localized on a (3+1)-dimensional brane(s).

- A simple cosmological model of this kind is based on the RSII model.

Tachyon inflation in an AdS braneworld

- Cosmology on the brane is obtained by allowing the brane to move in the bulk. Equivalently, the brane is kept fixed at z=0 while making the metric in the bulk time dependent.

- The fluctuation of the interbrane distance implies the existence of the radion.

- Radion – a massless scalar field that causes a distortion of the bulk geometry.

Tachyon inflation in an AdS braneworld

- The bulk spacetime of the extended RSII model in Fefferman-Graham coordinates is described by the metric

$$ds^2_{(5)} = G_{ab}dX^a dX^b = \frac{1}{k^2 z^2}\left[\left(1 + k^2 z^2 \eta(x)\right)g^{\mu\nu}dx^\mu dx^\nu - \frac{1}{\left(1 + k^2 z^2 \eta(x)\right)^2}dz^2\right]$$

- Inverse of the AdS curvature radius $- k$

- Radion field $- \eta(x)$

- Fifth coordinate $- z$

Tachyon inflation in an AdS braneworld

- Add dynamical 3-brane, i.e. tachyon field (in terms of induced metric).

- The action, after integrating out fifth coordinate z:

$$S = \int d^4 x \sqrt{-g} \left(-\frac{R}{16\pi G} + \frac{1}{2} g^{\mu\nu} \Phi_{,\mu} \Phi_{,\nu} \right) - \int d^4 x \sqrt{-g} \frac{\sigma}{k^4 \Theta^4} (1 + k^2 \Theta^2 \eta)^2 \sqrt{1 - \frac{g^{\mu\nu} \Theta_{,\mu} \Theta_{,\nu}}{(1 + k^2 \Theta^2 \eta)^3}}$$

- Radion field (canonical) $- \Phi$

- Tachyon field $- \Theta$

$$\eta = \sinh^2 \left(\sqrt{4/3\pi G} \Phi \right)$$

Tachyon inflation in an AdS braneworld

- In the absence of radion – tachyon condenzate:

$$S_{\text{br}}^{(0)} = -\int d^4x \sqrt{-g}\, \frac{\lambda}{\Theta^4}\, \sqrt{1 - g^{\mu\nu}\Theta_{,\mu}\Theta_{,\nu}}$$

- Going back, lagrangian we are playing with:

$$\mathcal{L} = \frac{1}{2} g^{\mu\nu}\Phi_{,\mu}\Phi_{,\nu} - \frac{\lambda\psi^2}{\Theta^4}\sqrt{1 - \frac{g^{\mu\nu}\Theta_{,\mu}\Theta_{,\nu}}{\psi^3}}$$

$$\psi = 1 + k^2\Theta^2\eta$$

Tachyon inflation in an AdS braneworld

- Hubble expansion rate H in standard cosmology (without brane):

$$H \equiv \frac{\dot{a}}{a} = \sqrt{\frac{8\pi G}{3} \mathcal{H}}$$

- Hubble expansion rate H in RS cosmology:

$$H \equiv \frac{\dot{a}}{a} = \sqrt{\frac{8\pi G}{3} \mathcal{H} \left(1 + \frac{2\pi G}{3k^2} \mathcal{H}\right)}$$

Tachyon inflation in an AdS braneworld

- Hamilton's equation:

$$\dot{\Phi} = \frac{\partial \mathcal{H}}{\partial \Pi_\Phi}$$

$$\dot{\Theta} = \frac{\partial \mathcal{H}}{\partial \Pi_\Theta}$$

$$\dot{\Pi}_\Phi + 3H\Pi_\Phi = -\frac{\partial \mathcal{H}}{\partial \Phi}$$

$$\dot{\Pi}_\Theta + 3H\Pi_\Theta = -\frac{\partial \mathcal{H}}{\partial \Theta}$$

Tachyon inflation in an AdS braneworld

- Dimensionless:

$$h = H \ / \ k, \, \phi = \Phi \ / \ (k\sqrt{\lambda}),$$

$$\pi_\phi = \Pi_\Phi \ / \ (k^2\sqrt{\lambda})), \, \theta = k\Theta, \quad \pi_\theta = \Pi_\Theta \ / \ (k^4\lambda)$$

$$\dot{\phi} = \pi_\phi$$

$$\dot{\theta} = \frac{\theta^4 \psi \pi_\theta}{\sqrt{1 + \theta^8 \pi_\theta^2 \ / \ \psi}}$$

$$\dot{\pi}_\phi = -3h\pi_\phi - \frac{\psi}{2\theta^2} \frac{4 + 3\theta^8 \pi_\theta^2 \ / \ \psi}{\sqrt{1 + \theta^8 \pi_\theta^2 \ / \ \psi}} \eta'$$

$$\dot{\pi}_\theta = -3h\pi_\theta + \frac{\psi}{\theta^5} \frac{4 - 3\theta^{10} \eta \pi_\theta^2 \ / \ \psi}{\sqrt{1 + \theta^8 \pi_\theta^2 \ / \ \psi}}$$

$$\kappa^2 = 8\pi\lambda G k^2$$

$$\dot{h} = -\frac{\kappa^2}{2} (\overline{\rho} + \overline{p}) \left(1 + \frac{\kappa^2}{6} \overline{\rho} \right)$$

$$\dot{N} = h$$

Tachyon inflation in an AdS braneworld

- Slow-roll parameters:

$$\epsilon_1 \simeq \frac{8\theta^2}{\kappa^2}\left(1+\frac{\kappa^2}{6\theta^4}\right)\left(1+\frac{\kappa^2}{12\theta^4}\right)^{-2}$$

$$\epsilon_2 \simeq \frac{8\theta^2}{\kappa^2}\left(1+\frac{\kappa^2}{12\theta^4}\right)^{-2}\left[1+\frac{\kappa^2}{4\theta^4}-\frac{\kappa^2}{6\theta^4}\left(1+\frac{\kappa^2}{6\theta^4}\right)^{-1}\right]$$
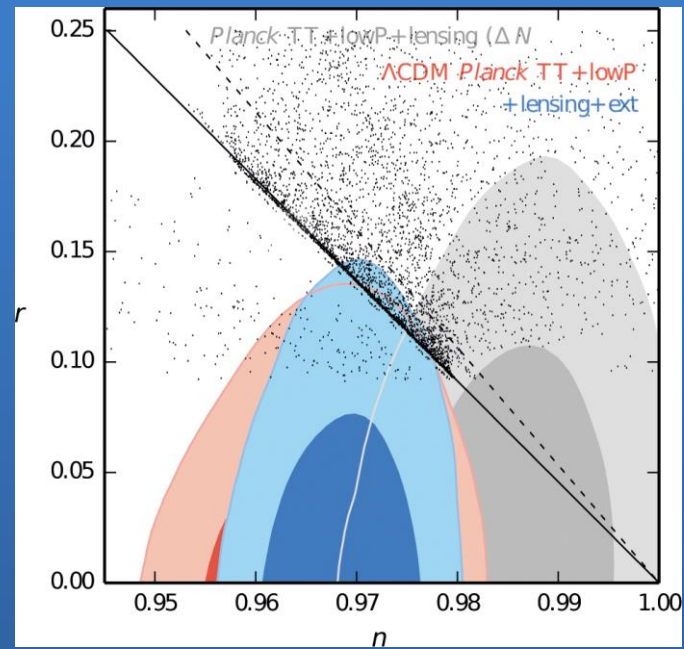
- Observational parameters:

$$r = 16\epsilon_1(\theta_i)\left[1-\frac{1}{6}\epsilon_1(\theta_i)+C\epsilon_2(\theta_i)\right]$$

$$n_s = 1-2\epsilon_1(\theta_i)-\epsilon_2(\theta_i)-\left[2\epsilon_1^2(\theta_i)+\left(2C+\frac{8}{3}\right)\epsilon_1(\theta_i)\epsilon_2(\theta_i)+C\epsilon_2(\theta_i)\epsilon_3(\theta_i)\right]$$

Tachyon inflation in an AdS braneworld

- Some numerical results:

$60 \leq N \leq 120, \ 1 \leq \kappa \leq 12 \text{ and } 0 \leq \phi_0 \leq 0.5$

Final remarks

- The n_s/r relation here is substantially different from the standard one and is closer to the best observational value.

- The model is based on the brane dynamics which results in a definite potential with one free parameter only.

- We have analized the simplest tachyon model. In principle, the same mechanism could lead to a more general tachyon potential if the AdS5 background metric is deformed by the presence of matter in the bulk.

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THANK YOU!
ХВАЛА!