Revisiting the decoupling effects in the running Cosmological Constant

Oleg Antipin (IRB, Zagreb)

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in collaboration with Blazenka Melic

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Outline

- RG running of the Cosmological Constant
- Case study: Standard Model
- Applications
- Conclusions

Cosmological constant

$$S_{HE} = -\frac{1}{16\pi G}\int d^4x\sqrt{-g}\,\{\,\mathcal{L}_{SM} + R + 2\Lambda + \xi\; H^\dagger H\, R\}$$

Vacuum energy density

$$\rho_\Lambda^{vac}(\mu) \equiv \frac{\Lambda(\mu)}{8\pi G(\mu)}$$

Induced vacuum energy density

$$V_0 = -\frac{1}{2}m^2\phi^2 + \frac{1}{8}\lambda\phi^4$$

$$\rho_{ind}(\mu) \equiv V_0(\langle\phi\rangle) = -\frac{m^4(\mu)}{2\lambda(\mu)}$$

Experimentally measured value at $\mu_c = \mathcal{O}(10^{-3})eV$

$$\rho_{phys} = \rho_\Lambda^{\text{vac}}(\mu_c) + \rho_{ind}(\mu_c) + \ldots = 10^{-47}\ \text{GeV}^4$$

Cosmological constant

$$S_{HE} = -\frac{1}{16\pi G}\int d^4x\sqrt{-g}\ \{\ \mathcal{L}_{SM} + R + 2\Lambda + \xi\ H^\dagger H\, R\}$$

Vacuum energy density

$$\rho_\Lambda^{vac}(\mu) \equiv \frac{\Lambda(\mu)}{8\pi G(\mu)}$$

Induced vacuum energy density

$$V_0 = -\frac{1}{2}m^2\phi^2 + \frac{1}{8}\lambda\phi^4$$

$$\rho_{ind}(\mu) \equiv V_0(\langle\phi\rangle) = -\frac{m^4(\mu)}{2\lambda(\mu)}$$

!

This represents “**mathematical**” scaling introduced during renormalization
which sometimes can be used to trace the scaling dependence with a physical quantities

The **physical** running (if there is at all) is not with μ but with some physical external momentum q
or some dynamical property of the background metric, e.g. Hubble scale

The physical scaling of the CC is unknown and we concentrate on its μ-dependence

Cosmological constant problem

$$\rho_{phys} = \rho_{\Lambda}^{\text{vac}}(\mu_c) + \rho_{ind}(\mu_c) + \ldots = 10^{-47}\ \text{GeV}^4$$

But $\rho_{ind}(\mu = 246\ GeV) \sim\ 10^8\ GeV^4$ $\Rightarrow$ fine-tuning

However

CC experimentally measured value at $\mu_c = \mathcal{O}(10^{-3})eV$

$\Rightarrow$ need to RG run to this low scale taking into account decoupling of the heavy particles

Decoupling effects

On dimensional grounds and for $m_{light} \ll \mu \ll m$

Appelquist-Carazzone theorem

$$\beta_{\rho_\Lambda^{vac}}\left(m_{light}, \frac{\mu}{m}\right) = a\, m_{light}^4 + b\left(\frac{\mu}{m}\right)^2 m^4 + c\left(\frac{\mu}{m}\right)^4 m^4 + \ldots$$

Decoupling does not hold in the mass-<u>independent</u> schemes like MS-bar

to analyze decoupling we need to work in a mass-<u>dependent</u> schemes

Cosmological Perspective

Sola,Shapiro,….

Running vacuum models (subclass of dynamical vacuum models)

$$\mu = H$$

The **physical** running of the CC (not derived from the first principles)

$$\rho_\Lambda(H) = \rho_\Lambda^0 + const \times M_P^2(H^2 - H_0^2) + \mathcal{O}(H^4)$$

and test it using current data…

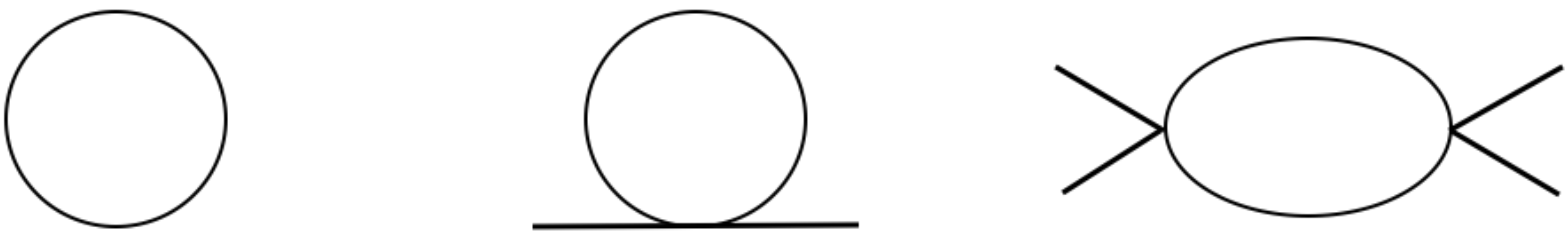
Instead, our goal is to learn how CC problem can provide constraints on the particle physics models…

Case study: Real scalar field

So far we had only classical parameters... now quantum

Toy model : $$L = \frac{1}{2}m^2\phi^2 + \frac{1}{8}\lambda\phi^4$$

One-loop effective potential:



(a) (b) (c)

$$V_{\text{eff}} = V_{vac}(m^2, \lambda, \rho_\Lambda^{\text{vac}}, \mu) + V_{scal}(\phi, m^2, \lambda, \mu)$$

Case study: Real scalar field

$\overline{MS}$ - scheme

$$V_{vac}(m^2, \lambda, \rho_\Lambda^{vac}, \mu) = \rho_\Lambda^{vac}(\mu) + \frac{m^4}{64\pi^2} \left(\ln \frac{m^2}{\mu^2} - \frac{3}{2}\right)$$

$$V_{scal}(\phi, m^2, \lambda, \mu) = \rho_{ind}(\mu) + \frac{1}{64\pi^2}(\frac{3}{2}\lambda\phi^2 - m^2)^2 \left[\ln \frac{3/2\lambda\phi^2 - m^2}{\mu^2} - \frac{3}{2}\right]$$

Various pieces satisfy RG equations: $\frac{dV_i}{d\mu} = 0$ $\Rightarrow$

$$\left(\mu\frac{\partial}{\partial\mu} + \beta_\lambda\frac{\partial}{\partial\lambda} + \gamma_m m^2\frac{\partial}{\partial m^2} + \beta_{\rho_\Lambda^{\rm vac}}\frac{\partial}{\partial\rho_\Lambda^{\rm vac}}\right)V_{vac}(m^2, \lambda, \rho_\Lambda^{\rm vac}, \mu) = 0$$

$$\left(\mu\frac{\partial}{\partial\mu} + \beta_\lambda\frac{\partial}{\partial\lambda} + \gamma_m m^2\frac{\partial}{\partial m^2} - \gamma_\phi\phi\frac{\partial}{\partial\phi}\right)V_{scal}(\phi, m^2, \lambda, \mu) = 0$$

Vacuum and induced CC running

$$\left(\mu\frac{\partial}{\partial\mu}+\beta_\lambda\frac{\partial}{\partial\lambda}+\gamma_m m^2\frac{\partial}{\partial m^2}+\beta_{\rho_\Lambda^{\rm vac}}\frac{\partial}{\partial\rho_\Lambda^{\rm vac}}\right)V_{vac}(m^2,\lambda,\rho_\Lambda^{\rm vac},\mu)\;=\;0$$

$$V_{vac}(m^2,\lambda,\rho_\Lambda^{vac},\mu)\;=\rho_\Lambda^{vac}(\mu)+\frac{m^4}{64\pi^2}\,\left(\ln\frac{m^2}{\mu^2}-\frac{3}{2}\right) \;\Rightarrow\; \boxed{\mu\frac{\partial\rho_\Lambda^{\rm vac}}{\partial\mu}\equiv\beta_{\rho_\Lambda^{\rm vac}}=\frac{m^4}{32\pi^2}}$$

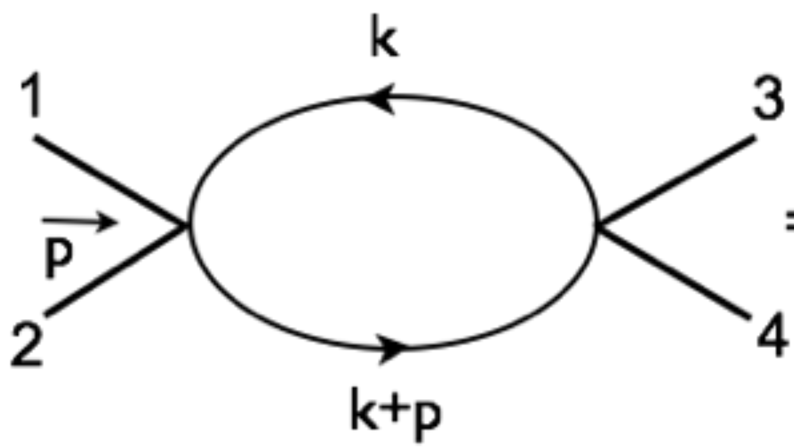
$$\left(\mu\frac{\partial}{\partial\mu}+\beta_\lambda\frac{\partial}{\partial\lambda}+\gamma_m m^2\frac{\partial}{\partial m^2}\right)V_{scal}(\langle\phi\rangle,m^2,\lambda,\mu)=0$$

$$\boxed{\mu\frac{\partial\rho_{ind}(\mu)}{\partial\mu}=\mu\frac{\partial}{\partial\mu}\left(-\frac{m^4(\mu)}{2\lambda(\mu)}\right)=\rho_{ind}(\mu)\,\left(2\frac{\beta_{m^2}}{m^2}-\frac{\beta_\lambda}{\lambda}\right)}$$

$\overline{MS}$ $\Rightarrow$ Valid only in the UV regime ! $\mu\gg m$

Case study: Real scalar field in $\overline{MS}$

Need a mass-dependent scheme, but first :



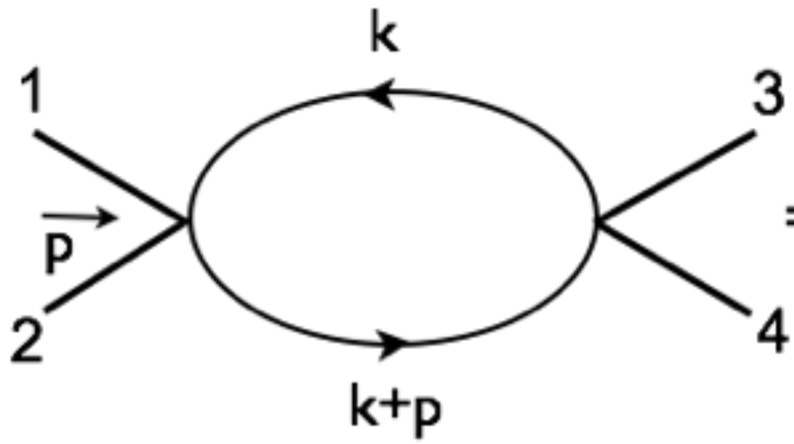
$$= \frac{(-3i\lambda)^2}{2} \int \frac{d^4k}{(2\pi)^4} \frac{i}{k^2 - m^2} \frac{i}{(k+p)^2 - m^2} \equiv (-3i\lambda)^2 i A(p^2)$$

$$A(p^2) = -\frac{1}{32\pi^2} \int_0^1 dx \left(\frac{2}{\epsilon} - \gamma_E + \log(4\pi) - \log[m^2 - x(1-x)p^2] \right)$$

$$A(p^2)_{\overline{MS}} = A(p^2) + c.t. = \frac{1}{32\pi^2} \int_0^1 dx \ \log \left(\frac{m^2 - x(1-x)p^2}{\mu^2_{\overline{MS}}} \right)$$

$$\left(\mu_{\overline{MS}} \frac{\partial}{\partial \mu_{\overline{MS}}} + \beta_\lambda^{\overline{MS}} \frac{\partial}{\partial \lambda} \right) G^{(4)}(m^2, \lambda, \mu_{\overline{MS}}) = 0 \quad \Rightarrow \quad \boxed{\beta_\lambda^{\overline{MS}} = \frac{9\lambda^2}{16\pi^2}}$$

Case study: Real scalar field in MOM



$$= \frac{(-3i\lambda)^2}{2} \int \frac{d^4k}{(2\pi)^4} \frac{i}{k^2 - m^2} \frac{i}{(k+p)^2 - m^2} \equiv (-3i\lambda)^2 i A(p^2)$$

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$$A(p^2)_{\mathrm{MOM}} = A(p^2) + c.t. = \frac{1}{32\pi^2} \int_0^1 dx \; \log \left(\frac{m^2 - x(1-x)p^2}{m^2 + x(1-x)\mu^2} \right)$$

$$\boxed{\beta_\lambda^{\mathrm{MOM}} = \frac{9\lambda^2}{16\pi^2} \int_0^1 \frac{x(1-x)\mu^2}{m^2 + x(1-x)\mu^2} dx}$$

Vacuum and induced CC running in MOM

$$\boxed{\mu \frac{\partial \rho_\Lambda^{\rm vac}}{\partial \mu}_{|\rm MOM} = \frac{m^4}{32\pi^2} \int_0^1 \frac{x(1-x)\mu^2 dx}{m^2 + x(1-x)\mu^2}}$$

$$\boxed{\mu \frac{\partial \rho_{ind}(\mu)}{\partial \mu} = \rho_{ind}(\mu) \left(2\frac{\beta_{m^2}}{m^2} - \frac{\beta_\lambda}{\lambda}\right) \int_0^1 \frac{x(1-x)\mu^2 dx}{m^2 + x(1-x)\mu^2}}$$

Valid in the UV and IR regime !

Case study: Standard Model in $\overline{MS}$

$$V[\mu, \lambda_i, m^2, \rho_\Lambda^{\text{vac}}; \phi] \equiv V_0 + V_1 + \cdots$$

Φ	i	n_i	κ_i	κ_i^m	c_i
$W^\pm$	1	6	$g^2/4$	0	5/6
Z^0	2	3	$(g^2+g'^2)/4$	0	5/6
t	3	−12	$y_t^2/2$	0	3/2
ϕ	4	1	$3\lambda/2$	1	3/2
χ_i	5	3	$\lambda/2$	1	3/2

$$V_0 = -\frac{1}{2}m^2\phi^2 + \frac{1}{8}\lambda\phi^4$$

$$V_1 = \sum_{i=1}^{5} \frac{n_i}{64\pi^2} M_i^4(\phi)\left[\log\frac{M_i^2(\phi)}{\mu^2} - c_i\right] + \rho_\Lambda^{\text{vac}} \qquad M_i^2(\phi) = \kappa_i\phi^2 - m^2\kappa_i^m$$

$$V(\langle\phi\rangle) = \rho_{ind} + \rho_\Lambda^{\text{vac}} + \sum_{i=1}^{5} \frac{n_i}{64\pi^2} M_i^4(\langle\phi\rangle)\left[\log\frac{M_i^2(\langle\phi\rangle)}{\mu^2} - c_i\right]$$

$$V_0(\langle\phi\rangle) = -\frac{m^4(\mu)}{2\lambda(\mu)} \qquad \langle\phi\rangle^2 = \frac{2m^2}{\lambda}$$

Case study: Standard Model in $\overline{MS}$

$$V(\langle\phi\rangle) = \rho_{ind} + \rho_\Lambda^{\rm vac} + \sum_{i=1}^{5} \frac{n_i}{64\pi^2} M_i^4(\langle\phi\rangle) \left[\log \frac{M_i^2(\langle\phi\rangle)}{\mu^2} - c_i\right]$$

$$\frac{dV(\langle\phi\rangle)}{d\mu} = 0 \quad \Rightarrow \quad \boxed{\mu \frac{\partial(\rho_{ind} + \rho_\Lambda^{\rm vac})}{\partial\mu} = \sum_i \frac{n_i}{32\pi^2} M_i^4(\langle\phi\rangle)}$$

$\overline{MS}$ Valid in the UV regime only !

Case study: Standard Model in MOM

$$\mu \frac{\partial \rho_\Lambda^{\rm vac}}{\partial \mu}_{|{\rm MOM}} = m^4 \sum_i \frac{n_i (\kappa_i^m)^2}{32\pi^2} \int_0^1 \frac{x(1-x)\mu^2 dx}{(M^2_{phys})_i + x(1-x)\mu^2}$$

$$\mu \frac{\partial \rho_{ind}(\mu)}{\partial \mu}_{|{\rm MOM}} = -\frac{\lambda \langle \phi \rangle^4}{32\pi^2} \sum_i n_i \kappa_i \left[\kappa_i^m - \frac{\kappa_i}{\lambda}\right] \int_0^1 \frac{x(1-x)\mu^2 dx}{(M^2_{phys})_i + x(1-x)\mu^2}$$

$$\boxed{\mu \frac{\partial (\rho_{ind} + \rho_\Lambda^{\rm vac})}{\partial \mu}_{|{\rm MOM}} = \sum_i \frac{n_i}{32\pi^2} (M^4_{phys})_i \int_0^1 \frac{x(1-x)\mu^2 dx}{(M^2_{phys})_i + x(1-x)\mu^2}}$$

Valid in the UV and IR regimes !

Applications : SM

$$\boxed{\mu\frac{\partial(\rho_{ind}+\rho_\Lambda^{\rm vac})}{\partial\mu}_{|\rm MOM} = \sum_i \frac{n_i}{32\pi^2}(M^4_{phys})_i \int_0^1 \frac{x(1-x)\mu^2 dx}{(M^2_{phys})_i + x(1-x)\mu^2}}$$

$$\int_0^1 \frac{x(1-x)\mu^2 dx}{(M^2_{phys})_i + x(1-x)\mu^2} = \begin{cases} 1 & \mu \gg (M^2_{phys})_i \\ \frac{\mu^2}{6(M^2_{phys})_i} - \frac{\mu^4}{30(M^4_{phys})_i} & \mu \ll (M^2_{phys})_i \end{cases}$$

$\Rightarrow$

$$\mu\frac{\partial(\rho_\Lambda^{\rm vac}+\rho_{ind})}{\partial\mu} \approx \sum_j \frac{n_j (m^4_{light})_j}{32\pi^2} + \frac{1}{32\pi^2}\sum_i n_i (M^4_{phys})_i \int_0^1 \frac{x(1-x)\mu^2 dx}{(M^2_{phys})_i + x(1-x)\mu^2} =$$

$$\mu\frac{\partial(\rho_\Lambda^{\rm vac}+\rho_{ind})}{\partial\mu} \approx \sum_j \frac{n_j (m^4_{light})_j}{32\pi^2} - \frac{\mu^2}{12(4\pi)^2}\boxed{\left[12M_t^2 - 6M_W^2 - 3M_Z^2 - M_H^2\right]} + \frac{\mu^4}{30(4\pi)^2} + \mathcal{O}\left(\frac{\mu^6}{(M^2_{phys})_i}\right)$$

Goldstone terms canceled in the sum !

Responsible for CC tuning

Applications: Massless theories

$$\boxed{\mu \frac{\partial(\rho_{ind} + \rho_\Lambda^{\text{vac}})}{\partial \mu}_{|\text{MOM}} = \sum_i \frac{n_i}{32\pi^2} (M^4_{phys})_i \int_0^1 \frac{x(1-x)\mu^2 dx}{(M^2_{phys})_i + x(1-x)\mu^2}}$$

$$\boxed{\mu \frac{\partial \rho_\Lambda^{\text{vac}}}{\partial \mu} \equiv \beta_{\rho_\Lambda^{\text{vac}}} = \frac{m^4}{32\pi^2}} \quad \Rightarrow \quad \rho_\Lambda^{\text{vac}} \quad \text{is} \quad \text{constant} \quad \Rightarrow$$

$$\mu \frac{\partial \rho_{ind}}{\partial \mu} = \frac{\langle \phi \rangle^4}{32\pi^2} \sum_i n_i \kappa_i^2 \int_0^1 \frac{x(1-x)\mu^2 dx}{(M^2_{phys})_i + x(1-x)\mu^2}$$

$$= \frac{\mu^2 \langle \phi \rangle^2}{12(4\pi)^2} \sum_i \kappa_i n_i + \frac{\mu^4}{20(4\pi)^2} = \frac{\mu^2 \langle \phi \rangle^2}{12(4\pi)^2} \boxed{\frac{\partial}{\partial \phi^2} \sum_i n_i M_i^2(\phi)} + \frac{\mu^4}{20(4\pi)^2}$$

$$M_i^2(\phi) = \kappa_i \phi^2$$

generalized
Veltman condition

In massless models fine-tuning of Higgs mass and CC are linked

Massless theories : SM + real scalar

$$V_0 = V_0^{SM} + \lambda_{HS} H^\dagger H S^2 + \frac{\lambda_S}{4} S^4$$

$$12M_t^2 - 6M_W^2 - 3M_Z^2 - M_S^2 = 0 \implies \lambda_{HS}(\mu) = 6y_t^2(\mu) - \frac{9}{4}g^2(\mu) - \frac{3}{4}g'^2(\mu) \approx 4.8$$

$$M_S^2(\phi) = \lambda_{HS}\phi^2 \approx (550\ GeV)^2$$

One-loop induced (Coleman-Weinberg) Higgs mass:

$$M_H^2 = \frac{3}{8\pi^2}\left[\frac{1}{16}(3g^4 + 2g^2g'^2 + g'^4) - y_t^4 + \frac{1}{3}\lambda_{HS}^2\right]v_{EW}^2 \approx (125\ GeV)^2$$

prediction!

Example of how running of the CC
may be connected to BSM physics

Applications: SM in constant curvature space

$$S_{HE} = -\frac{1}{16\pi G}\int d^4x\sqrt{-g}\,\{\,\mathcal{L}_{SM} + R + 2\Lambda + \xi\; H^\dagger H\, R\,\}$$

$$\boxed{\mu\frac{\partial(\rho_{ind} + \rho_\Lambda^{\rm vac} + \kappa R)}{\partial\mu} = \sum_i \frac{n_i}{32\pi^2}\mathcal{M}_i^4(\langle\phi\rangle)}$$

$$\mathcal{M}_i^2(\langle\phi\rangle) = \kappa_i\langle\phi\rangle^2 - m^2\kappa_i^m + \left(\kappa_i^R - \frac{1}{6}\right)$$

$$\langle\phi\rangle^2 = \frac{2(m^2 - \xi R)}{\lambda}$$

Φ	i	n_i	κ_i	κ_i^m	κ_i^R
$W^\pm$(ghost)	1	−2	$g^2/4$	0	1/2
$W^\pm$	2	8	$g^2/4$	0	1/2
Z^0(ghost)	3	−1	$(g^2+g'^2)/4$	0	1/2
Z^0	4	4	$(g^2+g'^2)/4$	0	1/2
t	5	−12	$y_t^2/2$	0	1/4
ϕ	6	1	$3\lambda/2$	1	1/2
χ_i	7	3	$\lambda/2$	1	1/2

$$\mu\frac{\partial(\rho_{ind} + \rho_\Lambda^{\rm vac} + \kappa R)}{\partial\mu} \approx \sum_j \frac{n_j(\mathcal{M}^4_{light})_j}{32\pi^2} - \frac{\mu^2}{12(4\pi)^2}\left[12m_t^2 - 6m_W^2 - 3m_Z^2 - m_H^2 - \frac{7}{3}R\right] + \frac{\mu^4}{30(4\pi)^2}$$

Conclusions

- We considered the RG running of the Cosmological Constant
- We showed that only RG running of the total (induced + vacuum) CC exhibits behaviour consistent with the decoupling theorem
- We provided a simple extension of the SM with addition of one massless real scalar where condition of absence of leading RG effect allowed us to predict the radiative Higgs mass correctly
- We also provided generalization to the constant curvature space

On a safe road to quantum gravity with matter

Hvar Island, Croatia, September 11-14 2018

http://hvar2018.irb.hr

The workshop aims at discussing the subject of asymptotic safety from both perturbative and non-perturbative perspectives, including modern developments in gravity and ordinary gauge theories.